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EXAMINATION OF THE STUDENTS' PREVIOUS KNOWLEDGE BEFORE AND AFTER THE FIRST YEAR OF SECONDARY EDUCATION

Jasmina Markoska[1)], Gjorgji Markoski[2)]

Abstract. The prerequisite knowledge of the students enrolled in the first year in the "Georgi Dimitrov" Skopje high school in the 2013/2014 and 2014/2015 school years was examined. The students enrolled came from 15 primary schools in Skopje. We believe that this examination is a good indicator of the quality of the knowledge necessary for understanding the mathematics curriculum in the first year of high school. A similar examination was made at the beginning of the mentioned school years to a group of students that started the second year of high school.

According to the mathematics curriculum for the first year of high school education from May 2001, in point 3 it is stated:

Prerequisite knowledge. For successful acquisition and mastering of the content of this subject, i.e. achieving the goals it prerequisite knowledge from the subjects mathematics in primary education is required.

How to measure the prerequisite knowledge of the students?

Most often by assigning problems and procedures to remain as a lasting knowledge and skills through teaching units and teaching in general. The simplest way of measuring the prerequisite knowledge of students are assignments in the form of diagnostic tests that the teacher administers at the beginning of each, or at least most of the curriculum topics. We think the best way of checking such prior knowledge is the beginning of the academic year, when students are free from the burden of regularly attending classes, reviewing of some basic procedures, recalling previously studied material etc. If our interest is permanent knowledge of the students, then the best period for measurement is after an extensive period of rest, such as the summer break.

How was choice of the problems for the diagnostic tests done?

In the absence of standardized tests for measurement, search was made through the content of primary school workbooks, textbooks, databases with problems for external examinations of students, online sources with similar measurements from other countries.

Thus the ideal choice should include those topics that are revisited and extended in the curriculum for the relevant year. For practical reasons, we chose

multiple choice problems, and in all tests there is one problem which requires the student to explain the procedure by which they came to the solution.
According to the mathematics curriculum for first year of high school, students, having 3 classes weekly, should master the following topics: According to the mathematics curriculum for first year of high school, students, having 3 classes weekly, should master the following topics:

1. Mathematical logic and sets
2. Basic sets of numbers
3. Algebraic Rational Expressions
4. Geometric figures in the plane
5. Proportionality
6. Linear functions, linear equations and inequalities
7. Systems of linear equations and inequalities
8. Powers and roots
9. Data analysis

Diagnostic testing of students in first year of high school in 2013/2014 and 2014/2015 school year is done through a test with 10 multiple choice problems with 5 offered choices, in two groups, due to the large number of students per class. We now present the problems from the test. After each problem there is a graphic representation of the chosen answers by students from both school years.

Problem 1

I group. Given the sets $A=\{a\,|\,a \text{ is even number from the third tenth}\}$ and $B=\{b\,|\,b\in\mathbb{N}, 10\le b<26\}$, then

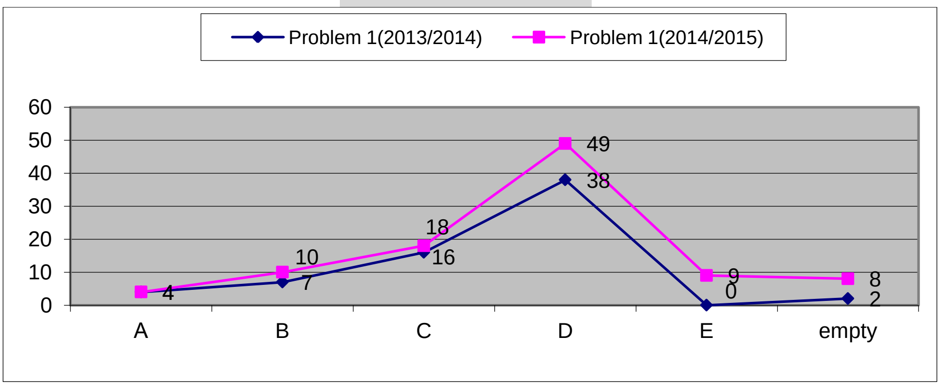
A) $A\cup B=\varnothing$ B) $A\cup B=B$ C) $A\cap B=\{22,24,26\}$

D) $A\cap B=\{20,22,24\}$ E) none of the above

II group. Given the sets $A=\{a\,|\,a \text{ is even number from the second tenth}\}$ and $B=\{b\,|\,b\in\mathbb{N}, 10\le b<26\}$, then

A) $A\cup B=\varnothing$ B) $A\cup B=B$ C) $A\cap B=\{12,14,16,18\}$

D) $A\cap B=\{10,12,14,16\}$ E) none of the above



Expected errors made by students are due to different representations of sets which leads to errors in operations with sets.

