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**Dedicated to the memory of**
**Prof. Dušan Adamović**
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# ON A MATHEMATICAL PAPER OF MILUTIN MILANKOVIĆ IN THE CONTEXT OF THE NEWER DIFFERENTIAL CALCULUS PAPERS

JOVAN V. MALEŠEVIĆ

**Abstract.** According to the paper [2] by Jovan Karamata, the paper [1] by Milutin Milanković ,, has entered many textbooks with certain changes “. In the context of the papers listed in references, the author gave an analogon to the mentioned paper of Milutin Milanković in Differential calculus as well as the numerous correlaries of the analogon. Therefore, the paper is an overview of analogon and correlaries, published in the year of the 100th anniversary of Milutin Milanković's paper [1] in 1909.

## 1. INTRODUCTION

Serbian mathematician Milutin Milanković is in his paper ,,Eine graphiche Darstellung der geometrischen Progressionen, Zeitschrift für mathematishen und narurw. Unterricht. XL. Heft 6/7 (1909), p. 22” [1] showed how convergency can be interpreted, ie. divergency of geometrical series. According to the paper by Jovan Karamata [2] ,,this concept with some changes was quoted in many textbooks”. This paper is being published as the tribute to the 100th Anniversary of the publishing of the Milanković's paper. In the context of that anniversary, the concept will be interpreted according to paper [2].

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Let $\overline{AB} = a$, $\angle CBA = 90^0$ and the straight line $AS$ and $BS'$ such that $\angle CAB = \angle CBD = \alpha$. For $\alpha < 45^0$ (fig. 1) the formed poligonal line $ABCDEFGH$ represents members of the geometrical series:

$$a, aq, aq^2, \ldots \ ,$$

where $q = \text{tg}\alpha < 1$. Lines $AS$ and $BS'$ are convergent and interesect at the point $L$. Geometric series

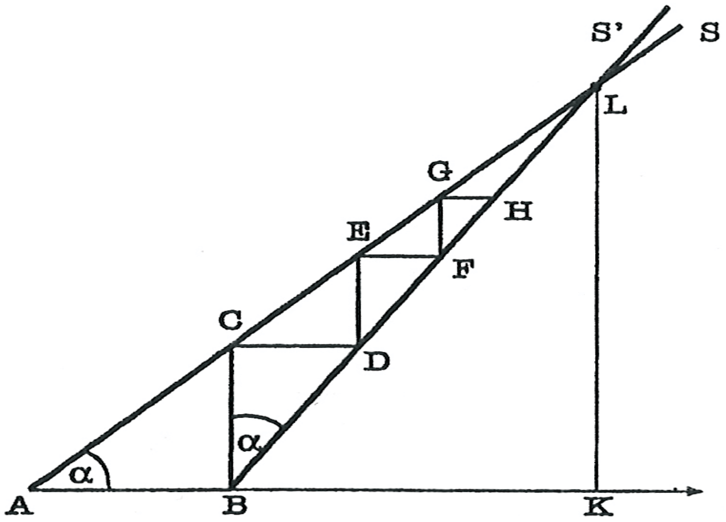


FIGURE 1

$$a + aq + aq^2 + \ldots$$

is convergent and its sum is

$$S = \overline{AK} + \overline{KL}.$$

If $\alpha \geq 45^0$, ie. $q \geq 1$, semi-linears $AS$ and $BS'$ diverge or are parallel; intersection does not exist, and geometric series diverge.

In the context of the mentioned paper by Jovan Karamata, there is also a paper by Mladen Berić [3]: Considerations on convergence and divergence of papers, presented at the Academy of Science, on November, 25th 1913; published in the Journal of Serbian Royal Academy after the First World War, in 1921.

The two papers mentioned, by M. Berić and J. Karamata, inspired the author to create the mentioned papers [5] and [6] in the field of differential calculus. Namely, the mentioned linears which intersect have an analogy in differential calculus, and gives a new concept and new mathematical facts. This paper would present one aspect of papers [5] and [6] in the context of the preoviously mentioned ones.

