COMMON FIXED POINTS IN b-DISLOCATED METRIC SPACES USING (E.A) PROPERTY

Kastriot Zoto¹, Ilir Vardhami², Jani Dine³ and Arben Isufati⁴

Abstract. In this paper, we prove coincidence and common fixed point results for one a pair of mappings that satisfy the (E.A) property and its generalized variants in the setup of b-dislocated metric spaces. Our results generalize and extend some existing results in the literature.

1. INTRODUCTION

The study of metric fixed point theory in b-metric space was introduced and studied by Bakhtin [4] and Czerwik [10]. After that a series of papers have been published with interesting results about fixed point and common fixed points for different classes of mappings such as single value and multi valued, involving a single map, two mappings, compatible and weakly compatible mappings in the framework of B-metric spaces. One another generalization is dislocated metric spaces considered by P. Hitzler and A. K. Seda in [5] who introduced this metric as a generalization of usual metric, and generalized the Banach contraction principle on this space. Further many papers has been given as in references [2,6,7,11,13,14,15].

Recently a generalization of b-metric space and dislocated metric space such as b-dislocated metric spaces was introduced and studied by N. Hussain et.al [7]. Also in [7] are presented some topological aspects and properties of b-dislocated metrics. Subsequently, some fixed point and common fixed point results have been investigated for different types of contractions in these spaces.

On the other hand, (E:A) property was introduced in 2002 by Aamri and Moutaawakil in [18]. Later, some authors employed this concept to obtain some new fixed point results, can see ([19, 20, 21, 22]).

2010 Mathematics Subject Classification. Primary: 47H10 Secondary: 55M20

Key words and phrases. (E.A) property; (E.A) Like property; b-dislocated metric space; weakly compatible maps; common fixed point.
In this paper, we prove results for a pair of mappings which satisfy the (E.A) and (E.A) Like property in \( b \)-dislocated metric spaces. We generalize some coincidence and fixed point theorems for mappings using the concepts of weakly compatible pair of mappings, as well as by using \( \psi \) -contractive conditions and linear type in a class of spaces such as \( b \)-dislocated metric spaces.

2. Preliminaries

**Definition 2.1** [6]. Let \( X \) be a nonempty set and a mapping \( d_I : X \times X \to [0, \infty) \) is called a dislocated metric (or simply \( d_I \)-metric) if the following conditions hold for any \( x, y, z \in X \):

i. If \( d_I(x, y) = 0 \), then \( x = y \)

ii. \( d_I(x, y) = d_I(y, x) \)

iii. \( d_I(x, y) \leq d_I(x, z) + d_I(z, y) \)

The pair \((X, d_I)\) is called a dislocated metric space (or \( d \)-metric space for short). Note that when \( x = y \), \( d_I(x, y) \) may not be 0.

**Definition 2.2**[8]. Let \( X \) be a nonempty set and a mapping \( b_d : X \times X \to [0, \infty) \) is called a \( b \)-dislocated metric (or simply \( b_d \)-dislocated metric) if the following conditions hold for any \( x, y, z \in X \) and \( s \geq 1 \):

a. If \( b_d(x, y) = 0 \), then \( x = y \)

b. \( b_d(x, y) = b_d(y, x) \)

c. \( b_d(x, y) \leq s[b_d(x, z) + b_d(z, y)] \)

The pair \((X, b_d)\) is called a \( b \)-dislocated metric space. And the class of \( b \)-dislocated metric space is larger than that of dislocated metric spaces, since a \( b \)-dislocated metric is a dislocated metric when \( s = 1 \).

**Example 2.3.** If \( X = \mathbb{R} \), then \( d_I(x, y) = |x| + |y| \) defines a dislocated metric on \( X \).

**Definition 2.4** [7] Let \((X, b_d)\) a \( b_d \)-metric space, and \( \{x_n\} \) be a sequence of points in \( X \). A point \( x \in X \) is said to be the limit of the sequence \( \{x_n\} \) if
Common fixed points in $b$-dislocated metric spaces using (E. A)...

\[ \lim_{n \to \infty} b_d(x_n, x) = 0 \] and we say that the sequence \( \{x_n\} \) is \( b_d \)-convergent to \( x \) and denote it by \( x_n \to x \) as \( n \to \infty \).

