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COMMON FIXED POINTS IN b-DISLOCATED METRIC SPACES USING (E.A) PROPERTY

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Abstract. In this paper, we prove coincidence and common fixed point results for one a pair of mappings that satisfy the (E.A) property and its generalized variants in the setup of b-dislocated metric spaces. Our results generalize and extend some existing results in the literature.

1. INTRODUCTION

The study of metric fixed point theory in b-metric space was introduced and studied by Bakhtin [4] and Czerwik [10]. After that a series of papers have been published with interesting results about fixed point and common fixed points for different classes of mappings such as single value and multi valued, involving a single map, two mappings, compatible and weakly compatible mappings in the framework of B-metric spaces. One another generalization is dislocated metric spaces considered by P. Hitzler and A. K. Seda in [5] who introduced this metric as a generalization of usual metric, and generalized the Banach contraction principle on this space. Further many papers has been given as in references [2,6,7,11,13,14,15].

Recently a generalization of b-metric space and dislocated metric space such as b-dislocated metric spaces was introduced and studied by N. Hussain et.al [7]. Also in [7] are presented some topological aspects and properties of b-dislocated metrics. Subsequently, some fixed point and common fixed point results have been investigated for different types of contractions in these spaces.

On the other hand, (E:A)property was introduced in 2002 by Aamri and Moutaawakil in [18]. Later, some authors employed this concept to obtain some new fixed point results, can see ([19, 20, 21, 22]).

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In this paper, we prove results for a pair of mappings which satisfy the (E.A) and (E.A) Like property in b-dislocated metric spaces. We generalize some coincidence and fixed point theorems for mappings using the concepts of weakly compatible pair of mappings, as well as by using ψ-contractive conditions and linear type in a class of spaces such as b- dislocated metric spaces.

2. PRELIMINARIES

Definition 2.1 [6]. Let X be a nonempty set and a mapping $d_l : X \times X \to [0,\infty)$ is called a dislocated metric (or simply d_l-metric) if the following conditions hold for any $x, y, z \in X$:

i. If $d_l(x,y) = 0$, then $x = y$

ii. $d_l(x,y) = d_l(y,x)$

iii. $d_l(x,y) \le d_l(x,z) + d_l(z,y)$

The pair (X, d_l) is called a dislocated metric space (or d-metric space for short). Note that when $x = y$, $d_l(x,y)$ may not be 0.

Definition 2.2[8]. Let X be a nonempty set and a mapping $b_d : X \times X \to [0,\infty)$ is called a b-dislocated metric (or simply b_d-dislocated metric) if the following conditions hold for any $x, y, z \in X$ and $s \ge 1$:

a. If $b_d(x,y) = 0$, then $x = y$

b. $b_d(x,y) = b_d(y,x)$

c. $b_d(x,y) \le s[b_d(x,z) + b_d(z,y)]$

The pair (X, b_d) is called a b-dislocated metric space. And the class of b-dislocated metric space is larger than that of dislocated metric spaces, since a b-dislocated metric is a dislocated metric when $s = 1$.

Example 2.3. If $X = R$, then $d_l(x,y) = |x| + |y|$ defines a dislocated metric on X.

Definition 2.4 [7]Let (X, b_d) a b_d-metric space, and $\{x_n\}$ be a sequence of points in X. A point $x \in X$ is said to be the limit of the sequence $\{x_n\}$ if

$\lim_{n\to\infty} b_d(x_n, x) = 0$ and we say that the sequence $\{x_n\}$ is b_d-convergent to x and denote it by $x_n \to x$ as $n \to \infty$.

The limit of a b_d-convergent sequence in a b_d-metric space is unique [8, Proposition 1.27].

Definition 2.5 [7]. A sequence $\{x_n\}$ in a b_d-metric space (X, b_d) is called a b_d-Cauchy sequence iff given $\varepsilon > 0$, there exists $n_0 \in N$ such that for all $n, m > n_0$, we have $b_d(x_n, x_m) < \varepsilon$ or $\lim_{n,m\to\infty} b_d(x_n, x_m) = 0$.

Every b_d-convergent sequence in a b_d-metric space is a b_d-Cauchy sequence.

Definition 2.6 [7]. A b_d-metric space (X, b_d) is called complete if every b_d-Cauchy sequence in X is b_d-convergent.