# I

In the paper of this author: On a theorem by G. Darboux and theorem by D. Trahan-a [4] (1980), the following definition was given: For the function $f$, continuous over segment $[a,b]$ , which has definite oneside differentation $f'(a)$ and $f'(b)$, it is said to belong to $E$ class, if

$$\text{min. or max.}\left\{f'(a), f'(b), \frac{f(b)-f(a)}{b-a}\right\} = \frac{f(b)-f(a)}{b-a} \tag{1}$$

$\left(f'(a), f'(b) \neq \frac{f(b)-f(a)}{b-a}\right)$. For $f'(a) = f'(b)$ it is said to be of subclass $E_1 \subset E$.

In the case

$$f'(a) \underset{(>)}{<} \frac{f(b)-f(a)}{b-a} \underset{(>)}{<} f'(b) \tag{2}$$

it is said to be class $E_2$ (a complementary class to class $E$ ($\overline{E}$ in the paper [5]).

Relations in (2) given on the linear, will be goemetrized in the plane (in system $XOY$). For the function $f$, for which $f''(x) \underset{(>)}{<} 0$ on the segment $[a,b]$ with the convex arc $\overset{\frown}{AB}$ and tangents $T_A$ and $T_B$ in points $A$ and $B$ respectively, by secant $AB$, along with the relation $f'(a) < \frac{f(b)-f(a)}{b-a} < f'(b)$ it is presented by fig. 2. The figure is well known from the absolute whole geometrical intepretation of Young's statement in differential calculus [6].

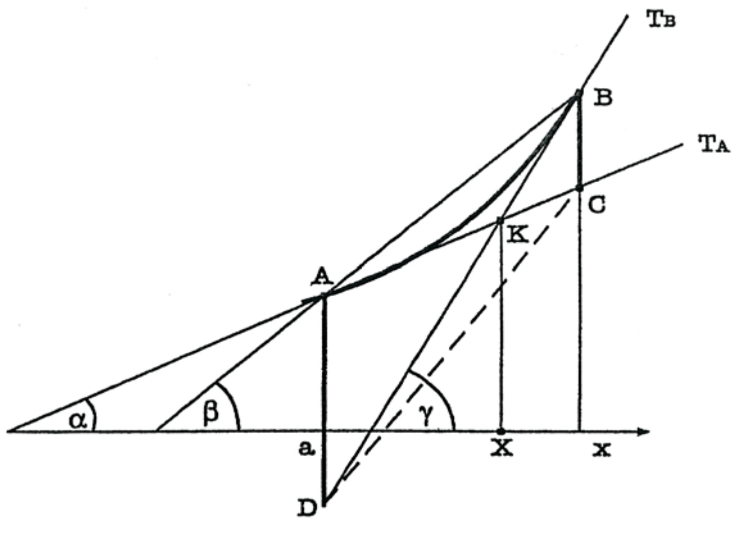


FIGURE 2

By Theorem 4 in the paper [5] it is confirmed that the projection of the interseciton $K$ and tangents $T_A$ and $T_B$ – points X, is the element of the interval $(a,x)$; namely:

$$f''(t) \underset{(>)}{<} 0 \Longrightarrow X \in (a,x), \qquad \forall t \in (a,x). \tag{3}$$

Next, by activation of the point K, projection X is reached, which is given by the following term:

$$X = \frac{xf'(x) - af'(a) - [f(x) - f(a)]}{f'(x) - f'(a)}. \tag{4}$$

The previous term is transformed into the term [4]

$$f(x) - f(a) = [p(x)f'(a) + q(x)f'(x)]\,(x-a), \tag{5}$$

where

$$p(x) = \frac{X-a}{x-a}, \quad q(x) = \frac{x-X}{x-a}, \quad p+q=1 \text{ and } p,\, q \in (0,1).$$

Coeficients $p(x)$ and $q(x)$, as we can already see, are normed distances between point $X$ and edges $a$ and $x$, respectively. By this, the first geometrization of tanget trapezoide $ABCD$ is finished.