The limit of a \( b_d \)-convergent sequence in a \( b_d \)-metric space is unique [8, Proposition 1.27].

**Definition 2.5** [7]. A sequence \( \{x_n\} \) in a \( b_d \)-metric space \( (X, b_d) \) is called a \( b_d \)-Cauchy sequence iff given \( \varepsilon > 0 \), there exists \( n_0 \in \mathbb{N} \) such that for all \( n, m > n_0 \), we have \( b_d(x_n, x_m) < \varepsilon \) or \( \lim_{n, m \to \infty} b_d(x_n, x_m) = 0 \).

Every \( b_d \)-convergent sequence in a \( b_d \)-metric space is a \( b_d \)-Cauchy sequence.

**Definition 2.6** [7]. A \( b_d \)-metric space \( (X, b_d) \) is called complete if every \( b_d \)-Cauchy sequence in \( X \) is \( b_d \)-convergent.

**Definition 2.7** [20]. Let \( f \) and \( g \) be two self mappings on a metric space \( (X, d) \). The mappings \( f \) and \( g \) are said to be compatible if

\[ \lim_{n \to \infty} d(fgx_n, gfx_n) = \lim_{n \to \infty} d(fx_n, gx_n) = 0 \]

whenever \( \{x_n\} \) is a sequence in \( X \) such that \( \lim_{n \to \infty} fx_n = \lim_{n \to \infty} gx_n = z \), for some \( z \in X \).

**Definition 2.8** [23]. Let \( f \) and \( g \) be self mappings of a set \( X \). Then, \( f \) and \( g \) are said to be weakly compatible if they commute at their coincidence point; that is \( fx = gx \) for some \( x \in X \) implies \( gfx = fgx \).

Some examples in the literature shows that in general a \( b \)-dislocated metric is not continuous.

**Lemma 2.9** [7]. Let \( (X, b_d) \) be a \( b \)-dislocated metric space with parameter \( s \geq 1 \). Suppose that \( \{x_n\} \) and \( \{y_n\} \) are \( b_d \)-convergent to \( x, y \in X \), respectively. Then we have

\[ \frac{1}{s^2} b_d(x, y) \leq \lim_{n \to \infty} \inf b_d(x_n, y_n) \leq \lim_{n \to \infty} \sup b_d(x_n, y_n) \leq s^2 b_d(x, y) \]

In particular, if \( b_d(x, y) = 0 \), then we have \( \lim_{n \to \infty} b_d(x_n, y_n) = 0 = b_d(x, y) \).
Moreover, for each \( z \in X \), we have
\[
\frac{1}{s} b_d(x, z) \leq \liminf_{n \to \infty} b_d(x_n, z) \leq \limsup_{n \to \infty} b_d(x_n, z) \leq s b_d(x, z)
\]
In particular, if \( b_d(x, z) = 0 \), then we have \( \lim_{n \to \infty} b_d(x_n, z) = 0 = b_d(x, z) \).

**Example 2.10.** If \( X = \mathbb{R}^+ \cup \{0\} \), then the function \( b_d(x, y) = (x + y)^2 \) defines a \( b \)-dislocated metric on \( X \) with parameter \( s = 2 \).

Consistent with [18,19] are the following definitions in a \( b \)-dislocated metric space.

**Definition 2.11.** Let \( X \) be a \( b \)-dislocated metric space. Selfmaps \( f \) and \( g \) on \( X \) are said to satisfy the (E.A)-property if there exists a sequence \( \{x_n\} \) in \( X \) such that \( \{fx_n\} \) and \( \{gx_n\} \) are \( b_d \) convergent to some \( t \in X \) and \( b_d(t, t) = 0 \), (equivalently \( \lim_{n \to \infty} b_d(fx_n, t) = \lim_{n \to \infty} b_d(gx_n, t) = b_d(t, t) = 0 \)).

**Definition 2.12.** Let \( f \) and \( g \) be two self-mappings of a \( b \)-dislocated metric space \((X, b_d)\). We say that \( f \) and \( g \) satisfy the (E.A) Like property if there exists a sequence \((x_n)\) such that \( \{fx_n\} \) and \( \{gx_n\} \) are \( b_d \) convergent to \( t \), for some \( t \in f(X) \) or \( t \in g(X) \), i.e. \( t \in f(X) \cup g(X) \) and \( b_d(t, t) = 0 \).

