NONLINEAR CONTRACTIONS AND FIXED POINTS IN COMPLETE DISLOCATED AND b-DISLOCATED METRIC SPACES

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Abstract. In this paper, we continue the study of complete dislocated and b-dislocated metric spaces and established some common fixed point theorems for one and two mappings. Our results generalizes and extend some existing results in the literature in a class effectively larger such as b-dislocated metric spaces, where the self distance for a point may not be equal to zero.

1. INTRODUCTION

The concept of $b$-metric space was introduced by Bakhtin [4] and extensively used by Czerwik in [10]. After that, several interesting results about the existence of a fixed point for single-valued and multi-valued operators in $b$-metric spaces have been obtained. Recently there are a number of generalizations of metric space. Some of them are the notions of dislocated metric spaces and $b$-dislocated metric spaces where the distance of a point in the self may not be zero. These spaces was introduced and studied by Hitzler and Seda [5], Nawab Hussain et.al [7]. Also in [7] are presented some topological aspects and properties of $b$-dislocated metrics. Subsequently, several authors have studied the problem of existence and uniqueness of a fixed point for single-valued and set-valued mappings and different types of contractions in these spaces.

The purpose of this paper is to unify and generalize some recent results in the setting of dislocated and $b$-dislocated metric spaces using a class of continuous functions $G_4$.

2. PRELIMINARIES

Definition 2.1 [6]. Let $X$ be a nonempty set and a mapping $d_l: X \times X \rightarrow [0, \infty)$ is called a dislocated metric (or simply $d_l$-metric) if the following conditions hold for any $x, y, z \in X$:

i. If $d_l(x, y) = 0$, then $x = y$

ii. $d_l(x, y) = d_l(y, x)$

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iii. \( d_I(x,y) \leq d_I(x,z) + d_I(z,y) \)

The pair \((X, d_I)\) is called a dislocated metric space (or \(d\)-metric space for short). Note that when \(x = y\), \(d_I(x,y)\) may not be 0.

**Example 2.2.** If \(X = R\), then \(d(x,y) = |x| + |y|\) defines a dislocated metric on \(X\).

**Definition 2.3** [6]. A sequence \((x_n)\) in \(d_I\)-metric space \((X,d_I)\) is called:
1. a Cauchy sequence if, for given \(\varepsilon > 0\), there exists \(n_0 \in N\) such that for all \(m,n \geq n_0\), we have \(d_I(x_m,x_n) < \varepsilon\) or \(\lim_{n,m \to \infty} d_I(x_n,x_m) = 0\),
2. convergent with respect to \(d_I\) if there exists \(x \in X\) such that \(d_I(x_n,x) \to 0\) as \(n \to \infty\).

In this case, \(x\) is called the limit of \((x_n)\) and we write \(x_n \to x\).

A \(d_I\)-metric space \(X\) is called complete if every Cauchy sequence in \(X\) converges to a point in \(X\).

**Definition 2.4** [8]. Let \(X\) be a nonempty set and a mapping \(b_d : X \times X \to [0, \infty)\) is called a \(b\)-dislocated metric (or simply \(b_d\)-dislocated metric) if the following conditions hold for any \(x,y,z \in X\) and \(s \geq 1\):

a. If \(b_d(x,y) = 0\), then \(x = y\),

b. \(b_d(x,y) = b_d(y,x)\),

c. \(b_d(x,y) \leq s[b_d(x,z) + b_d(z,y)]\).

The pair \((X, b_d)\) is called a \(b\)-dislocated metric space. And the class of \(b\)-dislocated metric space is larger than that of dislocated metric spaces, since a \(b\)-dislocated metric is a dislocated metric when \(s = 1\).

In [8] was showed that each \(b_d\)-metric on \(X\) generates a topology \(\tau_{b_d}\) whose base is the family of open \(b_d\)-balls \(B_{b_d}(x, \varepsilon) = \{y \in X : b_d(x,y) < \varepsilon\}\).

Also in [8] are presented some topological properties of \(b_d\)-metric spaces.

**Definition 2.5.** Let \((X, b_d)\) be a \(b_d\)-metric space, and \((x_n)\) be a sequence of points in \(X\). A point \(x \in X\) is said to be the limit of the sequence \((x_n)\) if \(\lim_{n \to \infty} b_d(x_n,x) = 0\) and we say that the sequence \((x_n)\) is \(b_d\)-convergent to \(x\) and denote it by \(x_n \to x\) as \(n \to \infty\).

