Математички Билтен
24 (L)
2000 (111-120)
Скопје, Македонија

ISSN 0351-336X

A CONTRIBUTION TO THE THEOREM ON LEVEL HOMOTOPY EQUIVALENCES

Nikita Shekutkovski

Abstract

In [7] it is proved the following result. If the members of an strictly commutative inverse sequence $\underline{X}$ of topological spaces are replaced by homotopy equivalent spaces, the new spaces can be organized as a coherent inverse system. Moreover, the two inverse sequences are isomorphic in the coherent category. In this paper this result is strengthened, by replacing the commutative inverse sequence $\underline{X}$ by an arbitrary coherent inverse sequence. The following result is proved also: For arbitrary coherent inverse sequencess $(X_m, p_{\underline{m}})$ and $(Y_m, q_{\underline{m}})$, if the maps $f_m : X_m \to Y_m$ and $f'_m : X_m \to Y_m$ are homotopic, and $f : (X_m, p_{\underline{m}}) \to (Y_m, q_{\underline{m}})$ is a coherent map for maps $f_m : X_m \to Y_m$, then there exists a coherent map $f'$for the maps $f'_m : X_m \to Y_m$ such that f and f' are coherently homotopic.

The following problem is considered in [7]. Let $\underline{X} = (X_n, p_{n,m})$ be a strictly commutative inverse sequence of topological spaces i.e. the maps $p_{n,m} : X_m \to X_n$, $n < m$ satisfy

$$p_{n,n+1}p_{n+1,n+2} = p_{n,n+2} \,.$$

Let $f_n : X_n \to Y_n$ be a homotopy equivalence with a homotopy inverse $g_n : Y_n \to X_n$ and let $g_n f_n \simeq 1_{X_n}$ by a homotopy H_n.

$$\begin{array}{ccccc} X_1 & \xleftarrow{p_{12}} & X_2 & \xleftarrow{p_{23}} & \cdots \\ f_1 \downarrow\uparrow g_1 & & f_2 \downarrow\uparrow g_2 & & \\ Y_1 & & Y_2 & & \end{array}$$

We can define an inverse sequence $\underline{Y} = (Y_n, q_{n,m})$ in a natural way: $q_{n,m} = f_n p_{n,m} g_m$ for $n < m$. The two inverse sequences cannot be compared in the category of commutative inverse systems pro-Top, since the inverse sequence $\underline{Y} = (Y_n, q_{n,m})$ is not strictly commutative.

The inverse sequence $\underline{Y} = (Y_n, q_{n,m})$ is an object of the category pro-HTop, since the maps $q_{k,m}$ and $q_{k,n}q_{n,m}$ are homotopic i.e. $q_{k,m} \simeq q_{k,n}q_{n,m}$. Moreover, the inverse sequences $\underline{X}$ and $\underline{Y}$ are isomorphic in pro-HTop (usually, Htop denotes the category of topological spaces and homotopy classes).

The question which arises is: is there a category stronger than pro-HTop where the two inverse sequences are isomorphic?

In the paper [7] it is given a positive answer to this question in the coherent category Coh.

The objects of this category are coherent inverse systems $(Y_a, q_{a_0a_1\ldots a_n}, A)$ defined as follows.

Definition: A triple $(Y_a, q_{\underline{a}}, A)$ is called a coherent inverse system over a directed set A if for $n \geq 2$, and $a_0 < a_1 < \ldots < a_n$, $\underline{a} = (a_0, a_1, \ldots, a_n)$, the map $q_{\underline{a}}: I^{n-1} \times Y_{a_n} \to Y_{a_0}$ satisfy the following two conditions

$$q_{\underline{a}}(t_1, \ldots, t_{j-1}, 0, t_{j+1}, \ldots, t_{n-1}, x) = q_{a_0 \ldots \hat{a}_j \ldots a_n}(t_1, \ldots, t_{j-1}, t_{j+1}, \ldots, t_{n-1}, x),$$

$$q_{\underline{a}}(t_1, \ldots, t_{j-1}, 1, t_{j+1}, \ldots, t_{n-1}, x) = q_{a_0 \ldots a_j}(t_1, \ldots, t_{j-1}, q_{a_j \ldots a_n}(t_{j+1}, \ldots, t_{n-1}, x)).$$