Problem 2

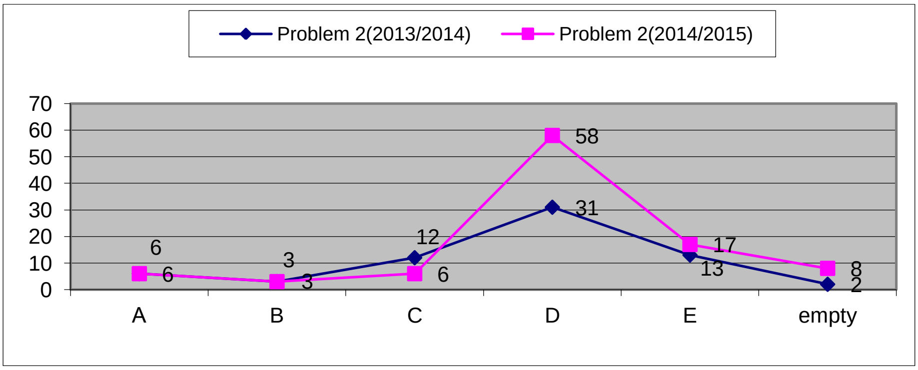
I group. Evaluate the expression $15:5\cdot 3-0:2+(3-10:2)=$

A) -3; B) 5; C) -1; D) 7; E) none of the above

II group. Evaluate the expression $12:4\cdot 3-0:5+(1-6:2)=$

A) -3; B) 5; C) -1; D) 7; E) none of the above

Expected errors $15:5\cdot 3=1$, $0:2=2$ and $3-10:2=(-7):2$



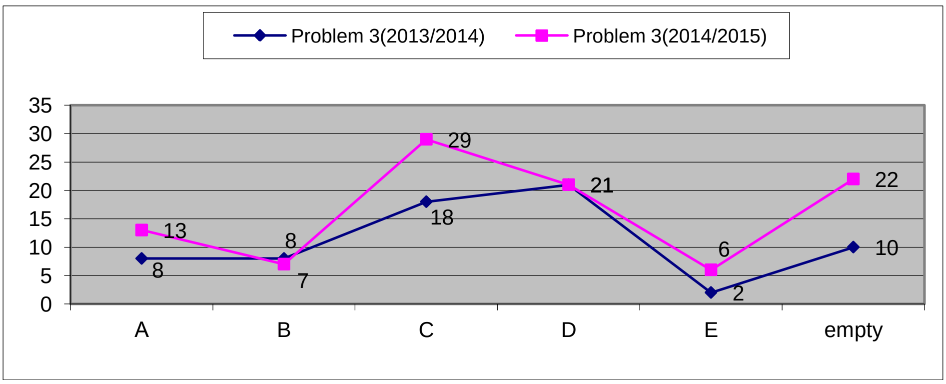
Problem 3.

I group. The sum of the complementary and the supplementary angles of the angle $\alpha=76^0$ is:

A) 118^0; B) 14^0; C) 180^0; D) 104^0; E) none of the above

II group. The sum of the complementary and the supplementary angles of the angle $\alpha=56^0$ is:

A) 158^0; B) 34^0; C) 180^0; D) 90^0; E) none of the above



Problem 4

I group. $(2x-y)(-2x-y)=$

A) $4x-y^2$; B) $4x^2-y^2$; C) $-4x^2+y^2$;

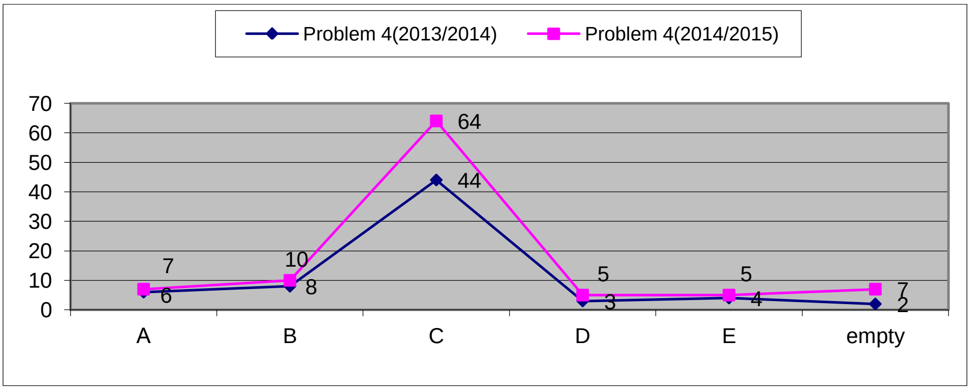
D) $4x^2-4xy+y^2$; E) none of the above

II group. $(3x-y)(-3x-y)=$

A) $9x-y^2$; B) $9x^2-y^2$; C) $-9x^2+y^2$;

D) $9x^2-6xy+y^2$; E) none of the above

Expected errors in performing operations with polynomials.