The limit of a \(b_d\)-convergent sequence in a \(b_d\)-metric space is unique [8, Proposition 1.27].
**Definition 2.6.** A sequence \((x_n)\) in a \(b_d\)-metric space \((X, b_d)\) is called a \(b_d\)-Cauchy sequence iff, given \(\varepsilon > 0\), there exists \(n_0 \in N\) such that for all \(n, m > n_0\), we have \(b_d(x_n, x_m) < \varepsilon\) or \(\lim_{n, m \to \infty} b_d(x_n, x_m) = 0\). Every \(b_d\)-convergent sequence in a \(b_d\)-metric space is a \(b_d\)-Cauchy sequence.

**Remark 2.7.** The sequence \((x_n)\) in a \(b_d\)-metric space \((X, b_d)\) is called a \(b_d\)-Cauchy sequence iff \(\lim_{n \to \infty} b_d(x_n, x_{n+p}) = 0\) for all \(p \in N^*\).

**Definition 2.8.** A \(b_d\)-metric space \((X, b_d)\) is called complete if every \(b_d\)-Cauchy sequence in \(X\) is \(b_d\)-convergent.

In general a \(b_d\)-metric is not continuous, as in Example 1.31 in [8] showed.

**Example 2.9.** Let \(X = R^+ \cup \{0\}\) and any constant \(\alpha > 0\). Define the function \(d_l : X \times X \to [0, \infty)\) by \(d_l(x, y) = \alpha(x + y)\). Then, the pair \((X, d_l)\) is a dislocated metric space.

**Lemma 2.10.** Let \((X, b_d)\) be a \(b\)-dislocated metric space with parameter \(s \geq 1\). Suppose that \((x_n)\) and \((y_n)\) are \(b_d\)-convergent to \(x, y \in X\), respectively. Then we have
\[
\frac{1}{s} b_d(x, y) \leq \lim \inf_{n \to \infty} b_d(x_n, y_n) \leq \lim \sup_{n \to \infty} b_d(x_n, y_n) \leq s^2 b_d(x, y)
\]
In particular, if \(b_d(x, y) = 0\), then we have \(\lim_{n \to \infty} b_d(x_n, y_n) = 0 = b_d(x, y)\). Moreover, for each \(z \in X\), we have
\[
\frac{1}{s} b_d(x, z) \leq \lim \inf_{n \to \infty} b_d(x_n, z) \leq \lim \sup_{n \to \infty} b_d(x_n, z) \leq s b_d(x, z)
\]
In particular, if \(b_d(x, z) = 0\), then we have \(\lim_{n \to \infty} b_d(x_n, z) = 0 = b_d(x, z)\).

Some examples in the literature show that in general a \(b\)-dislocated metric is not continuous.

**Example 2.11.** If \(X = R^+ \cup \{0\}\), then \(b_d(x, y) = (x + y)^2\) defines a \(b\)-dislocated metric on \(X\) with parameter \(s = 2\).

### 3. MAIN RESULT

We consider the set \(G_4\) of all continuous functions \(g : [0, \infty)^4 \to [0, \infty)\) with the following properties:
a) \( g \) is non-decreasing in respect to each variable

b) \( g(t, t, t, t) \leq t, t \in [0, \infty) \)

Some examples of these functions are as follows:
\[
\begin{align*}
g_1 & : g(t_1, t_2, t_3, t_4) = \max\{t_1, t_2, t_3, t_4\} \\
g_2 & : g(t_1, t_2, t_3, t_4) = \max\{t_1 + t_2 + t_3 + t_4\} \\
g_3 & : g(t_1, t_2, t_3, t_4) = \left[\max\{t_1 t_2 t_3 t_4\}\right]^\frac{1}{2} \\
g_4 & : g(t_1, t_2, t_3, t_4) = \left[\max\{t_1^p, t_2^p, t_3^p, t_4^p\}\right]^\frac{1}{p}, p > 0.
\end{align*}
\]

**Theorem 3.1.** Let \((X, d)\) be a complete \(b\)-dislocated metric space with parameter \(s \geq 1\) and \(T, S : X \to X\) two mappings satisfying the following contractive condition
\[
sd(Sx, Ty) \leq c \left[ d(x, y), d(x, Sx), d(y, Ty), \frac{d(x, Sx)d(y, Ty)}{1 + d(x, y)} \right] \tag{1}
\]
for all \(x, y \in X\) where \(g \in G_4\) and \(0 \leq c < 1\). Then \(T\) and \(S\) have a unique common fixed point and if \(u\) is a common fixed point of \(S\) and \(T\), then \(d(u, u) = 0\).

