Математички Билтен
19 (XLV)
1995 (25–36)
Скопје, Македонија

ISSN 0351–336X

POSITIVE RADIAL, SOLUTIONS OF CERTAIN SEMILINEAR ELLIPTIC EQUATIONS IN ANNULAR DOMAINS

B. Vrdoljak

Abstract

In this paper it is established the existence, behaviour and approximation of positive radially symetric solutions of semilinear elliptic equation (1) in an annulus Ω, when λ and F satisfy the corresponding sufficient conditions.

1. Introduction

Consider the semilinear elliptic equation

$$\Delta u(x) + \lambda \cdot F(u(x), |x|) \cdot u^q(x) = 0 \tag{1}$$

in an annulus $\Omega = \{x \in \mathbf{R}^n\colon A \leq |x| \leq B\}$, where $A, B \in \mathbf{R}^+ = (0, \infty)$, $n \geq 2$, F: $\mathbf{R}^\circ \times \mathbf{R}^+ \to \mathbf{R}^+$ is a C^1 function, $\mathbf{R}^\circ = [0, \infty)$, $\lambda > 0$ is a real parameter, $q \in \mathbf{R}^\circ$, A can be arbitrarily small and B arbitrarily large.

The equations of the form (1) occur in a wide variety of situations (see e.g. [1–5]). When $\lambda F = K(x)$ and $q = (n+2)/(n-2)$, (1) is known as the conformal scalar curvature equation in $\mathbf{R}^n$, $n \geq 3$. When $\lambda F \equiv 1$, (1) is known as the Lane–Emden equation in astrophysics or sometimes the Emden–Fowler equation, where u corresponds to the density of a single star. The Matukuma equation $\Delta u + u^q/(1+|x|^2) = 0$ in $\mathbf{R}^3$, $q > 1$, as a mathematical model to describe the dynamics of globular clusters of stars, where $u > 0$ is the gravitational potential.

MSC (1980): 35B40, 35J65
Key words: semilinear elliptic equation, radial solutions, existence, approximation

The equations of the form (1) were studied by many authors. Most results were obtained by variational techniqes or by shooting methods. In this paper we shall use the qualitative analysis theory and the topological retraction method (see e.q. [6–8]). We shall establish the existence and approximation of the certain radial solutions of (1) for all $A \le |x| \le B$.

Radial solutions of the equation (1) are the functions $u = u(r)$, $r = |x|$ satisfying the ordinary differential equation

$$u''(r) + (n-1)u'(r)/r + \lambda \cdot F\big(u(r), r\big) \cdot u^q(r) = 0, \quad r \in [A, B]. \tag{2}$$

For $n \ge 3$, setting

$$t = r^{2-n}, \quad u(t) = u(r) \quad \textit{and} \quad f\big(u(t), t\big) = F\big(u(r), r\big),$$

(2) can be rewritten as

$$u''(t) + \lambda \cdot p(t) \cdot f\big(u(t), t\big) \cdot u^q(t) = 0, \quad t \in [a, b] \tag{3}$$

($' = d/dt$), where

$$p(t) = (n-2)^{-2} t^{-k}, \quad k = 2 + 2/(n-2), \quad a = B^{2-n}, \quad b = A^{2-n}.$$

For $n = 2$, setting

$$t = -\ln r, \quad u(t) = u(r), \quad f\big(u(t), t\big) = F\big(u(r), r\big),$$

equation (2) can also be rewritten as (3) with

$$p(t) = e^{-2t}, \quad a = -\ln B, \quad b = -\ln A.$$

Now we establish the existence, behaviour and approximation of positive solutions $u(t)$ of equation (3) depending on zero, one or two parameters (integration constants) with the corresponding initial or boundary conditions.

2. Preliminaries

Let us rewrite the equation (3) as the equivalent system

$$u' = v, \quad v' = -\lambda p(t)\, f(u, t) u^q, \quad t' = 1, \tag{4}$$

which satisficies the conditions for existence and uniqueness of solutions of Cauchy problem in $\Sigma = D \times \mathbf{R} \times \mathbf{R}^+$, $\mathbf{R}^\circ \subset D \subset \mathbf{R}$.

