A GENERAL EXPLICIT FORMULA FOR THE PARTIAL FRACTION EXPANSION OF THE FUNCTION $\Phi(x)/F(x)$

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The purpose of this paper is to give a formula for the partial fraction expansion of the function $\phi(x)/f(x)$, where $\phi(x)$ aud $f(x)$ are polynomials of degres m_1 aud m_2 for which $m_1 < m_2$. The problem in a particular case has been considered recently.[1])

Let $f(x)$ has multiple zeros $a_1, a_2, \ldots a_n$ such that it may be written

$$f(x) = \prod_{r=1}^{n} (x-a_r)^{k_r}, \; k_r \in N \qquad \text{and} \qquad k_1+k_2+\ldots+k_n = m_2.$$

The function $\phi(x)/f(x)$ has a unique partial fraction expansion of the form

$$\frac{\phi(x)}{f(x)} = \sum_{\omega=1}^{\lambda} \sum_{\nu=1}^{n} \frac{A_\omega^\nu}{(x-a_\nu)^\omega}$$

where $\lambda = \max(k_1, k_2, \ldots k_n)$ and $A_\omega^\nu = 0, \; \omega > k_\nu, \; \nu = 1, 2, \ldots n$.

The coefficients A_ω^ν can be calculated from the formula

$$A_\omega^\nu = \frac{1}{(k_\nu-\omega)!} \left[\frac{\phi(x)}{f(x)}\right]^{(k_\nu-\omega)}_{x=a_\nu},$$

where

$$f_\nu(x) = (x-a_\nu)^{-k_\nu} f(x), \; \nu = 1, 2, \ldots n.$$

If $\phi(x) = 1$, we obtain

$$A_\omega^\nu = \prod_{s=1}^{n(\nu)} (a_\nu-a_s)^{-k_s} \sum_{\lambda_1 \ldots \lambda_n} C_{\lambda_1, \ldots \lambda_{\nu-1}, \lambda_{\nu+1} \ldots \lambda_n} \prod_{s=1}^{n(\nu)} (a_\nu-a_s)^{-\lambda_s}$$

with

$$C_{\lambda_1 \ldots \lambda_n} = \frac{(k_1)_{\lambda_1} \ldots (k_n)_{\lambda_n}}{\lambda_1! \ldots \lambda_n!}, \; \lambda_1+\cdots+\lambda_{\nu-1}+\lambda_{\nu+1}+\cdots+\lambda_n = k_\nu-\omega$$

$$(\alpha)_r = \alpha(\alpha+1)\ldots(\alpha+r-1), \; (\alpha)_0 = 1$$

$$n(\nu) \text{ is } s = 1, 2, \ldots \nu-1, \nu+1, \ldots n.$$

[1]) D. D. Adamović; D. S. Mitrinović; D. Ž. Djoković — Formule de decomposition d'une fraction rationelle en élements simples suivie de quelques applications, Publ. de la fac. d'electrotech de l'univ. Belgrade, № 65 (1963) p. 17.