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DISCRETE FLOW ON UNIFORM STRUCTURE OF SET $F(T, X)$

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Abstract. In this paper is proved that, if T is a discrete topological abelian group, and if X is a Hausdorff uniform space, then the mapping $\mu(f, t) = f_t$, defines a general discret flow on uniform structure of the set $F(T, X)$ related to the relative uniformity of pointwise convergence.

1. Preliminary remarks

Let T be a Hausdorff topological abelian group, $(Y, \mathcal{V})$-Hausdorff topological space, and $p_t : Y^T \to Y$, $t \in T$ natural projection. If we denote:

$$\begin{gathered} h_t : (Y^T)^2 \to Y^2, \forall t \in T \\ \forall (f, g) \in (Y^T)^2, h_t(f, g) = (f(t), g(t)), (f(t), g(t)) \in Y^2 \\ h_t^{-1}(V) = \{(f, g) \in (Y^T)^2 \,|\, (f(t), g(t)) \in V\}, \forall t \in T, \forall V \in \mathcal{V} \\ S = \bigcup_{t \in T} \{h^{-1}(V) \,|\, V \in \mathcal{V}\} \subset \mathcal{U} \end{gathered}$$

than the family $\mathcal{S}$ is the subbase for a uniformity $\mathcal{U} \subset P((Y^T)^2)$, (in Y^T), and the pair $(Y^T, \mathcal{U})$ is Hausdorff uniform space (because, the product of Hausdorff spaces is a Hausdorff space). If we denote

$$\begin{gathered} q_t = n_t \times n_t, n_t : X^T \to X, \forall t \in T \\ q_t^{-1}(B) = \{(f, g) \in X^2 \,|\, (f_t(s), g_t(s)) \in B\}, \forall t, s \in T, \forall B \in \mathcal{V} \\ \mathcal{M} = \{q_t^{-1}(B) \,|\, t, s \in T, B \in \mathcal{V}\} \quad \text{or} \\ \mathcal{M} = \bigcup_{t \in T} \{q_t^{-1}(B) \,|\, s \in T, B \in \mathcal{V}\} \subset \mathcal{N} \end{gathered}$$

than the family $\mathcal{M}$ is a subbase for auniformity $\mathcal{N} \subset P((X^T)^2)$, (in X^T), and the pair $(X^T, \mathcal{N})$, is Hausdorff uniform space (because, the product of Hausdorff uniform space is a Hausdorff space).

Let $F(T, X)$ be the set of all continuous functions on a topological Hausdorff space T to a uniform Hausdorff space $(X^T, \mathcal{N})$ and $i : F(T, X) \to X^T$. We can endow the subset $F(T, X)$ with the relative uniformity of pointwise convergence on T. If we denote:

$$h : (F(T,\, X))^2 \to (X^T)^2$$
$$\forall\,(f,\, g) \in (F(T,\, X))^2,\; h(f,\, g) = (f_t(s),\, g_t(s)),\; (f(s),\, g(s)) \in B$$
$$h^{-1}(B) = B \cap (F(T,\, X))^2 = W(B),\; \forall\, B \in \mathcal{N}$$
$$W(B) = h^{-1}(B) = \{(f,\, g) \in (F(T,\, X))^2 \,|\, (f_t(s),\, g_t(s)) \in B\},\; \forall\, t, s \in T, \forall\, B \in \mathcal{V}$$
$$\mathcal{B} = \{W(B) \,|\, B \in \mathcal{N}\} \subset \mathcal{P}$$

than the family $\mathcal{B}$ is a base for a uniformity $\mathcal{P} \subset P(F(T,\, X))^2$, (in $F(T,\, X)$) which is called the relative uniformity of pointwise convergence, and the pair $(F(T,\, X),\, \mathcal{P})$ is a Hausdorff space (because, each subspace of a Hausdorff space, is a Hausdprff space).

Definition. *Let H be a topological space and S is a discrete topological group. The mapping $\psi : H \times S \to H$ is said to be a general discrete flow on H, if satisfying the following axioms:*

(a_1) *(Identity property)*

$\psi(x,\, 0) = x,\, \forall\, x \in H$

(where 0 is the identity of S).