**Remark.** From the definitions 2.9-2.10, it is evident that a pair \((f, g)\) satisfying the (E.A) like property always enjoys the property (E.A) but the implication is not reversible.

**Definition 2.13 [6].** Let \( f \) and \( g \) be two self-mappings on a non-empty set \( X \) then,

1. Any point \( x \in X \) is said to be fixed point of \( f \) if \( fx = x \).
2. Any point \( x \in X \) is called coincidence point of \( f \) and \( g \) if \( fx = gx \), and we called \( u = fx = gx \) is a point of coincidence of \( f \) and \( g \).
3. A point \( x \in X \) is called common fixed point of \( f \) and \( g \) if \( fx = gx = x \).
3. Main Result

In this section, some common fixed point results for two mappings satisfying “max” type of contractive conditions and by using altering distance functions \( \psi \in \Psi \), in the framework of a \( b \)-dislocated metric space, are obtained.

Let \( \Psi \) denote the set of all continuous and non decreasing functions \( \psi : [0, \infty) \to [0, \infty) \) such that \( \psi(t) = 0 \) iff \( t = 0 \), and we start with the following theorem.

**Theorem 3.1** Let \( (X, b_d) \) be a \( b \)-dislocated -metric space with parameter \( s \geq 1 \) and \( f, g : X \to X \) are two self mappings such that for all \( x, y \in X \), constant \( 0 \leq c < 1 \) and \( \psi \in \Psi \),

\[
\psi(2s^2b_d(fx, fy)) \leq c\psi(\max\{b_d(gx, gy), b_d(fx, gx), b_d(fy, gy), \\
\frac{b_d(fx, gy) + b_d(gx, fy)}{2s}\})
\]  
(3.1)

Suppose that the pair \( (f, g) \) satisfies (E.A) Like property in \( X \). Then the pair \( (f, g) \) has a common point of coincidence in \( X \). Moreover if the pair \( (f, g) \) is weakly compatible then \( f \) and \( g \) have a unique common fixed point in \( X \).

**Proof.** Since \( f \) and \( g \) satisfy the E. A. Like Property therefore exists a sequence \( \{x_n\} \) in \( X \) such that \( \lim_{n \to \infty} fx_n = \lim_{n \to \infty} gx_n = t \) for some \( t \in f(X) \) or \( g(X) \).

Assume that \( \lim_{n \to \infty} fx_n = t \in g(X) \). Therefore, \( t = gu \) for some \( u \in X \).

From condition (3.1) we have:

\[
\psi(2s^2b_d(fu, fx_n)) \leq c\psi(\max\{b_d(gu, gx_n), b_d(fu, gu), b_d(fx_n, gx_n), \\
\frac{b_d(fu, gx_n) + b_d(gu, fx_n)}{2s}\})
\]  
(3.2)

Taking the upper limit as \( n \to \infty \) using lemma 2.9 and definition 2.11, we get

\[
\psi(2sb_d(fu, t)) = \psi(2s^2\frac{1}{s} b_d(fu, t)) \leq \psi(2s^2 \lim_{n \to \infty} \sup b_d(fu, fx_n))
\]  
\[
\leq c\psi(\lim_{n \to \infty} \sup \max\{b_d(t, t), b_d(fu, t), sb_d(t, t), \frac{sb_d(fu, t) + b_d(t, t)}{2s}\})
\]  
\[
\leq c\psi(\max\{0, b_d(fu, t), 0, \frac{b_d(fu, t)}{2}\})
\]