We shall consider the behaviour of the integral curves $\big(u(t), v(t), t\big)$ of system (4) with restect to the sets Σ and

$$\sigma = \{(u, v, t) \in \Sigma\colon\ h_1(t) < u < h_2(t),\ g_1(t) < v < g_2(t),\ t \in I\},$$

$I = (a, b)$, $b \leq \infty$. The funkctions h_i, $g_i \in C^1(I)$, $i = 1, 2$, will be effectively defined. The boundary surfaces of σ are

$$U_i = \{(u, v, t) \in C1\sigma: \quad H_i = (-1)^i[u - h_i(t)] = 0\},$$
$$V_i = \{(u, v, t) \in C1\sigma: \quad G_i = (-1)^i[v - g_i(t)] = 0\}, \quad i = 1, 2.$$

Let us denote the vector field defined by (4) by X. By means of sign of the scalar products

$$Pu_1 = (\nabla H_i, X) = (-1)^i(v - h_i') \qquad \text{on } U_i,$$
$$Pv_i = (\nabla G_i, X) = (-1)^{i+1}(\lambda p f u^q + g_i') \quad \text{on } \quad V_i, \ i = 1, 2,$$

we shall establish the behaviour of integral curves of (4) with respect to sets σ and Σ. The vectors ∇H_i and ∇G_i, $i = 1, 2$ are vectors of external normal on surfaces U_i and V_i, $i = 1, 2$.

In this paper we shall use the constants c, d, m, s, α, β, θ, ζ, $\mu \in \mathbf{R}^+$ and the sets $U = (U_1 \cup U_2) \backslash (V_1 \cup V_2)$ and $V = (V_1 \cup V_2) \backslash (U_1 \cup U_2)$.

3. The case $n \geq 3$

Consider equation (3) for $n \geq 3$, i.e., the equation

$$u''(t) + \lambda(n-2)^{-2} t^{-k} f(u, t) u^q(t) = 0, \tag{5}$$

where $k = 2 + 2/(n-2)$, $q \geq 0$.

Equation (5) is equivalent to the system

$$u' = v, \quad v' = -\lambda(n-2)^{-2} t^{-k} f(u, t) u^q, \quad t' = 1. \tag{6}$$

Let

$$\eta = \{(u, t) \in \mathbf{R}^2: \ h_1(t) \leq u \leq h_2(t), \ t \in [a, b]\},$$

where the fukctions h_i will be defined in the following theorems.

Theorem **1**. Let

$$h_1 = \theta\alpha(t - a), \quad h_2 = \alpha(t - a), \tag{7}$$
$$\beta(b - a + 1) \leq \alpha < \beta/\theta, \quad 0 < \theta < 1, \tag{8}$$
$$0 < \lambda \leq \beta(n-2)^2 a^k d^{-1} \alpha^{-q} (b - a)^{-q} \tag{9}$$

and the function f satisfies the condition

$$0 < f(u, t) \leq d \quad \text{for} \quad (u, t) \in \eta. \tag{10}$$

Then equation (5) has a one–parameter family of solutions $u(t)$ satisfying the conditions

$$\begin{aligned} &\theta\alpha(t-a) < u(t) < \alpha(t-a) \qquad \forall\, t \in (a,\, b],\;\; u(a) = 0, \\ &\beta < u'(t) < \beta(b+1-t) \qquad \forall\, t \in [a,\, b),\;\; u'(b) = \beta. \end{aligned} \tag{11}$$

Proof. Let consider the behaviour of integral curves of system (6) at points of the surfaces U_i, and V_i, $i = 1, 2$, where the functions h_i are defined by (7) and g_i by

$$g_1 = \beta, \qquad g_2 = \beta(n+1-t). \tag{12}$$

In view of the conditions of Theorem **1**, for the corresponding scalar products Pu_i and Pv_i we have

$$\begin{aligned} Pu_1 &= -v + h_1' \le -g_1 + h_1' = -\beta + \theta\alpha < 0 \quad \text{on } U_1, \\ Pu_2 &= v - h_2' \le g_2 - h_2' = \beta(b+1-t) - \alpha < 0 \quad \text{on } U_2, \\ Pv_1 &= \lambda(n-2)^{-2}t^{-k}fu^q + g_1' = \lambda(n-2)^{-2}t^{-k}fu^q > 0 \text{ on } V_1, \\ Pv_2 &= -\lambda(n-2)^{-2}t^{-k}fu^q - g_2' > -\lambda(n-2)^{-2}a^{-k}d\alpha^q(b-a)^q + \beta \ge 0 \text{ on } V_2. \end{aligned}$$

Consecvently, the corresponding set U is a set of points of strict entrance and the corresponding set V is a set of points of strict exit of integral curves of (6) with respect to the sets σ and Σ. Moreover, we can note that points of the curves $L_{ij} = U_i \cap V_j$, $i, j = 1, 2$ are not points of exit and that all points of exit are points of strict exit.