Definition 2.7 [20]. Let f and g be two self mappings on a metric space (X, d). The mappings f and g are said to be compatible if

$$\lim_{n\to\infty} d(fgx_n, gfx_n) = \lim_{n\to\infty} d(fx_n, gx_n) = 0$$

whenever $\{x_n\}$ is a sequence in X such that $\lim_{n\to\infty} fx_n = \lim_{n\to\infty} gx_n = z$, for some $z \in X$.

Definition 2.8 [23]. Let f and g be self mappings of a set X. Then, f and g are said to be weakly compatible if they commute at their coincidence point; that is $fx = gx$ for some $x \in X$ implies $gfx = fgx$.

Some examples in the literature shows that in general a b-dislocated metric is not continuous.

Lemma 2.9 [7]. Let (X, b_d) be a b-dislocated metric space with parameter $s \geq 1$. Suppose that $\{x_n\}$ and $\{y_n\}$ are b_d-convergent to $x, y \in X$, respectively. Then we have

$$\frac{1}{s^2} b_d(x, y) \leq \lim_{n\to\infty} \inf b_d(x_n, y_n) \leq \lim_{n\to\infty} \sup b_d(x_n, y_n) \leq s^2 b_d(x, y)$$

In particular, if $b_d(x, y) = 0$, then we have $\lim_{n\to\infty} b_d(x_n, y_n) = 0 = b_d(x, y)$.

Moreover, for each $z \in X$, we have

$$\frac{1}{s} b_d(x,z) \leq \lim_{n\to\infty} \inf b_d(x_n,z) \leq \lim_{n\to\infty} \sup b_d(x_n,z) \leq s b_d(x,z)$$

In particular, if $b_d(x,z)=0$, then we have $\lim_{n\to\infty} b_d(x_n,z) = 0 = b_d(x,z)$.

Example 2.10. If $X = \mathbb{R}^+ \cup \{0\}$, then the function $b_d(x,y) = (x+y)^2$ defines a b -dislocated metric on X with parameter $s = 2$.

Consistent with [18,19] are the following definitions in a b-dislocated metric space.

Definition 2.11. Let X be a b-dislocated metric space. Selfmaps f and g on X are said to satisfy the (E.A)-property if there exists a sequence $\{x_n\}$ in X such that $\{fx_n\}$ and $\{gx_n\}$ are b_d convergent to some $t \in X$ and $b_d(t,t) = 0$, (equivalently $\lim_{n\to\infty} b_d(fx_n,t) = \lim_{n\to\infty} b_d(gx_n,t) = b_d(t,t) = 0$).

Definition 2.12. Let f and g be two self-mappings of a b -dislocated metric space (X, b_d) . We say that f and g satisfy the (E. A) Like property if there exists a sequence (x_n) such that $\{fx_n\}$ and $\{gx_n\}$ are b_d convergent to t , for some $t \in f(X)$ or $t \in g(X)$, i.e. $t \in f(X) \cup g(X)$ and $b_d(t,t) = 0$.

Remark. From the definitions 2.9-2.10, it is evident that a pair (f,g) satisfying the (E.A) like property always enjoys the property (E.A) but the implication is not reversible.

Definition 2.13 [6]. Let f and g be two self-mappings on a non-empty set X then,

(1) Any point $x \in X$ is said to be fixed point of f if $fx = x$.

(2) Any point $x \in X$ is called coincidence point of f and g if $fx = gx$, and we called $u = fx = gx$ is a point of coincidence of f and g .

(3) A point $x \in X$ is called common fixed point of f and g if $fx = gx = x$.

3. MAIN RESULT

In this section, some common fixed point results for two mappings satisfying "max" type of contractive conditions and by using altering distance functions $\psi \in \Psi$, in the framework of a b-dislocated metric space, are obtained.

Let Ψ denote the set of all continuous and non decreasing functions $\psi:[0,\infty)\to[0,\infty)$ such that $\psi(t)=0$ iff $t=0$, and we start with the following theorem.

Theorem 3.1 Let (X,b_d) be a b-dislocated -metric space with parameter $s\geq 1$ and $f,g:X\to X$ are two self mappings such that for all $x,y\in X$, constant $0\leq c<1$ and $\psi\in\Psi$,

$$\psi(2s^2 b_d(fx,fy)) \leq c\psi(\max\{b_d(gx,gy), b_d(fx,gx), b_d(fy,gy), \frac{b_d(fx,gy)+b_d(gx,fy)}{2s}\}) \tag{3.1}$$

Suppose that the pair (f,g) satisfies (E.A) Like property in X . Then the pair (f,g) has a common point of coincidence in X . Moreover if the pair (f,g) is weakly compatible then f and g have a unique common fixed point in X .