As a result we have,

\[
\psi(2sb_d(fu, t)) \leq c\psi(b_d(fu, t)).
\]  
(3.3)
By property of $\psi$, since $0 < c < 1$ and $s \geq 1$ the above inequality implies
$\psi(b_d(fu,t)) = 0$ that is $fu = t$.
Therefore we have that $u$ is a coincidence point of $f$ and $g$ ($fu = gu = t$).
The weak compatibility of $f$ and $g$ implies that,
$$ ft = fgu = gfu = gt $$
Let we show that $t$ is a fixed point of $f$. According to the condition 3.1, consider:
$$ \psi(2s^2b_d(ft,fx_n)) \leq c\psi(\max\{b_d(gt,gx_n),b_d(ft,gt),b_d(fx_n,gx_n),
\frac{b_d(ft,gx_n)+b_d(gt,fx_n)}{2s}\}) $$
(3.4)
Taking the upper limit as $n \to \infty$ and using lemma 2.9, we get
$$ \psi(2sb_d(ft,t)) = \psi(2s^2\frac{1}{s}b_d(ft,t)) \leq \psi(2s^2\limsup_{n\to\infty}b_d(ft,fx_n))
\leq c\psi(\limsup_{n\to\infty}\max\{b_d(gt,gx_n),b_d(ft,gt),b_d(fx_n,gx_n),
\frac{b_d(ft,gx_n)+b_d(gt,fx_n)}{2s}\}) $$
(3.5)
$$ \leq c\psi(\max\{sb_d(ft,t),b_d(ft,ft),0,\frac{sb_d(ft,t)+0}{2s}\})
\leq c\psi(2sb_d(ft,t)) $$
This inequality implies $\psi(2sb_d(ft,t)) = 0$, and as result $ft = gt = t$. Hence, $t$ is a common fixed point of $f$ and $g$.

**Uniqueness.** Let $t \neq t_1$ be two common fixed points of the mappings $f$ and $g$.
Then from (3.1) we have:
$$ \psi(2sb_d(ft,t_1)) \leq \psi(2s^2b_d(ft,ft_1)) $$
$$ \leq c\psi(\max\{b_d(gt,gt_1),b_d(ft,gt),b_d(ft_1,gt_1),\frac{b_d(ft,gt_1)+b_d(gt,ft_1)}{2s}\}) $$
(3.6)
$$ = c\psi(\max\{b_d(t,t_1),b_d(t,t_1),b_d(t,t_1),\frac{b_d(t,t_1)+b_d(t,t_1)}{2s}\})
\leq c\psi(2sb_d(t,t_1)) $$
This inequality implies that $\psi(2sb_d(t,t_1)) = 0$, since $0 \leq c < 1$. we get, $t = t_1$.
Hence the proof is complete.
The following example illustrates theorem.

**Example 3.2** Let $X = [0,1]$ and $b_d(x,y) = (x + y)^2$ for all $x, y \in X$ is a b-dislocated metric on $X$. Then $(X,b_d)$ be a b-dislocated metric space. We take the function $\psi(t) = t$ and define the mappings
Common fixed points in b-dislocated metric spaces using (E. A)...

\[ fx = \begin{cases} \frac{1}{10} x, & \text{if } x \in [0,1) \\ \frac{1}{12}, & \text{if } x = 1 \end{cases} \quad \text{and} \quad gx = \frac{1}{2} x. \]

If we consider the sequence \( \{x_n\} \), where \( x_n = \frac{1}{n} \) for all \( n \in \mathbb{N} \) it is clear that \( f \), \( g \) satisfy (E.A) Like property \( \lim_{n \to \infty} f x_n = \lim_{n \to \infty} g x_n = 0 \) for \( 0 \in f(X) \) or \( g(X) \).

For \( x, y \in [0,1) \) we have

\[
2s^2 b_d (fx, fy) = 8b_d \left( \frac{1}{10}, \frac{1}{10}, y \right) = 8 \left( \frac{1}{10} x + \frac{1}{10} y \right)^2 \\
= \frac{8}{25} \left( \frac{1}{2} x + \frac{1}{2} y \right)^2 \\
\leq ab_d (gx, gy)
\]

For \( y < x = 1 \) we have

\[
2s^2 b_d (f1, fy) = 8b_d \left( \frac{1}{12}, \frac{1}{10}, y \right) = 8 \left( \frac{1}{12} + \frac{y}{10} \right)^2 \\
\leq 8 \left( \frac{1}{10} + \frac{y}{10} \right)^2 \\
= \frac{8}{25} b_d (g1, gy) \leq ab_d (g1, gy) = ab_d (gx, gy)
\]

For \( x < y = 1 \) we have

\[
2s^2 b_d (fx, f1) = 8b_d \left( \frac{x}{10}, \frac{1}{12} \right) = 8 \left( \frac{x}{10} + \frac{1}{12} \right)^2 \\
\leq \frac{8}{25} \left( \frac{1}{2} x + \frac{1}{2} y \right)^2 \\
= \frac{8}{25} b_d (gx, g1) \leq ab_d (gx, g1) = ab_d (gx, gy)
\]