There follows further geometrization and relations. For segment $AD$ and $BC$ it is true that :

$$\overline{AD} = p\,|f'(x) - f'(a)|\,(x-a), \;\; \overline{BC} = q\,|f'(x) - f'(a)|\,(x-a). \tag{6}$$

From the previous relations, there follows the first equivalency:

$$\overline{AD} \underset{(>)}{<} \overline{BC} \iff p \underset{(>)}{<} q. \tag{7}$$

If in the terms $p$ and $q$:

$$p = \frac{X-a}{x-a} \quad \text{and} \quad q = \frac{x-X}{x-a} \tag{8}$$

$X$ is substituted by the value from (4), there follows the equivalency:

$$p \underset{(>)}{<} q \iff [f'(x) - f'(a)]\left[\frac{f'(a)+f'(x)}{2} - \frac{f(x)-f(a)}{x-a}\right] \underset{(>)}{<} 0. \tag{9}$$

Finally, the double equivalency is true:

$$\overline{AD} \underset{(>)}{<} \overline{BC} \Leftrightarrow p \underset{(>)}{<} q \Leftrightarrow [f'(x)-f'(a)]\left[\frac{f'(a)+f'(x)}{2} - \frac{f(x)-f(a)}{x-a}\right] \underset{(>)}{<} 0. \tag{10}$$

From the first equivalency in (10) for $p \underset{(>)}{<} q$ it follows that $\overline{AD} \underset{(>)}{<} \overline{BC}$ and then, the $AB$ and $DC$ are elongated and intersect to the right (left) from the Figure 2. It is said for $p > q$, to be right centrcity of the $\overset{\frown}{AB}$, and for $p < q$ to be left centricity of the $\overset{\frown}{AB}$ arc. For $p = q$ there is no centricity – lines $AB$ and $DC$ are parallel (Fig. 3). From the other equivalency, though, it follows that

$$\frac{f(x)-f(a)}{x-a} = \frac{1}{2}f'(a) + \frac{1}{2}f'(x). \tag{11}$$

This differential equation is a linear differential equation, which solution is a parabola.

$$f(x) = A_0x^2 \; (+Mx + N\;) \quad (A_0, M, N, -\text{constants}). \tag{12}$$

According to the mentioned parallelism in this case: $AB\|DC$ tangent trapes transforms into tangent paralellogram (Fig. 3). According to the convention, in this case it is said to be a parabolic centricity. Any two points $A$ and $B$ on the parabola (12) give a pair of parabolic points. The following relation proves a unique solution in this parabole case:

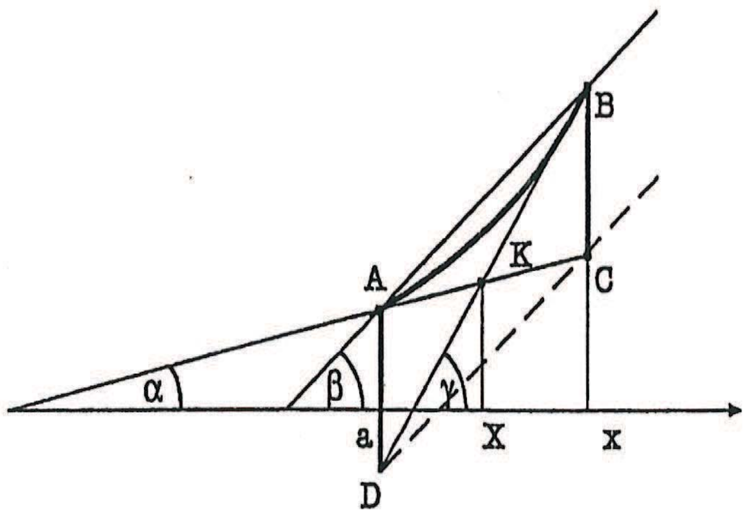


FIGURE 3

$$X_f = \frac{xf'(x) - af'(a) - [f(x) - f(a)]}{f'(x) - f'(a)} = \frac{a + x}{2}. \tag{13}$$