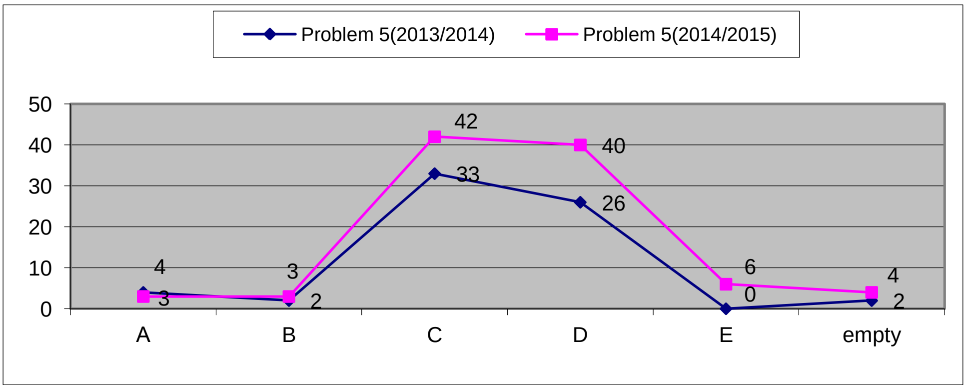
Problem 5

I group. If $2:3=x:6$, then

A) $x=1$ B) $x=2$ C) $x=3$ D) $x=4$ E) none of the above

II group. If $2:6=x:9$, then

A) $x=1$ B) $x=2$ C) $x=3$ D) $x=4$ E) none of the above



Problem 6

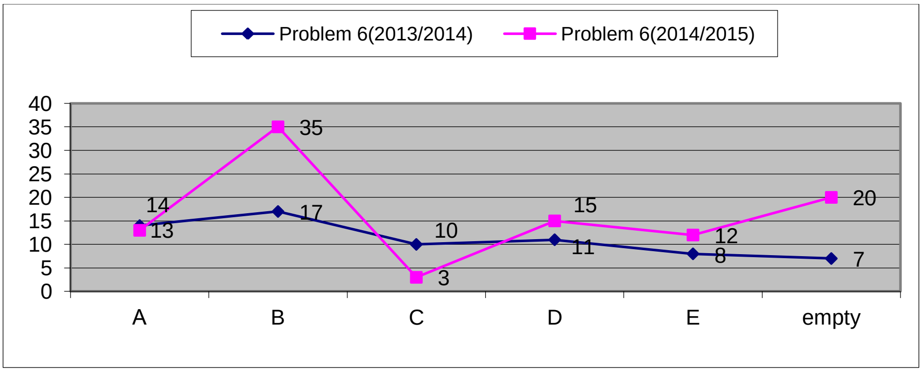
I group. What is the solution of the inequality $\frac{x}{3}-\frac{x+2}{6}<0,5$:

A) $x>0,25$ B) $x<1$ C) $x>-1$ D) $x<5$ E) none of the above

II group. What is the solution of inequality $\frac{x}{3}-\frac{x+2}{6}>0,5$:

A) $x<0,25$; B) $x<1$ C) $x>-1$ D) $x>5$ E) none of the above

Expected errors in simplifying the terms on the left hand side of the inequality.

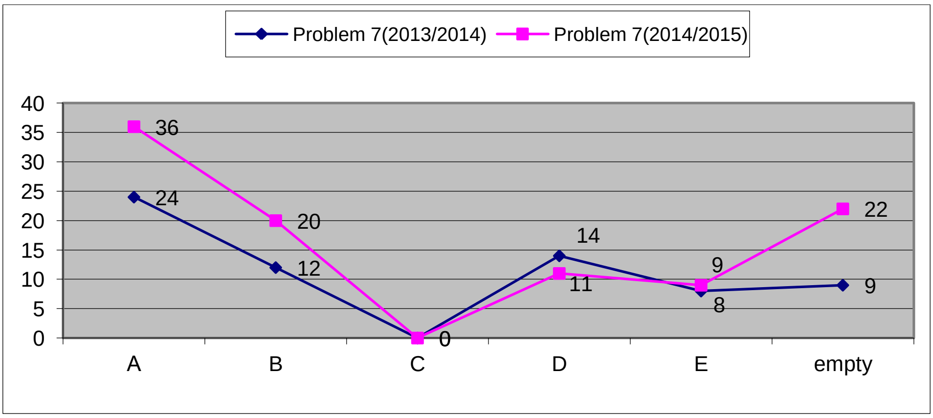


Problem 7

I and II group. During the summer break Trpe decided to read a book. On the first day he read 30% of the book, on the second day he read $\frac{2}{7}$ of the rest and on the third day he read the remaining 20 pages . How many pages does the book have?

A) 120 B) 100 C) 20 D) 60 ; E) none of the above

Expected errors in setting an equation under the given conditions and solving it.