For \( y = x = 1 \) we have

\[
2s^2 b_d (f1, f1) = 8b_d \left( \frac{1}{12}, \frac{1}{12} \right) = 8 \left( \frac{1}{12} + \frac{1}{12} \right)^2 \\
\leq \frac{8}{25} \left( \frac{1}{2} x + \frac{1}{2} y \right)^2 \\
= \frac{8}{25} b_d (g1, g1) \leq ab_d (gx, gy)
\]

As a result we have that,

\[
2s^2 b_d (fx, fy) \leq \frac{8}{25} b_d (gx, gy) \\
\leq c \max \{b_d (gx, gy), b_d (gx, fx), b_d (gy, fy), \frac{b_d (fx, gy) + b_d (fy, gx)}{2s}\}
\]

holds for all \( x, y \in X \), \( 0 \leq c < \frac{1}{2} \) and obviously \( x = 0 \) is the unique common fixed point of \( f \) and \( g \).

**Corollary 3.3.** Let \( (X, b_d) \) be a b-dislocated -metric space with parameter \( s \geq 1 \) and \( f, g : X \to X \) are two self mappings such that for all \( x, y \in X \), constant \( 0 \leq c < 1 \),

\[
2s^2 b_d (fx, fy) \leq c \max \{b_d (gx, gy), b_d (fx, gx), b_d (fy, fy), \frac{b_d (fx, gy) + b_d (fy, gx)}{2s}\}
\]
Suppose that the pair \((f, g)\) satisfies (E.A) Like property in \(X\). Then the pair \((f, g)\) has a common point of coincidence in \(X\). Moreover if the pair \((f, g)\) is weakly compatible then \(f\) and \(g\) have a unique common fixed point in \(X\).

**Proof.** Taking the altering distance function \(\psi(t) = t\) (identity function) in theorem 3.1.

**Theorem 3.4.** Let \((X, b_d)\) be a complete b-dislocated metric space with parameter \(s \geq 1\) and \(f, g : X \rightarrow X\) are two self mappings with \(f(X) \subseteq g(X)\), such that satisfy
\[
\psi(s^2 b_d(fx, fy)) \leq c\psi(\max\{b_d(gx, gy), b_d(fx, gx), b_d(fy, gy), \frac{b_d(fx, gy) + b_d(gx, fy)}{2s}\})
\]
for all \(x, y \in X\), where \(0 \leq c < 1\) and \(\psi \in \Psi\). Suppose that the pair \((f, g)\) satisfies (E.A) property and \(g(X)\) is \(b_d\)-closed in \(X\). Then the pair \((f, g)\) has a common point of coincidence in \(X\). Moreover if the pair \((f, g)\) is weakly compatible then \(f\) and \(g\) have a unique common fixed point in \(X\).

**Proof.** Since \(f\) and \(g\) satisfy the E.A. property, therefore there exists a sequence \(\{x_n\}\) in \(X\) such that \(\lim_{n \to \infty} fx_n = \lim_{n \to \infty} gx_n = t\) for some \(t \in X\). As \(g(X)\) is a \(b_d\)-closed subspace of \(X\); therefore, every convergent sequence of points of \(g(X)\) has a limit in \(g(X)\). Therefore,
\[
t = \lim_{n \to \infty} fx_n = \lim_{n \to \infty} gx_n = gu \text{ for some } u \in X
\]
This implies \(t = gu \in g(X)\) and in this conditions the pair \((f, g)\) satisfies (E.A) Like property and the proof follows from theorem 3.1.

**Theorem 3.5.** Let \((X, b_d)\) be a b-dislocated -metric space with parameter \(s \geq 1\) and \(f, g : X \rightarrow X\) are two self mappings such that,
\[
s^2 b_d(fx, fy) \leq \alpha b_d(gx, fy) + \beta b_d(gx, gy) + \gamma b_d(gy, fy) + \delta b_d(gx, fx) \quad (3.7)
\]
for all \(x, y \in X\) where the constants \(\alpha, \beta, \gamma, \delta\) are non negative and \(0 \leq \alpha + \beta + \gamma + \delta < \frac{1}{2}\).