Proof. Since f and g satisfy the E. A. Like Property therefore exists a sequence $\{x_n\}$ in X such that $\lim_{n\to\infty} fx_n = \lim_{n\to\infty} gx_n = t$ for some $t\in f(X)$ or $g(X)$.

Assume that $\lim_{n\to\infty} fx_n = t\in g(X)$. Therefore, $t = gu$ for some $u\in X$.

From condition (3.1) we have:

$$\psi(2s^2 b_d(fu,fx_n)) \leq c\psi(\max\{b_d(gu,gx_n), b_d(fu,gu), b_d(fx_n,gx_n), \frac{b_d(fu,gx_n)+b_d(gu,fx_n)}{2s}\}) \tag{3.2}$$

Taking the upper limit as $n\to\infty$ using lemma 2.9 and definition 2.11, we get

$$\psi(2sb_d(fu,t)) = \psi(2s^2 \tfrac{1}{s} b_d(fu,t)) \leq \psi(2s^2 \limsup_{n\to\infty} b_d(fu,fx_n))$$

$$\leq c\psi(\limsup_{n\to\infty} \max\{b_d(t,t), b_d(fu,t), sb_d(t,t), \frac{sb_d(fu,t)+b_d(t,t)}{2s}\})$$

$$\leq c\psi(\max\{0, b_d(fu,t), 0, \frac{b_d(fu,t)}{2}\})$$

As a result we have,

$$\psi(2sb_d(fu,t)) \leq c\psi(b_d(fu,t)) . \tag{3.3}$$

By property of ψ, since $0<c<1$ and $s\geq 1$ the above inequality implies $\psi(b_d(fu,t))=0$ that is $fu=t$.

Therefore we have that u is a coincidence point of f and g ($fu=gu=t$).

The weak compatibility of f and g implies that,

$$ft = fgu = gfu = gt$$

Let we show that t is a fixed point of f. According to the condition 3.1, consider:

$$\psi(2s^2 b_d(ft, fx_n)) \leq c\psi(\max\{b_d(gt, gx_n), b_d(ft, gt), b_d(fx_n, gx_n), \frac{b_d(ft, gx_n)+b_d(gt, fx_n)}{2s}\}) \tag{3.4}$$

Taking the upper limit as $n\to\infty$ and using lemma 2.9, we get

$$\begin{aligned}\psi(2sb_d(ft,t)) = \psi(2s^2 \tfrac{1}{s} b_d(ft,t)) &\leq \psi(2s^2 \limsup_{n\to\infty} b_d(ft, fx_n)) \\ &\leq c\psi(\limsup_{n\to\infty} \max\{b_d(gt, gx_n), b_d(ft, gt), b_d(fx_n, gx_n), \\ &\qquad \frac{b_d(ft, gx_n)+b_d(gt, fx_n)}{2s}\}) \qquad (3.5) \\ &\leq c\psi(\max\{sb_d(ft,t), b_d(ft, ft), 0, \frac{sb_d(ft,t)+0}{2s}\}) \\ &\leq c\psi(2sb_d(ft,t))\end{aligned}$$

This inequality implies $\psi(2sb_d(ft,t))=0$, and as result $ft=gt=t$. Hence, t is a common fixed point of f and g.

Uniqueness. Let $t\neq t_1$ be two common fixed points of the mappings f and g.

Then from (3.1) we have:

$$\begin{aligned}\psi(2sb_d(ft, ft_1)) &\leq \psi(2s^2 b_d(ft, ft_1)) \\ &\leq c\psi(\max\{b_d(gt, gt_1), b_d(ft, gt), b_d(ft_1, gt_1), \frac{b_d(ft, gt_1)+b_d(gt, ft_1)}{2s}\}) \qquad (3.6) \\ &= c\psi(\max\{b_d(t,t_1), b_d(t,t) b_d(t_1,t_1), \frac{b_d(t,t_1)+b_d(t,t_1)}{2s}\}) \\ &\leq c\psi(2sb_d(t,t_1))\end{aligned}$$

This inequality implies that $\psi(2sb_d(t,t_1))=0$, since $0\leq c<1$. we get, $t=t_1$. Hence the proof is complete.

The following example illustrates theorem.