Hence, according to the retraction method (see [6–8]) the system (6) has a one–parameter family of integral curves $\big(u(t), v(t), t\big)$ belonging to the set σ for all $t \in (a, b)$, i.e.,

$$h_1(t) < u(t) < h_2(t), \qquad g_1(t) < v(t) < g_2(t) \tag{13}$$

for all $t \in (a, b)$, where the functions h_i are defined by (7) and g_i by (12). These integral curves satisfy also the conditions

$$\begin{aligned} u(a) &= 0, \qquad \theta\alpha(b-a) < u(b) < \alpha(b-a), \\ v(b) &= \beta, \qquad \beta < v(a) < \beta(b-a+1), \end{aligned}$$

because the integral curves of system (6) are defined and continous on $[a, b]$.

Thus, equation (5) has a one–parameter family of solutions $u(t)$ which satisfy conditions (11).

Theorem **2**. Assume (7), (10) and

$$0 < \lambda \le (\beta - \theta\alpha)(n-2)^2 a^k d^{-1} \alpha^{-q} (b-a)^{-1-q}, \quad 0 < \theta\alpha < \beta < \alpha \tag{14}$$

Then equation (5) has a two–parameter family of solutions $u(t)$ satisfyng the conditions

$$\begin{aligned} &\theta\alpha(t-a) < u(t) < \alpha(t-a) \qquad \text{and} \\ &\beta - (\beta - \theta\alpha)(t-a)/(b-a) < u'(t) < \beta \quad \forall\, t \in (a,\, b], \end{aligned} \tag{15}$$

and a unique solution which satisfies the additional conditions

$$u(a) = 0, \qquad u'(a) = \beta. \tag{16}$$

Proof. Here is

$$\begin{aligned} q_1 =& \beta - (\beta - \theta\alpha)(t-a)/(b-a), \quad q_2 = \beta, \\ Pu_1 \leq& -g_1 + h_1' < -\theta\alpha + \theta\alpha = 0 \qquad \text{on } U_1, \\ Pu_2 \leq& g_2 - h_2' = \beta - \alpha < 0 \qquad \text{on } U_2, \\ Pv_1 =& \lambda(n-2)^{-2} t^{-k} f u^q - (\beta - \theta\alpha)/(b-a) < \\ &< \lambda(n-2)^{-2} a^{-k} d\alpha^q (b-a)^q - (\beta - \theta\alpha)/(b-a) \leq 0 \quad \text{on } V_1, \\ Pv_2 =& -\lambda(n-2)^{-2} t^{-k} f u^q < 0 \qquad \text{on } V_2. \end{aligned}$$

According to this all points of set $\partial\sigma$ are points of strict entrance of integral curves of system (6) and system (6) has a two–parameter family of solutions satisfying the corresponding conditions (13) for all $t \in (a,\, b]$, i.e., all solutions of (6) with the initial conditions

$$h_1(t_0) < u(t_0) < h_2(t_0), \qquad g_1(t_0) < v(t_0) < g_2(t_0)$$

for every $t_0 \in (a,\, b)$, satisfy also conditions (13) for all $t \in (t_0,\, b]$. Consequently, and since system (6) satisfies the conditions for existence and uniqueness of solutions, the solution with the initial conditions

$$u(a) = 0, \qquad v(a) = \beta \tag{17}$$

satisfies also conditions (13) for all $t \in (a,\, b]$.

Hence, equation (5) has a two–parameter family of solutions $u(t)$ which satisfy the conditions (15), and the unique solution with initial conditions (16) also satisfies conditions (15).

Now let us consider solutions $u(t)$ of equation (5) which are not monotone and $u(t) \geq m > 0$ on the bounded interval $[a,\, b]$.