Let us note that, according to the Statement 5 from the paper [5] the condition

$$L = \left[\frac{f(x) - f(a)}{x - a} - f'(a)\right]\left[\frac{f(x) - f(a)}{x - a} - f'(x)\right] < 0 \tag{14}$$

is sufficient and necessary for $X \in (a, x)$, ie. that the function $f$ from class $E_2$, which follows from the relation [5]:

$$\left[\frac{f(x){-}f(a)}{x{-}a}{-}f'(a)\right]\left[\frac{f(x){-}f(a)}{x{-}a}{-}f'(x)\right] = -(x{-}a)(x{-}X)\left(\frac{f(x){-}f(a)}{x{-}a}\right)^2. \tag{15}$$

Let us emphasize that the condition $f''(x) \underset{(>)}{<} 0$ fulfils the condition (14):

$$L{=}[f'(\xi_1){-}f'(a)][f'(\xi_1){-}f'(x)] = f''(\overline{\xi}_1)(\xi_1{-}a)\cdot f''(\overline{\xi}_2)(\xi_2{-}x){<}0,\ \xi_1{\in}(a, x). \tag{16}$$

## II

**1⁰.** Relation between $p$ and $q$ is equivalent to relations between $X$ and the mean $s = \dfrac{a+x}{2}$. Next, the mentioned relationships between $p$ and $q$ provide us with the relation between quotient $\overline{X} = \dfrac{f(x)-f(a)}{x-a}$ and the mean $S = \dfrac{f'(a)+f'(x)}{2}$. Thus, we have the following equivalencies:

$$p \underset{(>)}{<} q \Longleftrightarrow X \underset{(>)}{<} \frac{a+x}{2} = s \qquad \text{(Fig. 4).} \tag{17}$$


Figure 4

For $x < a$ is inverted - there is a symetry in relation to the mean $\dfrac{a+x}{2} = s$ (Fig. 5).


Figure 5

Next, the following equivalency is true

$$p \underset{(>)}{<} q \Longleftrightarrow \overline{X} \underset{(>)}{<} \frac{f'(a)+f'(x)}{2} = S \qquad \text{(Fig. 6).} \tag{18}$$

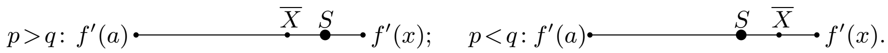


Figure 6

For $f'(x) < f'(a)$ is inverted − there is symetry in relation to the mean $S = \dfrac{f'(a)+f'(x)}{2}$ (Fig.7.)

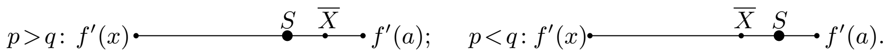


Figure 7

If the relation from one of the fig. 6 ie. 7 is known, thus the centricity of the arc $\overset{\frown}{AB}$ is determined according to (9). From the determined centricity the fig. 6, ie. 7, can be seen.The example is given in the following point 2.

**2⁰.** In papers [5] and [6] there are statements which regulate the kind of centricity of the arc on the segment $[q, b]$. From the result, by the inversion, the double relation of quotient is determined $\overline{X} = \dfrac{f(x)-f(a)}{x-a}$ towards the mean $S = \dfrac{f'(a)+f'(x)}{2}$ and one value $f'(a)$ or $f'(x)$.

Thus, in the case of the considered function $f(x) = \ln(1+x)$, $x > 0$, in the paper [5], according to the calculation:

$$f'(x) = \frac{1}{1+x},\ f^{(2)}(x) = -\frac{1}{(1+x)^2},\ f^{(3)}(x) = \frac{2}{(1+x)^3},\ f^{(2)}(x)\cdot f^{(3)}(x) < 0, \tag{19}$$

by Theorem 4 from paper [6], we conclude that it is the left arc centricity, which equals $p < q$. According to the feedback, one can see from the previous figure on the segment $[f'(x), f'(0)]$ (Fig. 8.):

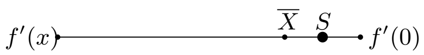


Figure 8

a double relation:

$$\frac{1}{1+x} < \frac{\ln(1+x)}{x} < \frac{1+\dfrac{1}{1+x}}{2},\quad x > 0, \tag{20}$$

ie.