For example in the special case $n = 2$, the map $q_{a_0a_1a_2} : I \times Y_{a_2} \to Y_{a_0}$ satisfies

$$q_{a_0a_1a_2}(0, x) = q_{a_0a_2}(x), \quad q_{a_0a_1a_2}(1, x) = q_{a_0a_1}q_{a_1a_2}(x).$$

The same definition under the name projective system appears in [4]. There was announced the existence of the coherent category of these stystems and of coherent maps of order without an explicit formula for coherent maps.

The coherent category Coh whose objects are coherent systems and morphisms are homotopy classes of coherent maps was explicitely constructed by the author [6].

The other description of coherence is given by Cordier and Porter in more general situation. Their approach is not based on explicit formula for coherent maps and coherent homotopies. In the paper [1] they gave a positive answer to the same problems considered in this paper. The relation of their coherent theory and the category Coh is an open question, although representing the directed set A as a category [2], it is easily seen that their notion of coherent diagram over A in the category of topological spaces coincides with the notion of coherent inverse system over A.

For purposes of this paper it is enough to describe the level coherent category $\mathrm{Coh}^{\mathrm{A}}$. Level coherent maps are a special type of coherent maps. The *level coherent map* over A, $(f_{\underline{a}}^{j}):(X_a, p_{\underline{a}}, A) \to (Y_a, q_{\underline{a}}, A)$ consists of :

0) a map $f_{a_0}: X_{a_0} \to Y_{a_0}$ for any index a_0 in A;

1) a map $f_{a_0a_1}: \Delta^1 \times X_{a_1} \to Y_{a_0}$ for any pair of indices) $a_0 < a_1$ in) A such that

$$f_{a_0a_1}(0,x) = f_{a_0}p_{a_0a_1}(x),$$

$$f_{a_0a_1}(1,x) = q_{a_0a_1}f_{a_1}(x).$$

2) In the general case, for a strictly increasing sequence $\underline{a}=(a_0, a_1, \ldots, a_n)$, $a_0 < a_1 < \ldots < a_n$, there is a set $f_{\underline{a}}$ of 2^{n-1} maps

$$\{f_{\underline{a}}^{j_0j_1\ldots j_k} \mid 0 = j_0 < j_1 < \ldots < j_k = n\} = f_{\underline{a}}.$$

The map $f_{\underline{a}}^{j_0j_1\ldots j_k}: I^{n-k} \times \Delta^k \times X_{a_n} \to Y_{a_0}$ must satisfy the following boundary conditions:

$$f_{\underline{a}}^{j_0j_1\ldots j_k}(\tau, \partial_i t, x) =$$

$$\begin{cases} q_{a_0\ldots a_{j_1}}(\tau_1, \ldots, \tau_{j_1-1}, f_{a_{j_1}\ldots a_n}^{j_1-j_1\ldots j_k-j_1}(\tau_{j_1}, \ldots, \tau_{n-k}, t, x)), & i=0, \\ f_{\underline{a}}^{j_0\ldots \hat{j}_i\ldots j_k}(\partial^1_{j_i-i+1}\tau, t, x), & 1 \le i \le k-1, \\ f_{a_0\ldots a_{j_{k-1}}}^{j_0\ldots j_{k-1}}(\tau_1, \ldots, \tau_{j_{k-1}-k+1}, t, p_{a_{j_{k-1}}\ldots a_n}(\tau_{j_{k-1}-k+2}, \ldots, \tau_{n-k}, x)), & i=k, \end{cases}$$

for $(\tau_1, \ldots, \tau_{n-k}) \in I^{n-k}$, $t = (t_0, \ldots, t_{k-1}) \in \Delta^{k-1}$, $x \in X_{a_n}$, and

$$f_{\underline{a}}^{j_0j_1\ldots j_k}(\partial^0_{j-m}\tau, t, x) = f_{a_0\ldots \hat{a}_j\ldots a_n}^{j_0\ldots j_m j_{m+1}-1\ldots j_k-1}(\tau, t, x), \quad j_m < j < j_{m+1}$$