Theorem **3**. Let

$$0 < c \leq f(u,\, t) \leq d \ \text{ on } \eta, \tag{18}$$

$$0 < \zeta < 2(b-a), \tag{19}$$

$$(b/a)^k (d/c)[1 + (\alpha/4m)(b-a+\zeta)^2]^q \leq [4(b-a)+\zeta]/[4(b-a)-\zeta], \tag{20}$$

$$\begin{aligned} &(\alpha/c)\big[2 - \zeta/\big(2(b-a)\big)\big](n-2)^2 b^k m^{-q} \leq \lambda \leq \\ &\quad \leq (\alpha/d)\big[2 + \zeta/\big(2(b-a)\big)\big](n-2)^2 a^k [m + (\alpha/4)(b-a+\zeta)^2]^q. \end{aligned} \tag{21}$$

Then equation (5) has a unique solution satisfying the conditions

$$h_1(t) < u(t) < h_2(t), \qquad q_1(t) < u'(t) < g_2(t) \tag{22}$$

for all $t \in [a, b)$ and

$$u(b) = m, \qquad u'(b) = -\alpha(b - a + \zeta/2), \tag{23}$$

where

$$\begin{aligned} h_1 &= -\alpha[t^2 - (a+b)t + ab] + m, \\ h_2 &= -\alpha[t^2 - (a+b-\zeta)t + ab - b\zeta] + m, \\ g_1 &= -\alpha\big[2 - \zeta/\big(2(b-a)\big)\big](t-a) + \alpha(b-a-\zeta), \\ g_2 &= -\alpha\big[2 + \zeta/\big(2(b-a)\big)\big](t-a) + \alpha(b-a). \end{aligned} \tag{24}$$

Proof. Here we have

$$\begin{aligned} Pu_1 &\geq -g_2 + h_1' = \alpha\zeta(t-a)/[2(b-a)] > 0 \quad \text{on } U_1, \\ Pu_2 &\geq g_1 - h_2' = \alpha\zeta(t-a)/[2(b-a)] > 0 \quad \text{on } U_2 \\ Pv_1 &> \lambda c m^q (n-2)^{-2} b^{-k} - \alpha\big[2 - \zeta/\big(2(b-a)\big)\big] \geq 0 \quad \text{on } V_1, \\ Pv_2 &> -\lambda d[m + (\alpha/4)(b-a+\zeta)^2]^q (n-2)^{-2} a^{-k} + \\ &\quad + \alpha\big[2 + \zeta/\big(2(b-a)\big)\big] \geq 0 \quad \text{on } V_2. \end{aligned}$$

Thus, the corresponding set $\partial\sigma$ is a set of points of strict exit of integral curves of system (6). According to the retraction method, system (6) has at least one solution which satisfies the corresponding conditions (13) for all $t \in (a, b)$. This solution also satisfies the conditions

$$\begin{aligned} m < u(a) < m + \alpha\zeta(b-a), &\qquad u(b) = m, \\ \alpha(b-a-\zeta) < v(a) < \alpha(b-a), &\qquad v(b) = -\alpha(b-a+\zeta/2), \end{aligned}$$

because the conditions for existence and uniqueness of solutions are valid. Therefore, equation (5) has exactly one solution satisfying conditions (22) and (23).

Now consider solutions of (5) on the unbounded interval $[a, \infty)$ with the initial condition $u(a) = 0$ or $u(a) = m > 0$.

Theorem 4. Let

$$\begin{aligned} &0 < \theta < 1, , \quad s = 2/(n-2), \\ &0 < \lambda \leq \theta a^s\, s(s+1)(n-2)^2 d^{-1} \alpha^{1-q}, \\ &0 < f(u,t) \leq d \quad \text{for} \quad 0 \leq u \leq \alpha, \ t \geq a. \end{aligned} \tag{25}$$

The equation (5) has a one–parameter family of solutions satisfying the conditions

$$0 < u(t) < \alpha[1 - (a/t)^s] \qquad \forall\, t \in (a, \infty) \quad \text{and} \quad u(a) = 0\,,$$
$$0 < u'(t) < \theta\, \alpha\, s\, a^s\, t^{-s-1} \qquad \forall\, t \in [a, \infty)\,.$$

Proof. Let us note that

$$\begin{aligned}
&h_1 = 0, \quad h_2 = \alpha[1 - (a/t^s], \quad g_1 = 0, \quad g_2 = \theta\, \alpha\, s\, a^s t^{-s-1} \\
&Pu_1 < -q_1 + h_1' = 0 \quad \text{on} \quad U_1 \setminus L, \quad L = U_1 \cap V_1\,, \\
&Pu_2 \le g_2 - h_2' = (\theta - 1)\, s a^s\, t^{-s-1} < 0 \quad \text{on} \quad U_2\,, \\
&Pv_1 = \lambda(n-2)^{-2} t^{-k}\, f u^q > 0 \quad \text{on} \quad V_1 \setminus L\,, \\
&Pv_2 > t^{-k}[\theta\, \alpha\, s(s+1)a^s - \lambda\, d\alpha^q (n-2)^{-2}] \ge 0 \quad \text{on} \quad V_2\,, \\
&Pu_1 = Pv_1 = 0 \quad \text{on} \quad L\,,
\end{aligned}$$

where L is an integral curve of (6). All points of exit are points of strict exit and here we can use the proof of Theorem **1**.