Example 3.2 Let $X=[0,1]$ and $b_d(x,y)=(x+y)^2$ for all $x,y\in X$ is a b-dislocated metric on X. Then (X,b_d) be a b-dislocated metric space. We take the function $\psi(t)=t$ and define the mappings

$$fx=\begin{cases}\frac{1}{10}x, & if\ x\in[0,1)\\ \frac{1}{12}, & if\ x=1\end{cases} \quad \text{and } gx=\frac{1}{2}x.$$

If we consider the sequence $\{x_n\}$, where $x_n=\frac{1}{n}$ for all $n\in N$ it is clear that f, g satisfy (E.A) Like property $\lim_{n\to\infty} fx_n=\lim_{n\to\infty} gx_n=0$ for $0\in f(X)$ or $g(X)$.

For $x,y\in[0,1)$ we have

$$\begin{aligned}2s^2b_d(fx,fy)&=8b_d(\tfrac{1}{10}x,\tfrac{1}{10}y)=8(\tfrac{1}{10}x+\tfrac{1}{10}y)^2\\&=\tfrac{8}{25}(\tfrac{1}{2}x+\tfrac{1}{2}y)^2\\&\le \alpha b_d(gx,gy)\end{aligned}$$

For $y<x=1$ we have

$$\begin{aligned}2s^2b_d(f1,fy)&=8b_d(\tfrac{1}{12},\tfrac{y}{10})=8(\tfrac{1}{12}+\tfrac{y}{10})^2\le 8(\tfrac{1}{10}+\tfrac{y}{10})^2\le \tfrac{8}{25}(\tfrac{1}{2}+\tfrac{y}{2})^2\\&=\tfrac{8}{25}b_d(g1,gy)\le \alpha b_d(g1,gy)=\alpha b_d(gx,gy)\end{aligned}$$

For $x<y=1$ we have

$$\begin{aligned}2s^2b_d(fx,f1)&=8b_d(\tfrac{x}{10},\tfrac{1}{12})=8(\tfrac{x}{10}+\tfrac{1}{12})^2\le \tfrac{8}{25}(\tfrac{x}{2}+\tfrac{1}{2})^2\\&=\tfrac{8}{25}b_d(gx,g1)\le \alpha b_d(gx,g1)=\alpha b_d(gx,gy)\end{aligned}$$

For $y=x=1$ we have

$$\begin{aligned}2s^2b_d(f1,f1)&=8b_d(\tfrac{1}{12},\tfrac{1}{12})=8(\tfrac{1}{12}+\tfrac{1}{12})^2\le \tfrac{8}{25}(\tfrac{1}{2}+\tfrac{1}{2})^2\\&=\tfrac{8}{25}b_d(g1,g1)\le \alpha b_d(gx,gy)\end{aligned}$$

As a result we have that,

$$\begin{aligned}2s^2b_d(fx,fy)&\le \tfrac{8}{25}b_d(gx,gy)\\&\le c\max\{b_d(gx,gy),b_d(gx,fx),b_d(gy,fy),\tfrac{b_d(fx,gy)+b_d(fy,gx)}{2s}\}\end{aligned}$$

holds for all $x,y\in X$, $0\le c<\frac{1}{2}$ and obviously $x=0$ is the unique common fixed point of f and g.

Corollary 3.3. Let (X,b_d) be a b-dislocated -metric space with parameter $s\ge 1$ and $f,g:X\to X$ are two self mappings such that for all $x,y\in X$, constant $0\le c<1$,

$$2s^2b_d(fx,fy)\le c\max\{b_d(gx,gy),b_d(fx,gx),b_d(fy,gy),\tfrac{b_d(fx,gy)+b_d(gx,fy)}{2s}\}$$

Suppose that the pair (f,g) satisfies (E.A) Like property in X . Then the pair (f,g) has a common point of coincidence in X . Moreover if the pair (f,g) is weakly compatible then f and g have a unique common fixed point in X .

Proof. Taking the altering distance function $\psi(t)=t$ (identity function) in theorem 3.1.

Theorem 3.4. Let (X,b_d) be a complete b-dislocated metric space with parameter $s\geq 1$ and $f,g:X\to X$ are two self mappings with $f(X)\subseteq g(X)$, such that satisfy

$$\psi(s^2 b_d(fx,fy))\leq c\psi(\max\{b_d(gx,gy),b_d(fx,gx),b_d(fy,gy),\frac{b_d(fx,gy)+b_d(gx,fy)}{2s}\})$$

for all $x,y\in X$, where $0\leq c<1$ and $\psi\in\Psi$. Suppose that the pair (f,g) satisfies (E.A) property and $g(X)$ is b_d -closed in X . Then the pair (f,g) has a common point of coincidence in X . Moreover if the pair (f,g) is weakly compatible then f and g have a unique common fixed point in X .