for $(\tau_1, \ldots, \tau_{n-k-1}) \in I^{n-k-1}$, $t = (t_0, \ldots, t_k) \in \Delta^k$, $x \in X_{a_n}$ (we identify the notations $f_{a_0}^{0} \equiv f_{a_0}$ and also $f_{a_0a_1}^{01} \equiv f_{a_0a_1}$). We denote a level coherent map over A, by $(f_{\underline{a}}^{j}): (X_a, p_{\underline{a}}, A) \to (Y_a, q_{\underline{a}}, A)$. Two level coherent maps $(f_{\underline{a}}^{j})$, $(f'^{j}_{\underline{a}})$ are homotopic if there exists a level coherent map $(F_{\underline{a}}^{j})$ such that the map $F_{\underline{a}}^{j}$ connects the maps $f_{\underline{a}}^{j}$, $f'^{j}_{\underline{a}}$.

The category having as objects coherent inverse systems over A, and as morphisms level homotopy classes of level coherent maps is denoted by Coh^A - the level coherent category over A. The level coherent category over the set of natural numbers N is denoted by Coh^N. The objects of this category are coherent inverse sequences denoted by $\underline{X} = (X_n, p_{n,m})$.

In this paper, the result from [7] is strengthened replacing the commutative inverse sequence $\underline{X} = (X_n, p_{n,m})$, by a coherent inverse sequence $(X_m, p_{\underline{n}})$ (Theorem 1).

Also, it is proved the following result:

Let $f = (f_{\underline{m}}^{j}): (X_n, p_{\underline{n}}) \to (Y_m, q_{\underline{m}})$ be a level coherent map between arbitrary coherent inverse sequences $(X_n, p_{\underline{n}})$ and $(Y_m, q_{\underline{m}})$. If the maps $f_m: X_m \to Y_m$ and $f'_m: X_m \to Y_m$ are homotopic by a homotopy $F_m: X_m \to Y_m$, then there exists a coherent map $f' = (f'^{j}_{\underline{m}}): (X_n, p_{\underline{n}}) \to (Y_m, q_{\underline{m}})$ and a coherent homotopy $F = (F_{\underline{m}}^{j}): (X_n, p_{\underline{n}}) \to (Y_m, q_{\underline{m}})$ which connects maps f and f'(Theorem 2).

In the proofs of these theorems it is used the Lemma of Vogt [8] about homotopy equivalences and the notion of strong fundamental sequences introduced in [3] by Lisitsa, for defining strong shape theory for metric compacta. As shown in [5], it is one of the several equivalent approaches to the strong shape theory of metric compacta.

The level strong homotopy classes of level strong fundamental sequences and non-commutative inverse sequences form a category as shown in [7]. As shown there, this is a very simple description of level coherent category Coh^N ([7], Theorem 2).

We repeat the description of this category - the level category of strong fundamental sequences.

Let $\underline{X} = (X_n, p_{n,m})$ and $\underline{Y} = (Y_n, q_{n,m})$ be inverse sequences commutative up to homotopy. A strong fundamental sequence from $\underline{X}$ to $\underline{Y}$ consists of a map $f_n: X_n \to Y_n$ and a homotopy $f_{n,n+1}: I \times X_{n+1} \to Y_n$ such that

$$f_{n,n+1}(0, x) = f_n p_{n,n+1}(x)\,,$$

$$f_{n,n+1}(1, x) = q_{n,n+1} f_{n+1}(x)\,.$$

Two strong fundamental sequences $(f_n, f_{n,n+1})$ and $(f'_n, f'_{n,n+1}): \underline{X} \to \underline{Y}$ are coherently homotopic if there exists a strong fundamental sequence $(F_n, F_{n,n+1}): I \times \underline{X} \to \underline{Y}$ (where $I \times \underline{X} = (I \times X_n, 1 \times p_{n,n+1}))$ such that $F_n: I \times X_n \to Y_n$ satisfy

$$F_n(0,x) = f_n(x)\,, \quad F_n(1,x) = f'_n(x)$$

and maps $F_{n,n+1}: I \times I \times X_{n+1} \to Y_n$ satisfy

$$F_{n,n+1}(t,0,x) = f_{n,n+1}(t,x)\,, \quad F_{n,n+1}(t,1,x) = f'_{n,n+1}(t,x)\,.$$

This is an equivalence relation and we denote $(f_n, f_{n,n+1}) \simeq (f'_n, f'_{n,n+1})$.