$$\frac{x}{1+x} < \ln(1+x) < \frac{2x+x^2}{2(1+x)}. \tag{21}$$

For the function $f(x) = \ln(l+x)$ and the case that $x \in (-1, 0)$ there is:

$$f'(x) = \frac{1}{1+x} > 1 = f'(0) \ \text{ and } \ p < q,$$

so we use the figure:

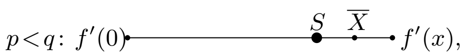


Figure 9

according to which

$$1 > \frac{\ln(1+x) - \ln(1+0)}{x-0} > \frac{\dfrac{1}{1+x}+1}{2}, \tag{22}$$

from which there follows

$$x > \ln(1+x) > \frac{2x+x^2}{2(1+x)}. \tag{23}$$

Let us note the application of the Theorem 1 [6] by the verification of the relation [7]:

$$\operatorname{tg} x > x + \frac{x^3}{3},\ x \in \left(0, \frac{\pi}{2}\right). \tag{24}$$

Namely, starting from the function $F(x) = \operatorname{tg}^2 x$ there is

$$F'(x) = \frac{2\operatorname{tg} x}{\cos^2 x} = \frac{2\sin x}{\cos^3 x},$$

$$F''(x) = 2\frac{\cos^4 x + 3\cos^2 x \sin^2 x}{\cos^4 x} = 2\frac{\cos^2 x + 3(1-\cos^2 x)}{\cos^4 x} = 2\frac{3 - 2\cos^2 x}{\cos^4 x} \geq 2.$$

(sign of equality is true for $x = 0$). Next , according to the relation (43) in [6] it is true

$$\int\limits_{-h}^{h} \mathrm{tg}^2 x\, dx > \sum_{s=0}^{1} \frac{F^{(2s)}(0)(2h)^{2s+1}}{2^{2s}(2s+1)!} = \frac{F^{(2)}(0)(2h)^3}{2^2 3!} = \frac{2}{3}\, h^3.$$

It is also true

$$\int\limits_{-h}^{h} \mathrm{tg}^2 x\; dx = 2\int\limits_{0}^{h} tg^2 x\, dx = 2\int\limits_{0}^{h} \frac{1-\cos^2 x}{\cos^2 x}\, dx = 2\int\limits_{0}^{h} \frac{1}{\cos^2 x}\, dx - 2h = 2\,\mathrm{tg}\, h - 2h$$

and hence

$$2\,\mathrm{tg}\, h - 2h > \frac{2}{3}\, h^3, \quad \text{ie.} \quad \mathrm{tg}\, h > h + \frac{1}{3}\, h^3.$$

In the context of „$p-q$" transformations, we state that in the paper [5] it is emphasized from the textbook examples ([8] and [9]), that „$p-q$" transformations can be successfully used for calculation of the border value functions as well.

**$3^0$.** In the context of the certain arc centricity (right, ie. left), there will be some characteristic point discussed on the segment $[a, x]$ such as classic means and their complementary means [10]:

$$x_1 = \frac{2ax}{a+x},\ x_2 = \sqrt{ax},\ x_3 = a + x - \sqrt{ax},\ x_4 = \frac{a^2+x^2}{a+x}. \tag{25}$$

From the correspondign quotient [6] (Theorems 2 and 3):

$$\frac{f_i(x) - f_i(a)}{x-a} = p_{f_i}(x) {f_i}'(a) + q_{f_i} x) {f_i}'(x),\ \ i = \overline{1,\, 4}\,, \tag{26}$$

we determine the functions $f_i$(x) which have the mentioned points respectively, as centralized points which $X_i$ ($i = \overline{1,\, 4}$) analogously to the case of parabolic centricity $\Big(X_0 = \dfrac{a+x}{2}\Big)$.