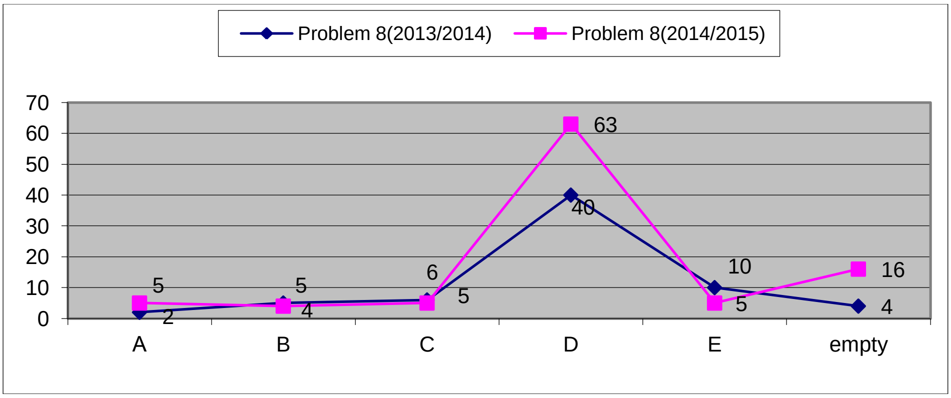
Problem 8

I group. If the ordered pair $(2,6)$ is a solution of the equation $(4x-2)k-1=y-k$, then $k=$

A) 10 ; B) 0 ; C) $\frac{3}{23}$; D) 1 ; E) none of the above

II group. If the ordered pair $(1,2)$ is a solution of the equation $(4x-2)k-1=y-k$, then $k=$

A) 10 ; B) 0 ; C) $\frac{3}{23}$; D) 1 ; E) none of the above



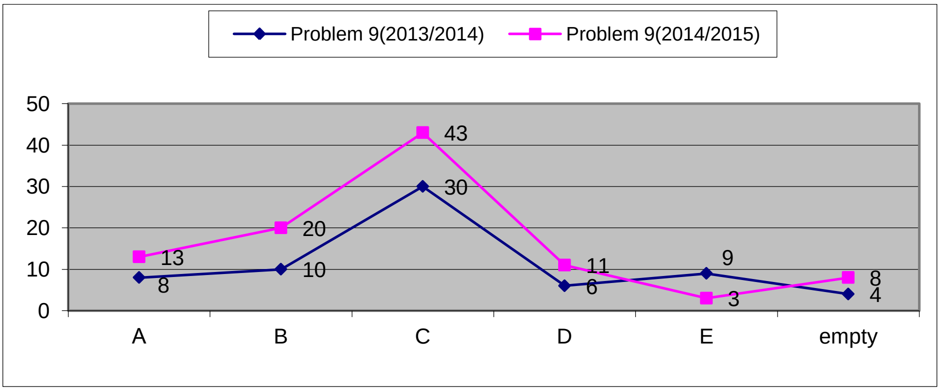
Problem 9.

I group. The number $235*74$ is divisible by 9 if $*$ is replaced with:

A) 0 B) 9 C) 6 D) 5 E) none of the above

II group. The number $745*23$ is divisible by 9 if $*$ is replaced with:

A) 0 B) 9 C) 6 D) 5 E) none of the above



Problem 10

I group. After simplifying all powers, the expression $\frac{(a^3 \cdot a^2)^5 \cdot a^7}{a^2 \cdot (a^4)^3}$ is equal to:

A) a^{18} B) a^8 C) a^{22} D) 1 E) none of the above

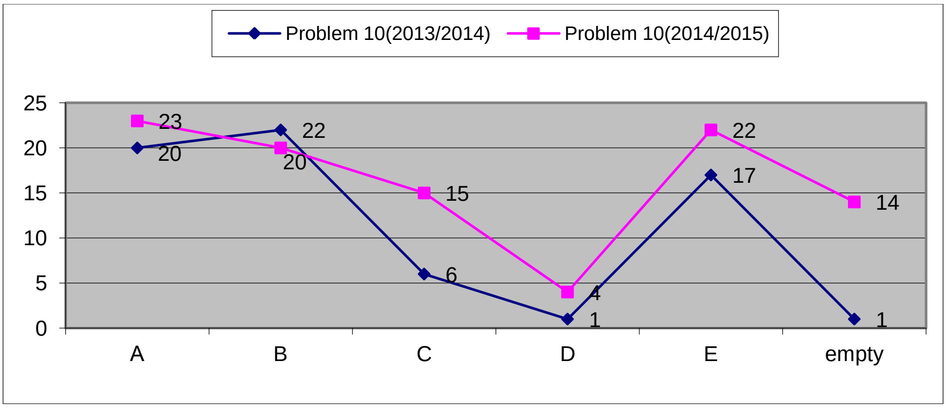
II group. After simplifying all powers, the expression $\frac{(a^3 \cdot a^2)^5 \cdot a^7}{a^2 \cdot (a^3)^4}$ is equal to:

A) a^{18}; B) a^8 C) a^{22} D) 1; E) none of the above

Expected errors include

$$a^2 \cdot (a^4)^3 = a^{2 \cdot 4^3} = a^{2 \cdot 64} = a^{128} \text{ or } a^2 \cdot (a^3)^4 = a^{2 \cdot 3^4} = a^{2 \cdot 81} = a^{162},$$

$$(a^3 \cdot a^2)^5 \cdot a^7 = a^{6 \cdot 5 \cdot 7} = a^{210} \text{ and finally } \frac{a^{210}}{a^{128}} = a^{210:128}, \text{ i.e. } \frac{a^{210}}{a^{162}} = a^{210:162}.$$



The following table shows the percentage of correctly solved problems.

number of problem	2013/2014-67 students	2014/2015-98 students
1	0%	9,18%
2	46,27%	59,18%
3	11,94%	13,27%
4	65,67%	65,31%
5	76,12%-51 students	74,49%-73 students
6	16,42%	15,31%
7	11,94%	9,18%
8	59,70%	64,29%
9	44,78%	43,88%
10	29,85%	23,47%

From the table, it can be concluded that problems 1, 3, 6 and 7 were solved by less than 20% of students . These are the main focus in the topics: Sets and operations with sets, angles and types of angles, linear inequalities in one variable, and word problems. Most of the students correctly solved problems 4, 5 and 8. Curiously, the students come from ten different primary schools and almost all have been excellent in mathematics in elementary education. Many of these students have confirmed the excellence through external examination.