The composition of level strong fundamental sequences $(f_n, f_{n,n+1})$: $\underline{X} \to \underline{Y}$ and $(g_n, g_{n,n+1}): \underline{Y} \to \underline{Z}$ is the strong fundamental sequence $(h_n, h_{n,n+1}): \underline{X} \to \underline{Z}$ defined by

$$h_n = g_n f_n$$

and

$$h_{n,n+1}(t,x) = \begin{cases} g_n f_{n,n+1}(2t,x), & 0 \le t \le \frac{1}{2}, \\ g_{n,n+1}(2t-1, f_{n+1}(x)), & \frac{1}{2} \le t \le 1\,. \end{cases}$$

Theorem 1. Let $(X_{\underline{m}}, p_{\underline{m}})$ be a coherent inverse sequence, let $f_m: X_m \to Y_m$ be a homotopy equivalence with a homotopy inverse $g_m: Y_m \to X_m$. Let $\underline{Y} = (Y_n, q_{n,m})$ be the inverse system defined by $q_{n,m} = f_n p_{n,m} g_m$, for $n < m$. Then there is a coherent inverse system $(Y_m, q_{\underline{m}})$ such that for $\underline{m} = (n,m)$, $q_{\underline{m}} = q_{n,m}$, and $(X_m, p_{\underline{m}})$ and $(Y_m, q_{\underline{m}})$ are isomorphic

1) in the level coherent category Coh^N

2) in the coherent category Coh.

Proof: First we construct the coherent inverse system $(Y_m, q_{\underline{m}})$ such that for $\underline{m} = (n,m)$, $q_{\underline{m}} = q_{n,m}$.

We will consider a more general situation: Let $(X_a, p_{\underline{a}}, A)$ be a commutative inverse system over a directed set A. Let $f_a: X_a \to Y_a$ be a homotopy equivalence with a homotopy inverse $g_a: Y_a \to X_a$ and $g_a \simeq f_a\, 1_{X_a}$ by a homotopy H_a.

We can define a map $q_{a_0 a_1}: Y_{a_1} \to Y_{a_0}$ for any pair of indices $a_0 < a_1$ by $q_{a_0 a_1} = f_{a_0} p_{a_0 a_1} g_{a_1}$. For $a_0 < a_1 < a_2$, we define a map $q_{a_0 a_1 a_2}: I \times Y_{a_2} \to Y_{a_0}$ by

$$q_{a_0 a_1 a_2}(t_1, x) = \begin{cases} f_{a_0} p_{a_0 a_1 a_2}(2t_1, g_{a_2}(x))\,, & 0 \le t_1 \le \frac{1}{2}, \\ f_{a_0} p_{a_0 a_1} H_{a_1}(2t_1 - 1, p_{a_1 a_2} g_{a_2}(x))\,, & \frac{1}{2} \le t_1 \le 1\,. \end{cases}$$

For $n \geq 2$, we define a map $q_{\underline{a}}: I^{n-1} \times Y_{a_n} \to Y_{a_0}$ in the following way. We define a partitioning of the cube I^{n-1} into 2^{n-1} smaller cubes. These cubes are defined for any sequence $0 < i_1 < \ldots < i_k < n$ and $0 \leq k \leq n-1$ by

$$\left\{(t, \ldots, t_{-}) \mid 0 \leq t \leq \frac{1}{2}, \ldots, 0 \leq t \leq \frac{1}{2} \; ; \; \frac{1}{2} \leq t \leq 1 \; , \; i \notin i, \ldots, i\right\}.$$