Thus for the case of harmonic mean:

$$\left(\begin{array}{l} p_{f_1}(x) = \dfrac{X_f - a}{x-a} = \dfrac{\frac{2ax}{a+x} - a}{x-a} = \dfrac{ax - a^2}{(x-a)(x+a)} = \dfrac{a}{a+x},\ q_{f_1}(x) = \dfrac{x}{a+x};\ \text{ or} \\ \text{from } X_{f_1} = aq + px\colon\ \dfrac{2ax}{a+x} = \dfrac{x}{a+x} a + \dfrac{a}{a+x} x \Longrightarrow q_{f_1} = \dfrac{x}{a+x},\ p_{f_1} = \dfrac{a}{a+x} \end{array}\right) \tag{27}$$

the quotient has a development:

$$\frac{f_1(x) - f_1(a)}{x-a} = \frac{a}{a+x} {f_1}'(a) + \frac{x}{a+x} {f_1}'(x). \tag{28}$$

In such a way we get an equation, by $f_1(x) = y(x)$, ${f_1}'(x) = y'(x)$:

$$[y(x) - y(a)]\,(x+a) = ay'(a)(x-a) + xy'(x)(x-a),$$

actually, linear differential equation:

$$y'(x) - \frac{y(x)(a+x)}{x(x-a)} = -\frac{y(a)(x+a)}{x(x-a)} - \frac{ay'(a)}{x} \tag{29}$$

where

$$P(x) = -\frac{a+x}{x(x-a)},\ Q(x) = -\frac{x+a}{x(x-a)}\,y(a) - \frac{ay'(a)}{x}.$$

By solving the formula:

$$y = e^{-\int P(x)\,dx}\left[C + \int Q(x)\,e^{\int P(x)\,dx}\,dx\right], \tag{30}$$

there is

$$f_1(x) = \frac{A}{x}\,(+Mx+N)\ \ (A, M, N - \text{constants}). \tag{31}$$

At the end there is the verification of the relation:

$$X_{f_1} = \frac{x{f_1}'(x) - a{f_1}'(a) - [f_1(x) - f_1(a)]}{{f_1}'(x) - {f_1}'(a)} = \frac{2ax}{a+x}.$$

We treat the other cases analogously. Thus, in the case of other geometrical means $X_{f_2} = \sqrt{ax}$, there is a relation

$$\frac{f_2(x) - f_2(a)}{x-a} = \frac{\sqrt{a}}{\sqrt{a}+\sqrt{x}}\,{f_2}'(a) + \frac{\sqrt{x}}{\sqrt{a}+\sqrt{x}}\,{f_2}'(x), \tag{32}$$

which leads to a linear differential equation with the solution

$$f_2(x) = B\sqrt{x}\,(+Mx+N)\ \ (B, M, N - \text{constants}). \tag{33}$$

Thus for the functions

$$f_1(x) = \frac{A}{x}\,(+Mx+N)\ \text{ and }\ f_2(x) = B\sqrt{x}\,(+Mx+N), \tag{34}$$

with the left arc centricity, it is proved that they are unique which give harmonic ie. geometric centricity of the quotient respectively.

Theorem 3 [6] takes into consideration the complementary means:

$$X_{f_3} = a+x-\sqrt{ax}\ \text{ and }\ X_{f_4} = \frac{a^2+x^2}{a+x}, \tag{35}$$

which brings us to the following quotient development:

$$\frac{f_3(x) - f_3(a)}{x-a} = \frac{x}{a+x}{f_3}'(a) + \frac{a}{a+x}{f_3}'(x) \tag{36}$$

and

$$\frac{f_4(x) - f_4(a)}{x-a} = \frac{\sqrt{x}}{\sqrt{a}+\sqrt{x}}{f_4}'(a) + \frac{\sqrt{a}}{\sqrt{a}+\sqrt{x}}{f_4}'(x), \tag{37}$$

respectively. The solutions provide the functions respectively:

$$f_3(x) = Ce^{\frac{x}{a}}(x-a)^2\,(+Mx+N) \tag{38}$$

and

$$f_4(x) = De^{2\sqrt{\frac{x}{a}}}(\sqrt{x}-\sqrt{a})^2\,(+Mx+N)\ \ (C, D, M, N - \text{constants}). \tag{39}$$

In the context of theorem 3 [6] for the obtained functions (38) and (39) with the right arc centricity, they are proved to be unique, which gives complementary harmonic ie. complementary geometric centricity of the quotient, respectively.