**Proof.** Let \(x_0\) be an arbitrary point in \(X\). Define the sequence \((x_n)\) as follows:
\[
x_1 = S(x_0), x_2 = T(x_1), \ldots, x_{2n} = T(x_{2n-1}), x_{2n+1} = S(x_{2n}), \ldots
\]
if we assume that for some \(n \in N\), \(x_{2n+1} = x_{2n}\) then \(x_{2n} = x_{2n+1} = Sx_{2n}\) and also using the contractive condition of theorem we will have that \(x_{2n+1} = x_{2n}\) is a fixed point of \(T\).

Then we assume that for \(n \in N\), \(x_{2n+1} \neq x_{2n}\). By condition (1) we have:
\[
sd(x_{2n+1}, x_{2n+2}) = sd(Sx_{2n}, Tx_{2n+1})
\]
\[
\leq cg\left[d(x_{2n}, x_{2n+1}), d(x_{2n}, Sx_{2n}), d(x_{2n+1}, Tx_{2n+1}), \frac{d(x_{2n}, Sx_{2n})d(x_{2n+1}, Tx_{2n+1})}{1 + d(x_{2n}, x_{2n+1})}\right]
\]
\[
= cg\left[d(x_{2n}, x_{2n+1}), d(x_{2n}, x_{2n+1}), d(x_{2n+1}, x_{2n+2}), \frac{d(x_{2n}, x_{2n+1})d(x_{2n+1}, x_{2n+2})}{1 + d(x_{2n}, x_{2n+1})}\right]
\]
\[
\leq cd(x_{2n+1}, x_{2n}).
\]

Thus
\[
d(x_{2n+1}, x_{2n+2}) \leq \frac{c}{s} d(x_{2n}, x_{2n+1}) \tag{2}
\]

Similarly by condition (1) we have:
\[
sd(x_{2n}, x_{2n+1}) = sd(Tx_{2n-1}, Sx_{2n})
\]
\[
= sd(Sx_{2n}, Tx_{2n-1})
\]
\[
\leq cg\left[d(x_{2n}, x_{2n-1}), d(x_{2n}, Sx_{2n}), d(x_{2n-1}, Tx_{2n-1}), \frac{d(x_{2n}, Sx_{2n})d(x_{2n-1}, Tx_{2n-1})}{1 + d(x_{2n}, x_{2n-1})}\right]
\]
\[
= cg\left[d(x_{2n}, x_{2n-1}), d(x_{2n}, x_{2n-1}), d(x_{2n-1}, x_{2n}), \frac{d(x_{2n}, x_{2n-1})d(x_{2n-1}, x_{2n})}{1 + d(x_{2n}, x_{2n-1})}\right]
\]
\[
\leq cd(x_{2n-1}, x_{2n}).
\]