On this part of the cube I^{n-1} the map $q_{\underline{a}}: I^{n-1} \times Y_{a_n} \to Y_{a_0}$, for the sequence $\underline{a} = (a_0, a_1, \ldots, a_n)$, $a_0 < a_1 < \ldots < a_n$, is defined by

$$q_{\underline{a}}(t_1, \ldots, t_{n-1}, x) =$$

$$= f_{a_0} p_{a_0 a_1 \ldots a_{i_1}}(2t_1, \ldots, 2t_{i_1-1}, H_{a_{i_1}}(2t_{i_1} - 1, p_{a_{i_1} \ldots a_{i_2}}(2t_{i_1+1}, \ldots$$

$$\ldots, 2t_{i_2-1}, H_{a_{i_2}}(2t_{i_2} - 1, \ldots, p_{a_{i_k} \ldots a_n}(2t_{i_k+1}, \ldots, 2t_{n-1}, g_{a_n}(x)))))$$

We have to verify that these maps are well defined. For $t_{i_\ell} = \frac{1}{2}$, $1 \leq \ell \leq n-1$, that follows from the equalities

$$p_{a_{i_{\ell-1}}} \ldots a_{i_\ell}(2t_{i_{\ell-1}+1}, \ldots, 2t_{i_\ell-1}, H_{a_{i_\ell}}(0, p_{a_{i_\ell} \ldots a_{i_{\ell+1}}}(2t_{i_\ell+1}, \ldots, 2t_{i_{\ell+1}-1}, y) =$$

$$= p_{a_{i_{\ell-1}} \ldots a_{i_\ell}}(2t_{i_{\ell-1}+1}, \ldots, 2t_{i_\ell-1}, p_{a_{i_\ell} \ldots a_{i_{\ell+1}}}(2t_{i_\ell+1}, \ldots, 2t_{i_{\ell+1}-1}, y)$$

$$= p_{a_{i_{\ell-1}} \ldots a_{i_{\ell+1}}}(2t_{i_{\ell-1}+1}, \ldots, 2t_{i_\ell-1}, 1, 2t_{i_\ell+1}, \ldots, 2t_{i_{\ell+1}-1}, y)$$

Similarly, for $t_j = \frac{1}{2}$, $i_\ell < j < i_{\ell+1}$, that follows from the following equalities

$$p_{a_{i_\ell}} \ldots a_{i_{\ell+1}}(2t_{i_\ell+1}, \ldots, 2t_{j-1}, 1, 2t_{j+1}, \ldots, 2t_{i_{\ell+1}-1}, y) =$$

$$= p_{a_{i_\ell} \ldots a_j}(2t_{i_\ell+1}, \ldots, 2t_{j-1}, p_{a_j \ldots a_{i_{\ell+1}}}(2t_{j+1}, \ldots, 2t_{i_{\ell+1}-1}, y)$$

$$= p_{a_{i_\ell} \ldots a_j}(2t_{i_\ell+1}, \ldots, 2t_{j-1}, H_{a_{i_j}}(0, p_{a_j \ldots a_{i_{\ell+1}}}(2t_{j+1}, \ldots, 2t_{i_{\ell+1}-1}, y)$$

For $t_{i_\ell} = 1$, $1 \leq \ell \leq n-1$, from

$$p_{a_{i_{\ell-1}} \ldots a_{i_\ell}}(2t_{i_{\ell-1}+1}, \ldots, 2t_{i_\ell-1}, H_{a_{i_\ell}}(1, p_{a_{i_\ell} \ldots a_{i_{\ell+1}}}(2t_{i_\ell+1}, \ldots, 2t_{i_{\ell+1}-1}, y) =$$

$$= p_{a_{i_{\ell-1}} \ldots a_{i_\ell}}(2t_{i_{\ell-1}+1}, \ldots, 2t_{i_\ell-1}, g_{i_\ell} f_{i_\ell} p_{a_{i_\ell} \ldots a_{i_{\ell+1}}}(2t_{i_\ell+1}, \ldots, 2t_{i_{\ell+1}-1}, y)$$

it follows that

$$q_{\underline{a}}(t_1,\ldots,t_{i_\ell-1},1,t_{i_\ell+1},\ldots,t_{n-1},x)=$$

$$=q_{a_0\ldots a_{i_\ell}}(t_1,\ldots,t_{i_\ell-1},q_{a_{i_\ell}\ldots a_n}(t_{i_\ell+1},\ldots,t_{n-1},x)).$$