According to the mathematics curriculum for the second year of high school education from May 2002, in point 3 it is stated:

Prerequisite knowledge. To achieve the set goals in of the course, prerequisite knowledge from the courses of mathematics in primary school and the first year of high school is required, particularly of the topics: *Integer powers*, *Rational algebraic expressions, Solving linear equations and problems that reduce to linear equations, Solving systems of linear equations in two variables and solving word problems that reduce to solving systems of linear equations in two variables, Solving linear inequalities, Solving system of linear inequalities in one variable, Working with data (presentation of data, mode, median, mean, range), Construction of triangles and quadrilaterals, Perimeter and area of plane figures Area and volume of solids.*

According to the mathematics curriculum for second year of high school, students, having 3 classes weekly, should master the following topics:
1. Right-triangle trigonometry
2. Complex numbers
3. Quadratic equations
4. Quadratic functions and quadratic inequalities
5. Construction of triangles and quadrilaterals
6. Area of plane figures
7. Elements of stereometry
8. Data analysis

Diagnostic testing of knowledge of students of the second year in 2014/2015 and 2015/2016 school was done through a test with 6 problems with 5 choices, 1 problem with 4 choices, and 3 open-ended problems. The test had two groups because of the large number of students per class. One of the open-ended problems is different in the two academic years so it is discarded in the analysis, although that sparked greatest interest among students to solve. All problems on the test are from topics already studied in primary school, extended in the course from the first year of high school. The level of the problems is higher, but in within curriculum and administered written examinations from the previous school year. We believe that the chosen problems can also be solved and students from primary school because they are from studied topics.

We now present the problems from the test. After each problem there is a graphic representation of the chosen answers by students from both school years.

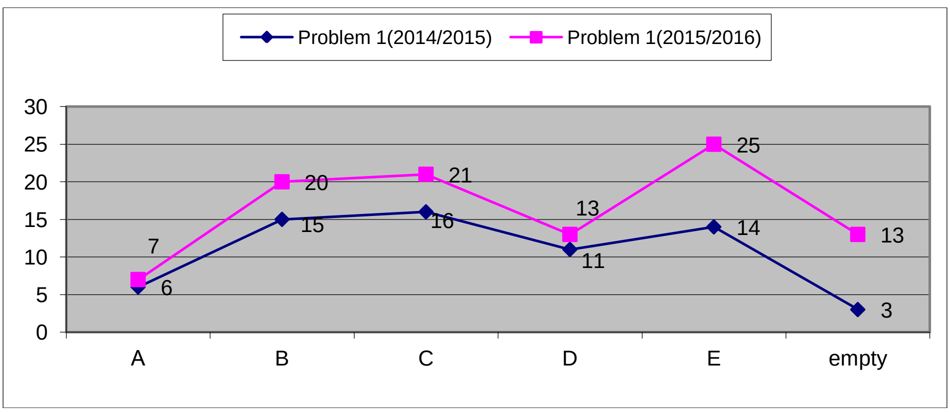
Problem 1

I group. Evaluate: $\frac{2^{12}}{2^6 \cdot 2^3} : \frac{8 \cdot 2^3 \cdot 2^5}{(2^3:4) \cdot 16}$

A) 1 B) 2 C) $\frac{1}{2}$ D) $\frac{1}{2^2}$ E) none of the above

II group. Evaluate $\frac{8 \cdot 2^3 \cdot 2^5}{(2^3:4) \cdot 16} : \frac{2^{12}}{2^6 \cdot 2^3}$

A) 1 B) 2 C) $\frac{1}{2}$ D) 2^2 E) none of the above



Problem 2.

I group. Dividing the solution of equation

$2x-2\cdot(1-2x+3\cdot(x-(x-4)))=x-1$ by 5 gives remainder of:

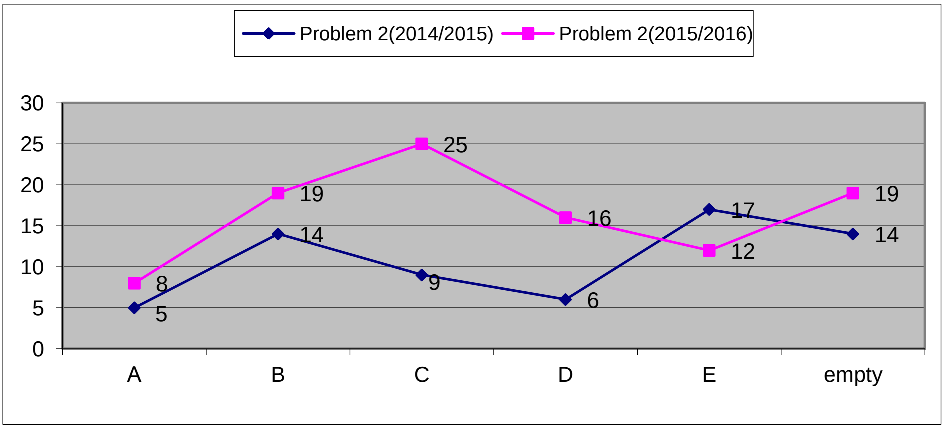
A) 4 B) 3 C) 2 D) 1 E) 0

II group. Dividing the solution of equation

$3x-3\cdot(1-2x+3\cdot(x-(x-4)))=x+1$ by 5 gives remainder of:

A) 4 B) 3 C) 2 D) 1 E) 0

Many students just solved the equation, but did not calculate the remainder. Our assumption is that they did not read the text of the problem carefully.



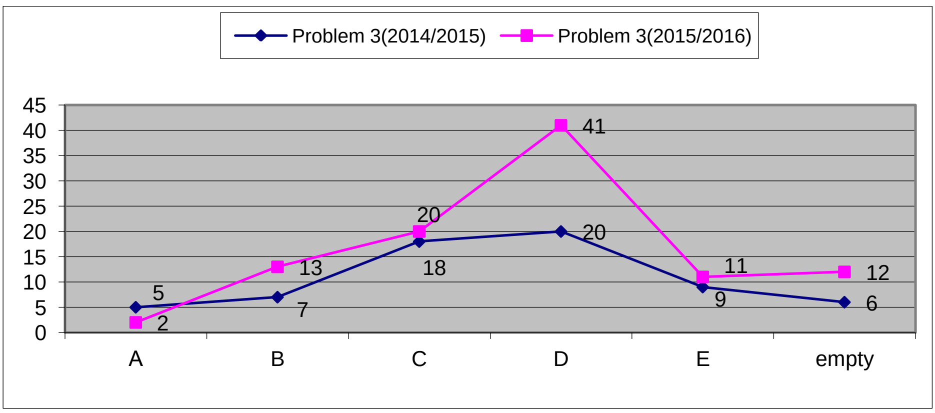
Problem 3

I group. If (x,y) is a solution of the system $\begin{cases}2x-y=10\\x+2y=15\end{cases}$, then $x-y=$

A) 1 B) 11 C) 7 D) 3 E) none of the above

II group. If (x,y) is a solution of the system $\begin{cases}2x-y=10\\x+2y=15\end{cases}$, then $x-y=$

A) 1 B) 2 C) 7 D) 11 E) none of the above



In this problem we have seen a number of students who solved just given system, but did not consider the additional request.

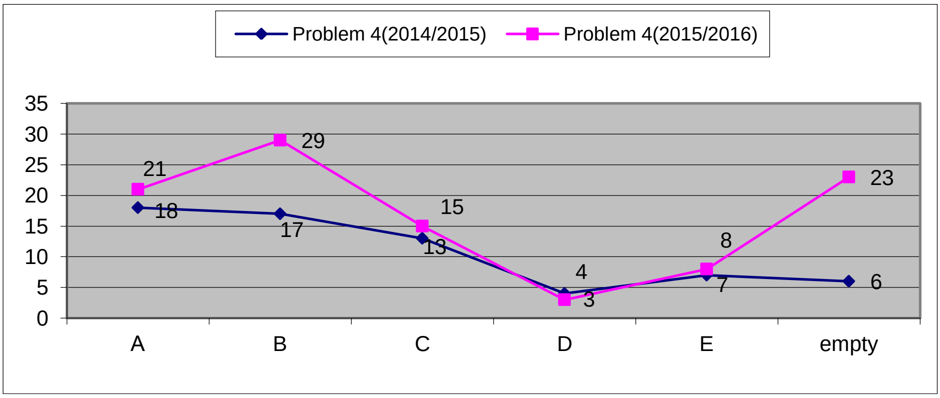
Problem 4

I group. The solution of the inequality $\frac{3x-1}{5}-\frac{x}{7}<1-\frac{x+1}{2}$ is an interval which contains:

A) 4 natural numbers B) 5 natural numbers C) 6 natural numbers
D) 7 natural numbers E) none of the above

II group. The solution of the inequality $\frac{3x-1}{5}-\frac{x+1}{2}<1-\frac{x}{7}$ is an interval which contains:

A) 4 natural numbers B) 5 natural numbers C) 6 natural numbers
D) 7 natural numbers E) none of the above