Thus
\[
d(x_{2n}, x_{2n+1}) \leq \frac{c}{s} d(x_{2n-1}, x_{2n}) \tag{3}
\]
Generally by conditions (2), (3) and denoting \( k = \frac{\xi}{s} \), we have
\[
d(x_{2n+1}, x_{2n+2}) \leq kd(x_{2n}, x_{2n+1}) \leq \ldots \leq k^2d(x_0, x_1) \text{ for } n \in \mathbb{N}.
\]
Since \( 0 \leq k < 1 \), taking limit for \( n \to \infty \) we have
\[
d(x_{2n+1}, x_{2n+2}) \to 0. \tag{4}
\]
Now, we prove that \((x_n)\) is a \( b_d \)-Cauchy sequence, and to do this let be \( m,n > 0 \) with \( m > n \), and using definition 2.4 (c) we have
\[
b_d(x_n, x_m) \leq s[b_d(x_n, x_{n+1}) + b_d(x_{n+1}, x_m)]
\]
\[
\leq sb_d(x_n, x_{n+1}) + s^2b_d(x_{n+1}, x_{n+2}) + s^3b_d(x_{n+2}, x_{n+3}) + \ldots
\]
\[
\leq sk^n b_d(x_0, x_1) + s^2k^{n+1} b_d(x_0, x_1) + s^3k^{n+2} b_d(x_0, x_1) + \ldots
\]
\[
= sk^n b_d(x_0, x_1) [1 + sk + (sk)^2 + (sk)^3 + \ldots]
\]
\[
\leq \frac{sk^n}{1-sk} b_d(x_0, x_1).
\]
On taking limit for \( n,m \to \infty \) we have \( b_d(x_n, x_m) \to 0 \) as \( ks < 1 \). Therefore \((x_n)\) is a \( b_d \)-Cauchy sequence in complete \( b \)-dislocated metric space \((X, b_d)\). So there is some \( u \in X \) such that \((x_n)\) dislocated converges to \( u \). Therefore the subsequences \( \{Sx_{2n}\} \to u \) and \( \{Tx_{2n+1}\} \to u \). Since \( T,S : X \to X \) are continuous mappings we get: \( Su = u \) and \( Tu = u \). Thus, \( u \) is a common fixed point of \( T \) and \( S \).

If consider that \( T \) is continuous and \( S \) not continuous we have that \( Tu = u \). Using the contractive condition of theorem we have,
\[
(sd(Su, Tx_{2n+1})) \leq cg \left[ d(u, x_{2n+1}), d(u, Su), d(x_{2n+1}, Tx_{2n+1}), \frac{d(u, Su)d(\varepsilon_{2n+1}, Tx_{2n+1})}{1+d(u, x_{2n+1})} \right]
\]
\[
\leq cg \left[ d(u, x_{2n+1}), d(u, Su), d(x_{2n+1}, Tx_{2n+1}), \frac{d(u, Su)d(\varepsilon_{2n+1}, x_{2n+2})}{1+d(u, x_{2n+1})} \right].
\]
Taking in upper limit as \( n \to \infty \), using lemma 2.10, property of \( g \) and result \( (4) \) we get
\[
sd(u, Su) \leq cg[0, d(u, Su), 0, 0].
\]
This inequality implies \( d(u, Su) \leq cd(u, Su) \) that means \( d(u, Su) = 0 \). Thus \( Su = u \) and \( u \) is a fixed point of \( S \).

If consider (c) we have that, \( u \) is a common fixed point of \( S \) and \( T \). Using the contractive condition of theorem, we obtain
\[
(sd(u, u)) = sd(Su, Tu)
\]
\[
\leq cg \left[ d(u, u), d(u, u), d(u, u), \frac{d(u, u)d(u, u)}{1+d(u, u)} \right]
\]
\[
= cd(u, u).
\]
The inequality above implies that \( d(u, u) \leq kd(u, u) \). So \( d(u, u) = 0, \) since \( 0 \leq k = \frac{\xi}{s} < 1 \)

**Uniqueness.** Let suppose that \( u \) and \( v \) are two common fixed points of \( T;S \). From condition (1) we have:
Replacing $v = u$ in (5) we get:

$$sd(u,v) = c g[d(u,v),d(u,u),d(v,v),\frac{d(u,u)d(v,v)}{1+d(u,v)}].$$

Replacing $v = u$ in (5) we get:

$$d(u,u) \leq c g[d(u,u),d(u,u),\frac{d(u,u)d(u,u)}{1+d(u,u)}] = cd(u,u),$$

i.e. $d(u,u) \leq \frac{c}{s} d(u,u) = kd(u,u)$. Since $0 \leq k < 1$ we obtain $d(u,u) = 0$. Similarly replacing $u = v$ in (5), we obtain $d(v,v) = 0$. Again from (5) have $d(u,v) = 0$, which implies $u = v$. Thus fixed point is unique.

**Corollary 3.2.** Let $(X,d)$ be a complete $b$-dislocated metric space with parameter $s \geq 1$ and $T,S : X \to X$ two mappings satisfying the following contractive condition

$$sd(Sx,Ty) \leq c g[d(x,y),d(x,Sx),d(y,Ty)]$$

for all $x,y \in X$ where $g \in G_3$ and $0 \leq c < 1$. Then $T$ and $S$ have a unique common fixed point and if $u$ is a common fixed point of $S$ and $T$, then $d(u,u) = 0$.