Suppose that the pair \((f, g)\) satisfies (E.A) Like property in \(X\). Then the pair \((f, g)\) has a common point of coincidence in \(X\). Moreover if the pair \((f, g)\) is weakly compatible then \(f\) and \(g\) have a unique common fixed point in \(X\).
**Proof.** Since \( f \) and \( g \) satisfy the (E. A.) Like Property, therefore exists a sequence \( \{x_n\} \) in \( X \) such that \( \lim_{n \to \infty} f x_n = \lim_{n \to \infty} g x_n = u \) for some \( u \in X \).

Assume that \( \lim_{n \to \infty} f x_n = t \in g(X) \). Therefore, \( t = u \) for some \( u \in X \).

From condition (3.7) we have:

\[
s^2 b_d(f u, f x_n) \leq \alpha b_d(g u, g x_n) + \beta b_d(f u, g u) + \gamma b_d(f x_n, g x_n) + \delta b_d(f u, g x_n)
\]

Taking the upper limit as \( n \to \infty \) in (3.8), and using lemma 2.9 we get

\[
\begin{align*}
\lim_{n \to \infty} s b_d(f u, f x_n) &= \lim_{n \to \infty} s^2 \frac{1}{s} b_d(f u, f x_n) \\
&= (\beta + \delta) s b_d(f u, f t) \\
&\leq (\alpha + \beta + \gamma + \delta) s b_d(f u, f t)
\end{align*}
\]

From this inequality since \( 0 \leq c < \frac{1}{2} \) and \( s \geq 1 \) have \( b_d(t, f u) = 0 \) implies \( f u = t \).

Therefore we have that \( u \) is a coincidence point of \( f \) and \( g \) (\( f u = g u = t \)).

The weak compatibility of \( f \) and \( g \) implies that,

\[
f t = f g u = g f u = g t
\]

Let we show that \( t \) is a common fixed point of \( f \). According to the condition 3.7, consider:

\[
s^2 b_d(f t, f x_n) \leq \alpha b_d(g t, g x_n) + \beta b_d(f t, g t) + \gamma b_d(f x_n, g x_n) + \delta b_d(f t, g x_n)
\]

Taking the upper limit as \( n \to \infty \) we get

\[
\begin{align*}
\lim_{n \to \infty} s b_d(f t, f x_n) &= \lim_{n \to \infty} s^2 \frac{1}{s} b_d(f t, f x_n) \\
&= (\beta + \delta) s b_d(f t, f t) \\
&\leq (\alpha + 2 \beta + \gamma + \delta) s b_d(f t, f t)
\end{align*}
\]

Since \( 0 \leq \alpha + \beta + \gamma + \delta < \frac{1}{2} \) and \( s \geq 1 \) this inequality implies \( b_d(f t, f t) = 0 \), and as result \( f t = g t = t \). Hence, \( t \) is a common fixed point of \( f \) and \( g \).

**Corollary 3.6.** Let \((X, b_d)\) be a complete b-dislocated metric space with parameter \( s \geq 1 \) and \( f, g : X \to X \) are two self mappings with \( f(X) \subseteq g(X) \), such that satisfy

\[
s^2 b_d(f x, f y) \leq k[b_d(g x, f y) + b_d(g x, g y) + b_d(g y, f y) + b_d(g x, f x)]
\]

for all \( x, y \in X \), where the constant \( 0 < k < 1 \). Suppose that the pair \((f, g)\) satisfies (E.A) property and \( g(X) \) is \( b_d \)-closed in \( X \). Then the pair \((f, g)\) has a common point of coincidence in \( X \). Moreover if the pair \((f, g)\) is weakly compatible then \( f \) and \( g \) have a unique common fixed point in \( X \).
Theorem 3.7. Let \((X, b_d)\) be a b-dislocated metric space with parameter \(s \geq 1\) and \(f, g : X \to X\) are two self mappings such that,

\[
s^2 b_d(fx, fy) \leq \alpha[b_d(fx, gy) + b_d(gx, fy)] + \beta[b_d(fx, gy) + b_d(gx, gy)] + \gamma[b_d(gx, fy) + b_d(gx, gy)] \tag{3.11}
\]

for all \(x, y \in X\) where the constants \(\alpha, \beta, \gamma, \delta > 0\) are non negative and \(0 \leq \alpha + \beta + \gamma < \frac{1}{2}\).