Theorem **5**. Suppose $s = 2/(n-2)$,

$$0 < \theta < \zeta < \mu < 1, \tag{26}$$

$$(1 + \alpha/m)^q\, d/c \le \mu/\zeta, \tag{27}$$

$$\zeta \alpha a^s s(s+1)(n-2)^2 c^{-1} m^{-q} \le \lambda \le \mu \alpha a^s s(s+1)(n-2)^2 (m+\alpha)^{-q}\, d^{-1}$$

$$0 < c \le f(u,\, t) \le d \quad \text{for} \quad 0 < m \le u \le m + \alpha, \quad t \ge a\,. \tag{28}$$

Then equation (5) has a one–parameter family of solutions $u(t)$ with the properties

$$u(a) = m\,,$$

$$m + \theta\alpha[1 - (a/t)^s] < u(t) < m + \alpha[1 - (a/t)^s] \qquad \forall t \in (a, \infty)\,,$$
$$\zeta\, \alpha\, s\, a^s\, t^{-s-1} < u'(t) < \mu \alpha s a^s t^{-s-1} \qquad \forall t \in [a, \infty)\,.$$

Proof. Here we have

$$h_1 = m + \theta\, \alpha[1 - (a/t)^s]\,, \qquad h_2 = m + \alpha[1 - (a/t)^s]\,,$$
$$g_1 = \zeta\, \alpha\, s\, a^s\, t^{-s-1}\,, \qquad g_2 = \mu\, \alpha\, s\, a^s\, t^{-s-1}\,,$$

$$Pu_1 \le -g_1 + h_1' = \alpha\, s\, a^s\, t^{-s-1}(\theta - \zeta) < 0 \quad \text{on} \quad U_1\,,$$
$$Pu_2 \le g_2 - h_2' = \alpha\, s\, a^s\, t^{-s-1}(\mu - 1) < 0 \quad \text{on} \quad U_2\,,$$
$$Pv_1 > t^{-k}[\lambda\, c\, m^q (n-2)^{-2} - \zeta\, \alpha\, s(s+1)a^s] \ge 0 \quad \text{on} \quad V_1\,,$$
$$Pv_2 > t^{-k}[\mu\, \alpha\, s(s+1)a^s - \lambda\, d(m+\alpha)^q (n-2)^{-2}] \ge 0 \quad \text{on} \quad V_2$$

and we can use the proof of Theorem **1**.

Using Theorems **1**, **2**, and **3** one can prove the following statements:

(i) If conditions (8), (9) and (10) are valid for the functions

$$h_1 = \theta\, \alpha(b-t), \qquad h_2 = \alpha(b-t)\,,$$

then equation (5) has a one–parameter family of solutions $u(t)$ with the properties

$$\theta\, \alpha(b-t) < u(t) < \alpha(b-t) \qquad \forall\, t \in [a,\, b), \qquad u(b) = 0\,,$$
$$-\,\beta(t-a+1) < u'(t) < -\beta \qquad \forall\, t \in (a,\, b], \qquad u'(a) = -\beta\,.$$

(ii) Suppose that

$$0 < \lambda \le \beta(n-2)^2 a^k d^{-1} \alpha^{-k} (b-a)^{-1-q}, \quad 0 < \beta < \alpha\,, \tag{29}$$
$$h_1 = 0, \qquad h_2 = \alpha(b-t)$$

and that condition (10) holds. Then equation (5) has a unique solution $u(t)$ satisfying the conditions

$$0 < u(t) < \alpha(b-t) \qquad \text{and}$$
$$-\beta < u'(t) < -\beta(t-a)/(b-a) \quad \forall\, t \in [a,\, b)\,,$$
$$u(b) = 0, \qquad u'(b) = -\beta\,.$$