Proof. Since f and g satisfy the E.A. property, therefore there exists a sequence $\{x_n\}$ in X such that $\lim_{n\to\infty} fx_n = \lim_{n\to\infty} gx_n = t$ for some $t\in X$. As $g(X)$ is a b_d -closed subspace of X; therefore, every convergent sequence of points of $g(X)$ has a limit in $g(X)$. Therefore,

$$t=\lim_{n\to\infty} fx_n = \lim_{n\to\infty} gx_n = gu \text{ for some } u\in X$$

This implies $t=gu\in g(X)$ and in this conditions the pair (f,g) satisfies (E.A) Like property and the proof follows from theorem 3.1.

Theorem 3.5. Let (X,b_d) be a b-dislocated -metric space with parameter $s\geq 1$ and $f,g:X\to X$ are two self mappings such that,

$$s^2 b_d(fx,fy)\leq \alpha b_d(gx,fy)+\beta b_d(gx,gy)+\gamma b_d(gy,fy)+\delta b_d(gx,fx) \quad (3.7)$$

for all $x,y\in X$ where the constants $\alpha,\beta,\gamma,\delta$ are non negative and $0\leq \alpha+\beta+\gamma+\delta<\frac{1}{2}$.

Suppose that the pair (f,g) satisfies (E.A) Like property in X . Then the pair (f,g) has a common point of coincidence in X . Moreover if the pair (f,g) is weakly compatible then f and g have a unique common fixed point in X .

Proof. Since f and g satisfy the (E. A.) Like Property, therefore exists a sequence $\{x_n\}$ in X such that $\lim_{n\to\infty} fx_n = \lim_{n\to\infty} gx_n = gu$ for some $u \in X$.

Assume that $\lim_{n\to\infty} fx_n = t \in g(X)$. Therefore, $t = gu$ for some $u \in X$.

From condition (3.7) we have:

$$s^2 b_d(fu, fx_n) \le \alpha b_d(gu, gx_n) + \beta b_d(fu, gu) + \gamma b_d(fx_n, gx_n) + \delta b_d(fu, gx_n) \quad (3.8)$$

Taking the upper limit as $n \to \infty$ in (3.8), and using lemma 2.9 we get

$$\begin{aligned} sb_d(fu,t) &= s^2 \cdot \tfrac{1}{s} b_d(fu,t) \le \alpha \cdot 0 + \beta s b_d(fu,t) + \gamma \cdot 0 + \delta s b_d(fu,t) \\ &= (\beta+\delta) s b_d(fu,t) \qquad (3.9) \\ &\le (\alpha+\beta+\gamma+\delta) s b_d(fu,t) \end{aligned}$$

From this inequality since $0 \le c < \frac{1}{2}$ and $s \ge 1$ have $b_d(t, fu) = 0$ implies $fu = t$.

Therefore we have that u is a coincidence point of f and g ($fu = gu = t$).

The weak compatibility of f and g implies that,

$$ft = fgu = gfu = gt$$

Let we show that t is a common fixed point of f. According to the condition 3.7, consider:

$$s^2 b_d(ft, fx_n) \le \alpha b_d(gt, gx_n) + \beta b_d(ft, gt) + \gamma b_d(fx_n, gx_n) + \delta b_d(ft, gx_n) \quad (3.10)$$

Taking the upper limit as $n \to \infty$ we get

$$\begin{aligned} sb_d(ft,t) &= s^2 \tfrac{1}{s} b_d(ft,t) \le \alpha s b_d(ft,t) + \beta b_d(ft, ft) + \gamma \cdot 0 + \delta s b_d(ft,t) \\ &\le (\alpha + 2\beta + \gamma + \delta) s b_d(ft,t) \end{aligned}$$

Since $0 \le \alpha+\beta+\gamma+\delta < \frac{1}{2}$ and $s \ge 1$ this inequality implies $b_d(ft,t) = 0$, and as result $ft = gt = t$. Hence, t is a common fixed point of f and g.