For $t_j=0$, $i_\ell<j<i_{\ell+1}$, from

$$p_{a_{i_\ell}\ldots a_{i_{\ell+1}}}(2t_{i_\ell+1},\ldots,2t_{j-1},0,2t_{j+1},\ldots,2t_{i_{\ell+1}-1},y)=$$

$$=p_{a_{i_\ell}\ldots\hat{a}_j\ldots a_{i_{\ell+1}}}(2t_{i_\ell+1},\ldots,2t_{j-1},2t_{j+1},\ldots,2t_{i_{\ell+1}-1},y)$$

it follows that

$$q_{\underline{a}}(t_1,\ldots,t_{j-1},0,t_{j+1},\ldots,t_{n-1},x)=$$

$$=q_{a_0\ldots\hat{a}_j\ldots a_n}(t_1,\ldots,t_{j-1},t_{j+1},\ldots,t_{n-1},x).$$

By this, it is verified that the triple $(Y_a,q_{\underline{a}},A)$ is a coherent inverse system over a directed set A.

Now, exactly as in [7], Theorem 1, we prove that $\underline{Y}=(Y_n,q_{n,m})$ and $(X_n,p_{n,m})$ are isomorphic in the level category of strong fundamental sequences. Then, from [7], Theorem 2 it follows that $(Y_m,q_{\underline{m}})$ and $(X_m,p_{\underline{m}})$ are isomorphic in the level coherent category Coh^N, and in the coherent category Coh.

Theorem 2. Let $(X_m,p_{\underline{m}})$ and $(Y_m,q_{\underline{m}})$ be arbitrary coherent inverse sequences and $f=(f^j_{\underline{m}})\colon(X_n,p_{\underline{n}})\to(Y_m,q_{\underline{m}})$ a level coherent map. If the maps $f_m\colon X_m\to Y_m$, and $f'_m\colon X_m\to Y_m$ are homotopic by a homotopy $F_m\colon X_m\to Y_m$, then there exists a level coherent map $f'=(f'^j_{\underline{m}})\colon(X_m,p_{\underline{m}})\to(Y_m,q_{\underline{m}})$ and a level coherent homotopy $F=(F^j_{\underline{m}})\colon(X_m,p_{\underline{m}})\to(Y_m,q_{\underline{m}})$ which connects coherent maps f and f'.

Proof: By [7], Theorem 2, there is a category isomorphism between level coherent category Coh^N and the level category of coherent inverse systems as objects and strong fundamental sequences as morphisms. This functor leaves the objects fixed and associates to the coherent homotopy class of coherent map $(f^j_{\underline{m}})$ the strong homotopy class of the strong fundamental sequence $(f_m,f_{m,m+1})$.

To prove the theorem it is enough to construct the strong fundamental sequence $(f'_m,f'_{m,m+1})$, and a strong level homotopy $(F_m,F_{m,m+1})$ connecting $(f_m,f_{m,m+1})$ and $(f'_m,f'_{m,m+1})$.

We define a map $F_{m,m+1}: I \times I \times X_m \to Y_m$, first on $((I \times 0) \cup (0, 1 \times I)) \times X_m$ by

$$F_{m,m+1}(t, 0, x) = f_{m,m+1}(t, x)$$

$$F_{m,m+1}(1, s, x) = q_{m,m+1} F_m(s, x)$$

$$F_{m,m+1}(0, s, x) = F_m(s, p_{m,m+1}(x)) .$$

The map is well defined on points $(0, 0, x)$, $x \in X_n$, since

$$f_{m,m+1}(0, x) = f_m p_{m,m+1}(x) = F_m(0, p_{m,m+1}(x)) ,$$

and on points $(1, 0, x)$, $x \in X_m$, since

$$f_{m,m+1}(1, x) = q_{m,m+1} f_m(x) = q_{m,m+1} F_m(x) .$$

Since there is a retraction $I \times I \to (I \times 0) \cup (0, 1 \times I)$ we can extend the map to a map $F_{m,m+1}: I \times I \times X_m \to Y_m$.