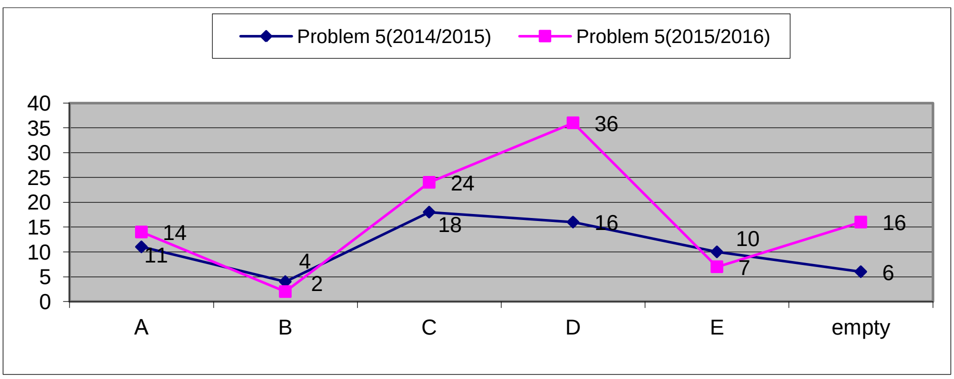
Problem 5

I group. A triangle can be constructed from segments whose length is:

A) 3cm, 3cm and 7cm B) 3cm, 4cm and 10cm
C) 2cm, 3cm and 5cm D) 3cm, 5cm and 7cm E) none of the above

II group. A triangle can be constructed from segments whose length is :

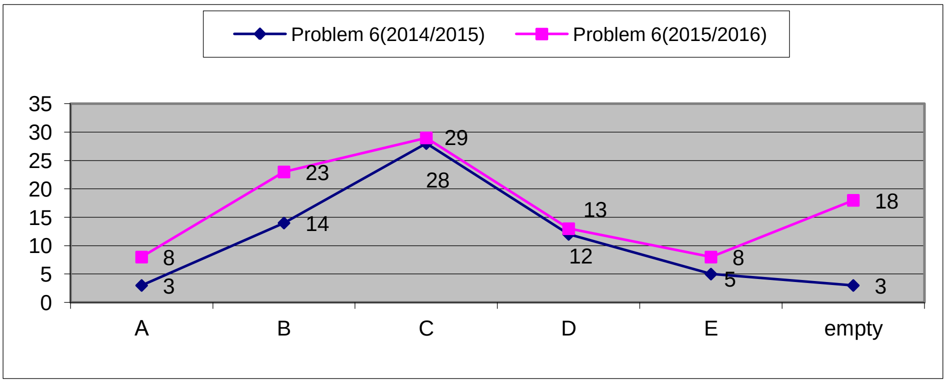
A) 4cm, 4cm and 9cm B) 4cm, 5cm and 10cm
C) 2cm, 3cm and 5cm D) 4cm, 5cm and 7cm E) none of the above



Problem 6

I and II group. The points $D((1,2))$, $A(3,-2)$, $B(5,-2)$, $C(5,2)$ are given on a coordinate plane. The area of the figure $ABCD$ is:

A) 10 B) 12 C) 14 D) 16 E) none of the above



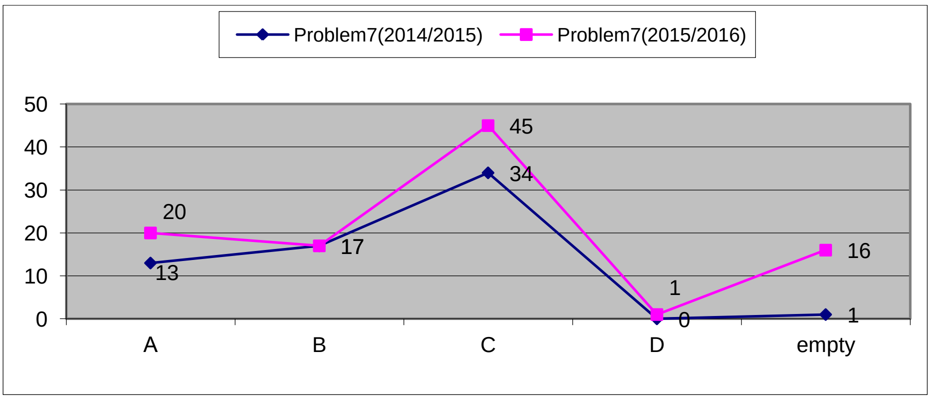
Problem 7

I group. If P is the magnitude of the area of a cube with a side 4cm and V is the magnitude of the volume of a cube with a side 4cm, then:

A) $P = V$ B) $P > V$ C) $P < V$ D) none of the above

II group. If P is the magnitude of the area of a cube with a side 6cm and V is the magnitude the volume of a cube with a side 6cm, then:

A) $P > V$ B) $P = V$ C) $P < V$ D) none of the above



The next two problems are open-ended.

Problem 8

I and II group. Jana has 15 banknotes of 10 denars, 4 banknotes of 50 denars and 3 banknotes of 100 denars. How should Jana exchange the money in banknotes of 10 denars and 100 denars so that she gets exactly one banknote of 100 denars more then banknotes of 10 denars?

Problem 9

I group. Simplify the expression $(1-\frac{b-a+1}{a+b})\cdot(a-\frac{2a^2+b}{2a-1})$.