Suppose that the pair \((f, g)\) satisfies (E.A) Like property in \(X\). Then the pair \((f, g)\) has a common point of coincidence in \(X\). Moreover if the pair \((f, g)\) is weakly compatible then \(f\) and \(g\) have a unique common fixed point in \(X\).

**Proof.** Since \(f\) and \(g\) satisfy the E. A. Like Property therefore exists a sequence \(\{x_n\}\) in \(X\) such that \(\lim_{n \to \infty} fx_n = \lim_{n \to \infty} gx_n = z \in f(X)\) or \(g(X)\).

Assume that \(\lim_{n \to \infty} fx_n = z \in g(X)\). Therefore, \(z = gu\) for some \(u \in X\).

From condition (3.11) we have:

\[
s^2 d(fu, fx_n) \leq \alpha[d(fu, gx_n) + d(gu, fx_n)] + \beta[d(fu, gx_n) + d(gu, gx_n)] + \gamma[d(gu, fx_n) + d(gu, gx_n)] \tag{3.12}
\]

Taking limit as \(n \to \infty\), we get

\[
sd(fu, z) = s^2 \frac{1}{s} d(fu, z) \leq \alpha[sd(fu, z) + 0] + \beta[sd(fu, z) + 0] + \gamma[0 + 0]
\]

\[
= (\alpha + \beta)d(fu, z)
\]

\[
\leq (2\alpha + 2\beta + 2\gamma)d(fu, z)
\]

From this inequality have

\[
d(fu, z) \leq \frac{2\alpha + 2\beta + 2\gamma}{s} d(fu, z) \tag{3.13}
\]

By (3.13) we get \(d(fu, z) = 0\) since \(0 \leq \frac{2\alpha + 2\beta + 2\gamma}{s} < 1\).

By property \(d_2\) have \(fu = z\). Hence \(fu = gu = z\). Using the weak compatibility we get \(fz = gz\).

Let we show that \(fz = z\). Again consider:

\[
d(fz, fx_n) \leq \alpha[d(fz, gx_n) + d(gz, fx_n)] + \beta[d(fz, gx_n) + d(gz, gx_n)] + \gamma[d(gz, fx_n) + d(gz, gx_n)]
\]

Taking the upper limit as \(n \to \infty\), we get
\[ sd(fz, z) = s^2 \cdot \frac{1}{s} d(fz, z) \leq \alpha \{ sd(fz, z) + sd(gz, z) \} + \beta \{ sd(fz, z) + sd(gz, z) \} \]
\[ + \gamma \{ d(gz, z) + d(gz, z) \} \]
\[ = \alpha \{ sd(fz, z) + sd(fz, z) \} + \beta \{ sd(fz, z) + sd(fz, z) \} \]
\[ + \gamma \{ sd(fz, z) + sd(fz, z) \} \]
\[ \leq (2\alpha + 2\beta + 2\gamma)sd(fz, z) \]

From this we have \( d(fz, z) = 0 \) since \( 0 \leq \alpha + \beta + \gamma < \frac{1}{2} \). Therefore \( d(fz, z) = 0 \)
\[ \Rightarrow fz = z \]

So \( fz = z = gz \). Hence, \( z \) is a common fixed point of \( f \) and \( g \).

**Uniqueness.** Clearly, as in theorem 3.1 we can show that fixed point is unique.

**Remark 3.8** As a consequence of theorem 3.1 and 3.3 for taking
1) the parameter \( s = 1 \)
2) the parameter \( s = 1 \) and the identity mapping \( fx = x \)
3) the parameter \( s = 1 \) and the function \( \psi(t) = t \);
we can establish many other corollaries in the setting of dislocated metric spaces

**COMPETING INTERESTS**

Authors have declared that no competing interests exist.

**References**


[13] M. Arshad, A. Shoaib and P. Vetro; *Common fixed points of a pair of Hardy Rogers type mappings on a closed ball in ordered dislocated metric spaces*, Journal of function spaces and applications, vol 2013, article id 638181


\[1,3,4\) Faculty of Natural Sciences, University of Gjirokastra, Gjirokastra, Albania  
E-mail address: zotokastriot@yahoo.com  

\[2\) Faculty of Natural Sciences, University of Tirana, Tirana, Albania