**Corollary 3.3.** Let $(X,d)$ be a complete dislocated metric space and $T,S : X \to X$ two mappings satisfying the following contractive condition

$$d(Sx,Ty) \leq c g[d(x,y),d(x,Sx),d(y,Ty),\frac{d(x,Sx)d(y,Ty)}{1+d(x,y)}],$$

for all $x,y \in X$ where $g \in G_4$ and $0 \leq c < 1$. Then $T$ and $S$ have a unique common fixed point and if $u$ is a common fixed point of $S$ and $T$, then $d(u,u) = 0$.

The following example supports our theorem.

**Example 3.4.** Let $X = [0,1]$ and $d(x,y) = x + y$, for all $x,y \in X$. It is clear that $d$ is a dislocated metric on $X$. We define the self mappings $S,T : X \to X$ as follows

$$Sx = \begin{cases} \frac{1}{8} x, & x \in [0,1) \\ \frac{1}{6}, & x = 1 \end{cases}$$ and $$Tx = \begin{cases} \frac{1}{5} x, & x \in [0,1) \\ \frac{1}{3}, & x = 1 \end{cases}.$$ Note that $S$ and $T$ are discontinuous maps. Now we will show that the contractive condition of 3.3 is satisfied for constant $c \in (0,1)$ and taking the function $g(t_1,t_2,t_3,t_4) = \max\{t_1,t_2,t_3,t_4\}$. We have the following cases:

Case 1. Note that for all $x,y \in [0,1)$, we have

$$d(Sx,Ty) = d\left(\frac{1}{8} x, \frac{1}{5} y\right) = \frac{1}{8} + \frac{1}{5} \leq \frac{1}{5} (x + y) = \frac{1}{5} d(x,y)$$

Case 2. Note that for $x = y = 1$, we have

$$d(Sx,Ty) = d\left(\frac{1}{8} x, \frac{1}{5} y\right) = \frac{1}{8} + \frac{1}{5} \leq \frac{1}{5} (x + y) = \frac{1}{5} d(x,y)$$
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\[ d(Sx, Ty) = d(S1, T1) = d(\frac{1}{6}, \frac{1}{3}) = \frac{1}{6} + \frac{1}{3} = \frac{1}{2} = \frac{1}{4} \cdot 2 = \frac{1}{4} d(x, y). \]

Case 3. For \( x \in [0, 1) \) and \( y = 1 \), we have

\[ d(Sx, Ty) = d(\frac{x}{6}, \frac{1}{3}) = \frac{x}{6} + \frac{1}{3} \leq \frac{1}{3} (x + 1) = \frac{1}{3} d(x, y). \]

Case 4. For all \( y \in [0, 1) \) and \( x = 1 \), we have

\[ d(Sx, Ty) = d(\frac{1}{6}, \frac{y}{3}) = \frac{1}{6} + \frac{y}{3} = \frac{5 + 6y}{30} \leq \frac{1}{4} (1 + y) = \frac{1}{4} d(x, y). \]

Thus all conditions of corollary 3. are satisfied and \( x = 0 \) is a unique common fixed point of \( S \) and \( T \).

Also we note that this theorem is not available in a usual metric space if \( d(x, y) = |x - y| \) and in \( b \)-metric space \( d(x, y) = |x - y|^2 \) because if consider points \( x = y = 1 \) we will have

\[ d(S1, T1) = |\frac{1}{6} - \frac{1}{3}| = \frac{1}{6} > cd(1, 1) = 0 \]

and

\[ d(S1, T1) = |\frac{1}{6} - 1|^2 = (\frac{1}{6})^2 = \frac{1}{36} > cd(1, 1) = 0. \]

So the contractive condition is failed in two cases.

**Corollary 3.5.** Let \((X, d)\) be a complete dislocated metric space and \( S : X \to X \) a self-mapping satisfying the following contractive condition

\[ d(Sx, Sy) \leq c g[d(x, y), d(x, Sx), d(y, Sy), \frac{d(x, Sx) + d(y, Sy)}{1 + d(x, y)}] \]

for all \( x, y \in X \) where \( g \in G_4 \) and \( 0 \leq c < 1 \). Then, \( S \) has a unique fixed point and \( d(u, u) = 0 \).