II group. Simplify the expression $(\frac{b-a+1}{a+b}-1)\cdot(\frac{2a^2+b}{2a-1}-a)$.

number of problem	2014/2015-65 students	2015/2016-99 students
1	20,9%	25,25%
2	25,37%	12,12%
3	29,85%	41,41%
4	15,38%	11,11%
5	23,88%	36,36%
6	20,9%	23,23%
7	25,37%	17,17%
8	10,77%	19,19%
9	1,54%	1,01%

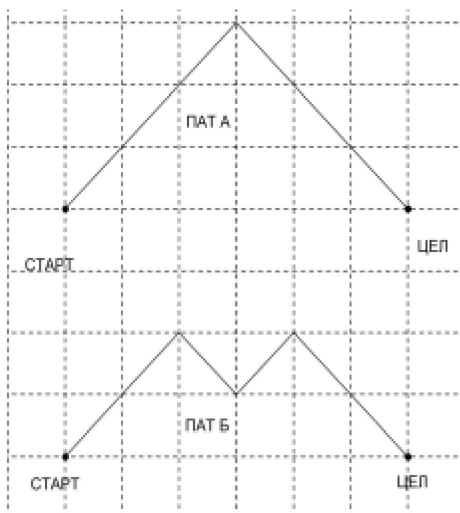
From the table we immediately note that no problem is solved by more than 50% of students. Most surprisingly, the ninth task is solved only by one student of both generations. The formulations of the problems are familiar to students from regular written examinations. The number of students who correctly solved the problems during the relevant examinations is large. Unfortunately in this test, that number is very small.

The following question remains: Why students at some specific time of the academic year achieve success, while a few months later show poor mastery of even basic content from the topics covered?

Experience shows that a large number of students only study for written examinations, and that does not result in retention of knowledge. We believe that this phenomenon is, on one hand, due to the small number of classes in mathematics per week, and, on the other hand, the large number of subjects in the first and second year of high school.

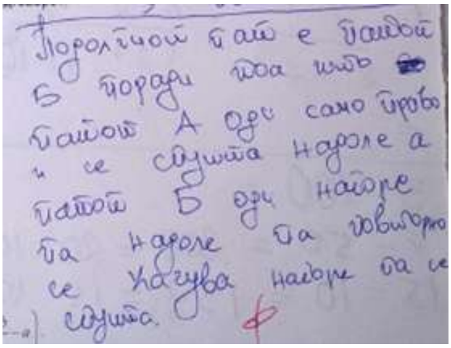
Probably the "success" is also due the large number of topics covered in the subject of mathematics in the two years.

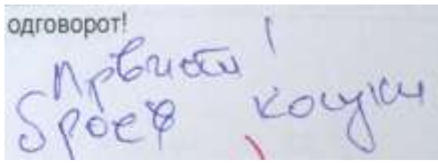
Our idea behind this analysis is not to find culprits in the teaching process. We started with the impression that fewer of the students retain the knowledge, procedures and methods acquired in the teaching process, and this paper is a result of the attempt to verify that impression. Finally, the problem that spiked greatest interest among students, which we have omitted from the analysis, is given below,

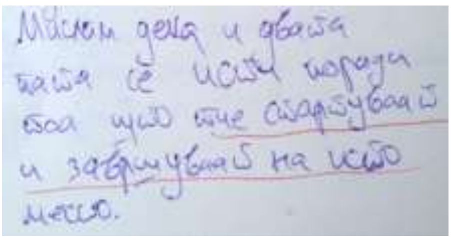


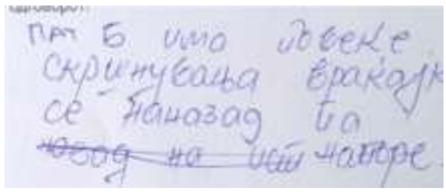
along with some of the solutions by the students.
Problem 10, in the test in the academic year 2014/2015, is: Which is the longer road ! Explain your reasoning !

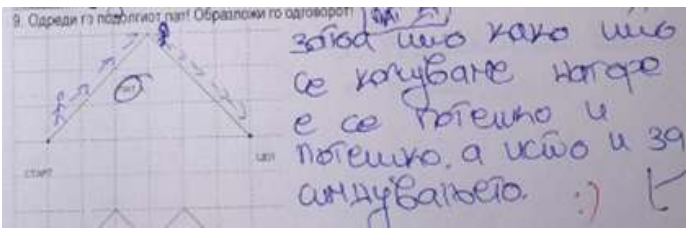
In the academic year 2015/2016 the problem is: There are 3 boxes with marbles: the first has 13, second 10, and the third 4. With just 3 transfers arrange the marbles in the boxes so that in each box there s equal numbers of marbles. The transfer from one box to another is done by moving exactly half of the number of marbles in the second box, to the first box.











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[1] SUGS ”Georgi Dimitrov”, Skopje,
R. Macedonia
jmarkoska@gmail.com

[2] Faculty of Natural Sciences and
Mathematics Ss. Cyril and Methodius
University, Skopje, R. Macedonia
gorgim@pmf.ukim.mk