Corollary 3.6. Let (X, b_d) be a complete b-dislocated metric space with parameter $s \ge 1$ and $f, g : X \to X$ are two self mappings with $f(X) \subseteq g(X)$, such that satisfy

$$s^2 b_d(fx, fy) \le k[b_d(gx, fy) + b_d(gx, gy) + b_d(gy, fy) + b_d(gx, fx)]$$

for all $x, y \in X$, where the constant $0 < k < 1$. Suppose that the pair (f, g) satisfies (E.A) property and $g(X)$ is b_d -closed in X. Then the pair (f, g) has a common point of coincidence in X. Moreover if the pair (f, g) is weakly compatible then f and g have a unique common fixed point in X.

Theorem 3.7. Let (X,b_d) be a b-dislocated -metric space with parameter $s \geq 1$ and $f,g : X \to X$ are two self mappings such that,

$$s^2 b_d(fx, fy) \leq \alpha[b_d(fx,gy)+b_d(gx,fy)]+\beta[b_d(fx,gy)+b_d(gx,gy)] \\ +\gamma[b_d(gx,fy)+b_d(gx,gy)] \quad (3.11)$$

for all $x,y \in X$ where the constants $\alpha,\beta,\gamma,\delta > 0$ are non negative and $0 \leq \alpha+\beta+\gamma < \frac{1}{2}$.

Suppose that the pair (f,g) satisfies (E.A) Like property in X. Then the pair (f,g) has a common point of coincidence in X. Moreover if the pair (f,g) is weakly compatible then f and g have a unique common fixed point in X.

Proof. Since f and g satisfy the E. A. Like Property therefore exists a sequence $\{x_n\}$ in X such that $\lim_{n\to\infty} fx_n = \lim_{n\to\infty} gx_n = z \in f(X)$ or $g(X)$.

Assume that $\lim_{n\to\infty} fx_n = z \in g(X)$. Therefore, $z = gu$ for some $u \in X$.

From condition (3.11) we have:

$$s^2 d(fu, fx_n) \leq \alpha[d(fu,gx_n)+d(gu,fx_n)]+\beta[d(fu,gx_n)+d(gu,gx_n)] \\ +\gamma[d(gu,fx_n)+d(gu,gx_n)[\ ] \quad (3.12)$$

Taking limit as $n \to \infty$, we get

$$sd(fu,z) = s^2 \cdot \tfrac{1}{s} d(fu,z) \leq \alpha[sd(fu,z)+0]+\beta[sd(fu,z)+0]+\gamma[0+0]$$
$$= (\alpha+\beta)d(fu,z)$$
$$\leq (2\alpha+2\beta+2\gamma)d(fu,z)$$

From this inequality have

$$d(fu,z) \leq \frac{2\alpha+2\beta+2\gamma}{s} d(fu,z) \quad (3.13).$$

By (3.13) we get $d(fu,z) = 0$ since $0 \leq \frac{2\alpha+2\beta+2\gamma}{s} < 1$.

By property d_2 have $fu = z$. Hence $fu = gu = z$. Using the weak compatibility we get $fz = gz$.

Let we show that $fz = z$. Again consider:

$$d(fz, fx_n) \leq \alpha[d(fz,gx_n)+d(gz,fx_n)]+\beta[d(fz,gx_n)+d(gz,gx_n)] \\ +\gamma[d(gz,fx_n)+d(gz,gx_n)]$$

Taking the upper limit as $n \to \infty$, we get

$$\begin{aligned} sd(fz,z) &= s^2 \cdot \tfrac{1}{s} d(fz,z) \le \alpha[sd(fz,z)+sd(gz,z)] + \beta[sd(fz,z)+sd(gz,z)] \\ &\quad +\gamma[d(gz,z)+d(gz,z)] \\ &= \alpha[sd(fz,z)+sd(fz,z)] + \beta[sd(fz,z)+sd(fz,z)] \\ &\quad +\gamma[sd(fz,z)+sd(fz,z)] \\ &\le (2\alpha+2\beta+2\gamma)sd(fz,z) \end{aligned}$$

From this we have $d(fz,z)=0$ since $0 \le \alpha+\beta+\gamma < \frac{1}{2}$. Therefore $d(fz,z)=0$ $\Rightarrow fz = z$.

So $fz = z = gz$. Hence, z is a common fixed point of f and g.

Uniqueness. Clearly, as in theorem 3.1 we can show that fixed point is unique.

Remark 3.8 As a consequence of theorem 3.1 and 3.3 for taking

1) the parameter $s=1$
2) the parameter $s=1$ and the identity mapping $fx = x$
3) the parameter $s=1$ and the function $\psi(t)=t$;

we can establish many other corollaries in the setting of dislocated metric spaces

COMPETING INTERESTS

Authors have declared that no competing interests exist.

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