**Example 3.6.** Let \( X = [0, 10] \) and \( d(x, y) = \frac{1}{2} (x + y) \), for all \( x, y \in X \). It is clear that \( d \) is a dislocated metric on \( X \) and \((X, d)\) is complete. Also \( d \) is not a metric on \( X \).

We define the self-mapping \( S : X \to X \) by

\[ Sx = \begin{cases} x - 1, & x \neq 0 \\ 0, & x = 0 \end{cases} \]

and take the function \( g(t_1, t_2, t_3, t_4) = \max\{t_1, t_2, t_3, t_4\} \) and also choose the constant \( c = \frac{9}{10} \). For \( x, y \in \{0, 1, \ldots, 10\} \), we have the following cases.

Case 1. For \( x = y = 0 \) have \( d(Sx, Sy) = d(0, 0) = 0 \)

Case 2. If \( x = y > 0 \), then

\[ d(Sx, Sy) = d(x - 1, x - 1) = x - 1 \leq \frac{9}{10} x = \frac{9}{10} d(x, y). \]

Case 3. If \( x > y = 0 \), then

\[ d(Sx, Sy) = d(x - 1, 0) = \frac{1}{2} (x - 1) \leq \frac{9}{10} \frac{x}{2} = \frac{9}{10} d(x, y). \]
Case 4. If \( x > y > 0 \), then
\[
d(Sx, Sy) = d(x - 1, y - 1) = \frac{1}{2} (x + y - 2) \leq \frac{9}{10} \frac{1}{2} (x + y) = \frac{9}{10} d(x, y).
\]
Thus all conditions of theorem are satisfied and \( S \) has a unique fixed point in \( X \). Also we note that for \( x = 1 \) and \( y = 10 \) the contractive condition is failed in the usual metric.

**Theorem 3.7.** Let \((X, d)\) be a complete \( b \)-dislocated metric space and \( T, S : X \to X \) two self-mappings satisfying the condition:
\[
sd(Sx, Ty) \leq c \max \{d(x, y) + d(x, Sx), d(x, Sx) + d(y, Ty), \\
d(x, y) + d(y, Ty), d(y, Ty) + \frac{d(x, Sx) d(y, Ty)}{1 + d(x, y)}\}
\]
for all \( x, y \in X \) and \( 0 \leq 2c < 1 \). Then \( T \) and \( S \) have a unique common fixed point in \( X \).

**Proof.** This theorem is corollary of theorem 3.1 if we use the function \( g_2 \in G_4 \).

**Theorem 3.8.** Let \((X, d)\) be a complete \( b \)-dislocated metric space and \( T, S : X \to X \) two self mappings satisfying the condition:
\[
s^p d^p (Sx, Ty) \leq c \max \{d^p (x, y), d^p (x, Sx), d^p (y, Ty), \frac{d(x, Sx) d(y, Ty)}{1 + d(x, y)}\}
\]
for all \( x, y \in X \) and \( 0 \leq c < 1 \). Then \( T \) and \( S \) have a unique common fixed point in \( X \).

**Proof.** This theorem is taken as a corollary of theorem 1, if we use the function \( g_4 \in G_4 \).

**Theorem 3.9.** Let \((X, d)\) be a complete \( b \)-dislocated metric space and \( T, S : X \to X \) two self-mappings satisfying the condition:
\[
s^2 d^2 (Sx, Ty) \leq c \max \{d(x, y) d(x, Sx), d(x, Sx) d(y, Ty), d(x, y) d(y, Ty), \frac{d(x, Sx) d(y, Ty)}{1 + d(x, y)}\}
\]
for all \( x, y \in X \) and \( 0 \leq 2c < 1 \). Then \( T \) and \( S \) have a unique common fixed point in \( X \).

**Proof.** This theorem is corollary of theorem 1, if we use the function \( g_3 \in G_4 \).

**Remark 3.10.** Results of the above theorems and corollaries are extended and unified of some classical fixed point results in metric spaces and generalization of results of the authors [1,2,9,10,18,19] and other results in dislocated metric spaces.

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