(iii) Assume (18), (19), (20) and (21). Then equation (5) has a one–parameter family of solutions satisfying conditions (22) for all $t \in (a,\, b)$ and

$$u'(a) = \alpha(b - a - \zeta/2)\,, \qquad u(b) = m\,,$$

where the functions h_i are defined by (24) and the functions g_i are

$$g_1 = -\alpha\big[2 + \zeta/\big(2(b-a)\big)\big](t-a) + \alpha(b - a - \zeta/2)\,,$$
$$g_2 = -\alpha\big[2 - \zeta/\big(2(b-a)\big)\big](t-a) + \alpha(\beta - a - \zeta/2)\,.$$

(iv) Assume (18), (19), (20), (21) and let

$$\begin{aligned} h_1 &= -\alpha[t^2 - (a+b)t + ab] + m\,,\\ h_2 &= -\alpha[t^2 - (a+b+\zeta)t + a(b+\zeta)] + m\,,\\ g_1 &= -\alpha\big[2 - \zeta/\big(2(b-a)\big)\big](t-a) + \alpha(b-a)\,,\\ g_2 &= -\alpha\big[2 + \zeta/\big(2(b-a)\big)\big](t-a) + \alpha(b-a+\zeta)\,. \end{aligned} \tag{30}$$

Then equation (5) has a one-parameter family of solutions with properties (22) for all $t \in (a, b)$ and

$$u(a) = m\,, \qquad u'(b) = -\alpha(b - \alpha - \zeta/2)\,.$$

(v) Assume (18), (19), (20), (21) and let the funkctions h_i are defined by (30) and

$$g_1 = -\alpha\big[2 + \zeta/\big(2(b-a)\big)\big](t-a) + \alpha(b - a + \zeta/2)\,,$$
$$g_2 = -\alpha\big[2 - \zeta/\big(2(b-a)\big)\big](t-a) + \alpha(b - a + \zeta/2)\,.$$

Then equation (5) has a two-parameter family of solutions satisfying conditions (22) for all $t \in (a, b]$ and

$$u(a) = m\,, \qquad u'(a) = \alpha(b - a + \zeta/2)\,.$$

In cases (i) and (iii) we have

$$Pu_i > 0 \quad \text{on} \quad U_i\,, \qquad Pv_i < 0 \quad \text{on} \quad V_i, \quad i = 1, 2,$$

in case (iv)

$$Pu_i < 0 \quad \text{on} \quad U_i\,, \qquad Pv_i > 0 \quad \text{on} \quad V_i, \quad i = 1, 2.$$

It means that the two opposite sides of $C1\sigma$ are a set of strict exit, and the other two sides are a set of strict exit. Hence, here we can use proof of Theorem **1**.

In case (v) is

$$Pu_i < 0 \quad \text{on} \quad U_i\,, \qquad pv_i < 0 \quad \text{on} \quad V_i\,, \quad i = 1, 2$$

and we can use proof of Theorem **2**.

For statement (ii) it is necessary to note that

$$Pu_i > 0 \quad \text{on} \quad U_i\,, \quad i = 1, 2, \qquad Pv_2 > 0 \quad \text{on} \quad V_2\,,$$
$$Pv_1 > 0 \quad \text{on} \quad V_1 \setminus K\,, \qquad P\dot{v}_1 = 0 \quad \text{on} \quad K = U_1 \cap V_1$$

and that all points of exit are points of strict exit. Here proof of Theorem **3** can be used.

4. The case $n = 2$

In this case for equation (3) we have

$$u''(t) + \lambda \cdot e^{-2t} f(u, t)\, u^q(t) = 0 \tag{31}$$

and for system (4) we have

$$u' = v\,, \qquad v' = -\lambda \cdot e^{-2t} f(u, t) u^q\,, \qquad t' = 1\,.$$

Let us first notice that the following theorem is valid.

Theorem **6**. For equation (31) the conclusions of Theorems **1**–**3** and statements (i)–(v) hold, whenever in conditions (9), (14) and (29) we have e^{2a} instead of $(n-2)^2 a^k$, and in conditions (20) and (21) the constants $(b/a)^k$, $(n-2)^2 a^k$ and $(n-2)^2 b^k$ are replaced by $e^{2(b-a)}, e^{2a}$ and e^{2b} respectively.

Instead of Theorems **4** and **5** one can prove the following two theorems.