We define a map $f'_{m,m+1}: I \times X_m \to Y_m$ by putting

$$f'_{m,m+1}(t, x) = F_{m,m+1}(t, 1, x) .$$

Then from the definition, the map $F_{m,m+1}: I \times I \times X_m \to Y_m$ connects the maps $f_{m,m+1}$ and $f'_{m,m+1}$ and is a strong fundamental sequence. From

$$f'_{m,m+1}(0, x) = F_{m,m+1}(0, 1, x) = F_m(1, p_{m,m+1}(x)) = f'_m p_{m,m+1}(x)$$

and

$$f'_{m,m+1}(1, x) = F_{m,m+1}(1, 1, x) = q_{m,m+1} F_{m+1}(1, x) = q_{m,m+1} f'_{m,m+1}(x)$$

one concludes that $(f'_m, f'_{m,m+1})$ is a strong fundamental sequence. It follows the pair $(F_m, F_{m,m+1})$ is a strong homotopy connecting the strong fundamental sequences $(f_m, f_{m,m+1})$ and $(f'_m, f'_{m,m+1})$.

References:

[1] J. M. Cordier and J.Porter: *Maps between homotopy coherent diagrams*, Topology Appl. **28**, 255-275 (1988).

[2] R. Fritcsh: *A functorial description of coherent mappings between inverse systems*, Glasnik Mat. **34**(54), 19-22 (1999).

[3] Ю. Лисица: *О точности последоватеьности спектральньих гомотопических групп в теории шейпов*, Доклады Академии Наук СССР, **236**, Но 1, 23-26 (1977), (Soviet Math. Dokl. **18**, 1186-1190 (1977).

[4] Ю. Лисица: *Теоремы двойственности и двойственные категории шейпов и кошейпов*, Доклады Академии Наук СССР, **263**, No 1, 532-536 (1981).

[5] Yu. Lisica, S. Mardešic: *Coherent homotopy and strong shape for compact metric spaces*, Glasnik Mat. Vol. **20** (**40**), 159-167 (1985).

[6] N. Shekutkovski: *Equivalence of coherent theories*, Topology Appl. **75**, 113-123 (1997).

[7] N. Shekutkovski: *From inverse to coherent systems*, Topology Appl. (2000) Special Issue - Proceedings of Conference in Dubrovnik, 1998, October 4-11, pp. 1-17.

[8] R. M. Vogt: *A note on homotopy equivalences*, Proc. Amer. Math. Soc. **32**, 627-629 (1972).

ПРИЛОГ КОН ТЕОРЕМАТА ЗА НИВО ХОМОТОПСКИ ЕКВИВАЛЕНЦИИ

Никита Шекутковски

Резиме

Во [7] е докажан следниот резултат. Ако членовите на инверзна низа од тополошки простори се заменат со хомотопски еквивалентни простори, од новите простори може да се формира кохерентна инверзна низа таква што двете инврзни низи да се изоморфни во кохерентната категорија. Во овој труд резултатот е подобрен, со замена на комутативната инверзна низа $\underline{X}$ со произволна кохерентна инверзна низа. Покажан е и следниот резултат: во произволни кохерентни инверзни низи $(X_m, p_{\underline{m}})$ и $(Y_m, q_{\underline{m}})$, ако пресликувањата $f_m : X_m \to Y_m$ и $f'_m : X_m \to Y_m$ се хомотопни и $f : (X_m, p_{\underline{m}}) \to (Y_m, q_{\underline{m}})$ е кохерентно пресликување за пресликувањата $f_m : X_m \to Y_m$, тогаш постои кохерентно пресликување f' за пресликувањата $f'_m : X_m \to Y_m$ такво што f и f' се кохерентно хомотопни.

Institute of Mathematics

Faculty of Mathematics and Natural Sciences

St. Cyril and Methodius University

P.O. Box 162, 1000 Skopje,

MACEDONIA

e-mail: nikita@iunona.pmf.ukim.edu.mk