(a_2) *(Group property)*

$\psi(\psi(x,\, t),\, s) = \psi(x,\, t \oplus s),\, \forall\, x \in H\ \&\ \forall\, t, s \in S,$

(where $\oplus$ is the group operation of S).

(a_3) *(Continuity property)*

The mapping $\psi : H \times G \to H$ is continuous in H. In other words, for each neighborhood N of point $\psi(x,\, t)$, there exists a neighborhood E of $x \in H$ such that $\psi(E,\, t) \subseteq N$.

2. The result

Let $F(T,\, X)$ be a set of all continuous functions $f : T \to X$. The subset $F(T,\, X)$ we can endow with the relative uniformity of pointwise convergence on T. If the mapping $\mu : F \times T \to F$ is defined by:

$$\forall\,(f,\, t) \in F \times T,\; \mu(f,\, t) = f_t$$

where $f_t(s) = f(t \oplus s), \forall\, t, s \in T$, then are satisfied the theorems:

Theorem. *Let T be a discrete topological abelian group and $F(T,\, X)$ is a Hausdorff topological space. The mapping $\mu(f,\, t) = f_t$, defines a general discret flow on uniform structure of set $F(T,\, X)$ related to the relative uniformity of pointwise convergence.*

Proof. We shall prove the axioms of discret flow:

(a_1) (Identity property). By the definition

$$\mu(f(s),\, t) = f_t(s) = f(t \oplus s),\; \forall\, t, s \in T$$
$$\mu(f(s),\, 0) = f_0(s) = f(0 \oplus s) = f(s),\; \forall\, s \in T$$
$$\mu(f,\, 0) = f,\; \forall\, f \in F(T,\, X)$$

(a_2) (Group property). For each t, $s \in T$ and for each $f \in F(T, X)$. By the definition

$$\begin{gathered}\mu(f(s), t) = f_t(s) = f(t \oplus s), \forall t, s \in T, \\ \mu(\mu(f(s), t), m) = \mu(f_t(s), m) = \mu(f(t \oplus s), m) = f_m(t \oplus s) \\ f_m(t \oplus s) = f(t \oplus s \oplus m) = f_{t \oplus s}(m) \\ \mu(\mu(f(s), t), m) = f_{t \oplus s}(m), \forall m \in T\end{gathered}$$

or

$$\left.\begin{array}{r}\mu(\mu(f, t), s) = f_{t \oplus s} \\ f_{t \oplus s} = \mu(f, t \oplus s)\end{array}\right\} \Rightarrow \mu(\mu(f, t), s) = \mu(f, t \oplus s).$$

(a_3) (Continuity property). Now let us show that the mapping $\mu : F \times T \to F$ is continuous in uniform structure of set $F(T, X)$. Assume that the set $F(T, X)$ has the $\mathcal{P}$ relative uniformity of pointwise convergence on T. That is

$$A \in \mathcal{P} \Leftrightarrow A = \{f \in M \cap F(T, X) \,|\, f_t(s) \in V\}, \; s, t \in T, \; V \in \mathcal{V}, \; M \in \mathcal{M}.$$

Let $f \in F(T, X)$ be an arbitrary point and let $\mathcal{N}_f$ be the family of all open neighborhoods of point f related to the relative uniformity of pointwise convergence. The family $\mathcal{N}_f$ can be directed with the binary relation $(\leq) \subseteq \mathcal{N}_f \times \mathcal{N}_f$ as follows:

$$\forall A_1, A_2 \in \mathcal{N}_f, \; A_1 \leq A_2 \Leftrightarrow A_1 \supseteq A_2.$$