Let us note that in Theorems 2 and 3 [6] from development of the quotient $\frac{f_i(x)-f_i(a)}{x-a}$, according to the Corollary 1 in [5], we can see the corespodent development

$$X_{f_i} = a \cdot q_{f_i} + x \cdot p_{f_i}, i = \overline{1,\, 4}, \tag{40}$$

with coefitients $q_{f_i}$ and $p_{f_i}$ by the means of procedure as the relation (27 ) mentioned in the brackets.

**4⁰.** Normed distances of points $X$ and $\overline{X}$ from points $\frac{a+x}{2}$ and $\frac{f'(a)+f'(x)}{2}$ respectively.

According to the definition of complementarity and normed in the context of the standard distance from $\frac{a+x}{2}$, follows

$$\begin{aligned} d_1(KX) \;\equiv\; d_{\frac{a+x}{2}}(KX) &= \left| \frac{X - \frac{a+x}{2}}{\frac{x-a}{2}} \right| = 2\left| \frac{qa+px-\frac{a+x}{2}}{\frac{x-a}{2}} \right| \\ &= 2\left| \frac{(2q-1)a+2(p-1)x}{2(x-a)} \right| = |p-q|, \end{aligned}$$

ie.

$$d_1(KX) = |p-q|. \tag{41}$$

Next, from

$$p(x) = \frac{f'(x)-\overline{X}}{f'(x)-f'(a)} \text{ and } q(x) = \frac{\overline{X}-f'(a)}{f'(x)-f'(a)},\ \overline{X} = \frac{f(x)-f(a)}{x-a} \tag{42}$$

$-$ $p(x)$ and $q(x)$ are standard distances of quotient $\overline{X}$ from the edges $f'(a)$ and $f'(x)$, respectively $-$ it follows that

$$|p(x)-q(x)| = \left| \frac{f'(x)+f'(a)-2\overline{X}}{f'(x)-f'(a)} \right|, \text{ ie. } \left| \frac{\overline{X} - \frac{f'(x)+f'(a)}{2}}{\frac{f'(x)-f'(a)}{2}} \right| = d_2(K\overline{X}). \tag{43}$$

From (41) and (43) it follows that

$$d_1(KX) = d_2(K\overline{X}) \tag{44}$$

with one signed correspondence using bijection $x \leftrightarrow f'(x)$.

Using the relations (41) and (43) according to the bijection, there are the following conclusions:

**a)** In the case of harmonic means $(H)$ and complementary harmonic mean $(KH)$, it is true, by the notation:

$$d_1(H) = d_1(KH) = \left| \frac{x-a}{x+a} \right|,\ d_2(\overline{H}) = d_2(K\overline{H}) = \left| \frac{f'(x)-f'(a)}{f'(x)+f'(a)} \right|. \tag{45}$$

**b)** In the case of the geometric means ($G$) and complementary geometric means ($KG$) it is true, by the notation:

$$d_1(G) = d_1(KG) = \left|\frac{\sqrt{x}-\sqrt{a}}{\sqrt{x}+\sqrt{a}}\right|, \; d_2(\overline{G}) = d_2(K\overline{G}) = \left|\frac{\sqrt{f'(x)}-\sqrt{f'(a)}}{\sqrt{f'(x)}+\sqrt{f'(a)}}\right|. \quad (46)$$

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# О ЈЕДНОМ МАТЕМАТИЧКОМ РАДУ МИЛУТИНА МИЛАНКОВИЋА У КОНТЕКСТУ НОВИЈИХ РАДОВА ИЗ ДИФЕРЕНЦИЈАЛНОГ РАЧУНА

**Јован В. Малешевић**

**РЕЗИМЕ**

Рад је посвећен стогодишњици објављивања Миланковићевог рада из 1909. године [1] који је ушао, са извесним изменама, у многе уџбенике. У раду су дати резултати аутора у контексту радова изнетих у референцама.

Футошка 60,
Нови Сад, Србија

E-mail: malesh@eunet.rs