Than, the ordering pair $(\mathcal{N}_f, \leq)$ becomes a directed family. Indeed, for two each open neighborhoods A_1, $A_2 \in \mathcal{N}_f$ of the point $f \in F(T, X)$, their intersection $A_3 = A_1 \cap A_2 \in \mathcal{N}_f$ is also an open neighborhood of the point f such that: $A_1 \leq A_3$, $A_2 \leq A_3$. The mapping $g : \mathcal{N}_f \to F(T, X)$ which is defined by:

$$\forall A \in \mathcal{N}_f, \; g(A) = g_A$$

defines the net $(g_A, A \in \mathcal{N}_f) \subset F(T, X)$ which converges at the unique point $f \in F(T, X)$. In other words, there exist an open neighborhood $A_0 \in \mathcal{N}_f$ of the point $f \in F(T, X)$ such that, for each open neighborhood $A \in \mathcal{N}_f$ of the point $f \in F(T, X)$ is fulfilled:

$$A \geq A_0 \Rightarrow g_A \in A \subseteq A_0$$

related to the $\mathcal{P} \subset P(F(T, X))$ relative topology of uniform pointwise convergence. The point f is unique, because the ordering pair $(F(T, X), \mathcal{P})$, is a Hausdorff space, related to the relative topology of uniform pointwise convergence. For continuity of the mapping $\mu : F \times T \to F$ in $F(T, X)$, it will be sufficient to show that the corresponding net $(\mu(g_A, t_0), A \in \mathcal{N}_f)$, (where $t_0 \in T$ is a fixed point), converges to a unique point $\mu(f, t_0) \in F(T, X)$ related to the relative topology of uniform pointwise convergence.

Suppose the contrary, that the corresponding net $(\mu(g_A, t_0), A \in \mathcal{N}_f) \subset F(T, X)$ converges to a unique point $\mu(f, t_0) \in F(T, X)$, but the mapping $\mu : F \times T \to F$ is discontinuous at the point $f \in F(T, X)$. In other words, there exists an open neighborhood $A_0 \in \mathcal{N}_{\mu(f, t_0)}$ of the point $\mu(f, t_0)$ in $F(T, X)$ such that, for each open neighborhood $A \in \mathcal{N}_f$ in $F(T, X)$ that satisfies the condition:

$$\left.\begin{array}{r}(\exists A_0 \in \mathcal{N}_{\mu(f, t_0)})(\forall A \in \mathcal{N}_f) \\ \mu(A, t_0) \not\subset A_0\end{array}\right\}$$

On the other hand, the net $(\mu(g_A,\, t_0),\, A \in \mathcal{N}_f) \subset F(T,\, X)$ converges to a unique point $\mu(f,\, t_0) \in F(T,\, X)$ relative to the $\mathcal{P} \subset P(F(T,\, X))$ topology of uniform pointwise convergence. This means that, there exist an open neighborhood $A_0 \in \mathcal{N}_{\mu(f,\, t_0)}$ of the point $\mu(f,\, t_0) \in F(T,\, X)$ such that $\mu(g_A,\, t_0) \in A_0$. Hence, we have:

$$\mu(g_A,\, t_0) \in \mu(A,\, t_0) \not\subset A_0.$$

Consequently,

$$\left.\begin{array}{c}(\exists\, A_0 \in \mathcal{N}_{\mu(f,\, t_0)})(\forall\, W \in \mathcal{N}_{\mu(f,\, t_0)}) \\ W \geq A_0 \Rightarrow \mu(A_0,\, t_0) \notin W \subseteq A_0\end{array}\right\}$$

The last condition shows that the net $(\mu(g_A,\, t_0),\, A \in \mathcal{N}_f) \subset F(T,\, X)$ does not converge to a unique point $\mu(f,\, t_0) \in F(T,\, X)$, which is impossible. This contradiction shows that the mapping $\mu : F \times T \to F$ is continuous in $F(T,\, X)$, related to the relative uniformity of pointwise convergence. □

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ДИСКРЕТЕН ТЕК ВО РАМНОМЕРНАТА СТРУКТУРА НА МНОЖЕСТВА $F(T,\, X)$

НЕКИ ДЕРВИШИ

Р е з и м е

Во оваа работа докажано е дека, ако T е дискретна абелова тополошка група, и ако X е рамномерен Хаусдорфов простор, тогаш пресликувањето $\mu(f,\, t) = f_t$ одредува еден генерален дискретен тек во рамномерната структура на множествата $F(T,\, X)$, во врска со релативната униформна конвергенција по точки.

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