Theorem **7**. If (25) and

$$0 < \lambda \leq 4\theta\, e^{2a}\, d^{-1}\, \alpha^{1-q}, \qquad 0 < \theta < 1$$

hold, then equation (31) has a one–parameter family of solutions satisfying the conditions

$$0 < u(t) < \alpha \left[1 - e^{-2(t-a)}\right] \qquad \forall\, t \in (a, \infty), \quad u(a) = 0,$$
$$0 < u'(t) < 2\theta\, e^{-2(t-a)} \qquad \forall\, t \in [a, \infty).$$

Theorem **8**. Assume (26), (27), (28) and

$$4\zeta\, \alpha\, e^{2a}\, c^{-1}\, m^{-q} \leq \lambda \leq 4\mu\, \alpha\, e^{2a}\, d^{-1}(m+\alpha)^{-q}.$$

Then equation (31) has a one–parameter family of solutions $u(t)$ satisfying the conditions

$$m+\theta\,\alpha \left[1 - e^{-2(t-a)}\right] < u(t) < m+\alpha \left[1-e^{-2(t-a)}\right] \;\; \forall\, t > a, \;\; u(a)=m,$$
$$2\zeta\, \alpha\, e^{-2(t-a)} < u'(t) < 2\mu\, \alpha\, e^{-2(t-a)} \quad \forall\, t \geq a.$$

Remark. Some results also hold for $q < 0$. In this case we have to modify certain conditions.

5. Approximation of solutions

We can note that the obtained results also contain the answers to the approximation of solutions $u(t)$ whose existence is established. The errors of the approximation for $u(t)$ and $u'(t)$ are defined by the functions

$$h(t) = h_2(t) - h_1(t), \qquad g(t) = g_2(t) - g_1(t)$$

respectively.

Let us point out some cases.

(a) In the case of Theorem **1** we have

$$h(t) = \alpha(1-\theta)(t-a), \quad g(t) = \beta(b-t), \qquad t \in (a, b).$$

The errors are sufficiently small, for every $t \in (a, b)$, when α and β are sufficiently small.

(b) In the case of statement (iii) we have

$$h(t) = \alpha\,\zeta(b-t), \quad g(t) = \alpha\,\zeta(t-a)/(b-a), \qquad t \in (a,b)\,.$$

The errors depend on $\alpha\,\zeta$ and we can note that those can be sufficiently small in many examples.

(c) In Theorem **8** we have

$$h(t) = \alpha(1-\theta)\left[1 - e^{-2(t-a)}\right], \quad g(t) = 2\alpha(\mu - \zeta)e^{-2(t-a)}, \qquad t > a\,.$$

Here

$$\max\{h(t)\} = \alpha(1-\theta), \quad \max\{g(t)\} = 2\alpha(\mu-\zeta), \qquad \forall\, t > a$$

can be arbitrarily small and $g(t)$ tend to zero as $t \to \infty$.

References

[1] Cheng, K. S., Cheren, J. L: *Existence of positive entire solutions of some semilinear elliptic equations*, J. Differential Equations **98**, 169–180, 1992

[2] Kwong, M. K., McLeod, J. B., Peletier, L. A., Troy, W. C.: *On ground state solutions of* $-\Delta u = u^p - u^q$, J. Differential Equations **95**, 218-239, 1992

[3] Li, Y.: *Asymptotic behaviour of positive solutions of equation* $\Delta u + K(x)u^p = 0$ in $\mathbb{R}^n$, J. Differential Equations **95**, 304–330, 1992

[4] Li, Y., Ni, W. M.: *On the existence and symetry properties of finite total mass solutions of the Matukama equation, the Edington equation and their generalizations*, Arch. Rational Mech. Anal. **108**, 175–194, 1989

[5] Merle, F., Peletier, A.: *Asymptotic behaviour of positive solutions of elliptic equations with critical and supercritical growth. I. The radial case*, Arch. Rational Mech. Anal. **112**, 1–19, 1990

[6] Vrdoljak, B.: *Negative radial solutions of certain semilinear elliptic equations*, Proc. VII Conf. on Appl. Math., Osijek, 209–217, 1989 (1990)

[7] Vrdoljak, B.: *Positive radial solutions of certain nonlinear elliptic equations*, Z. Angew. Math. Mech. **70**, 632–634, 1990

[8] Wazewski, T.: *Sur un principe topologique de l'examen de l'allure asymptotique des intégrales des équations différentialles ordinaires*, Ann. Soc. Polon. Math. **20**, 279–313, 1947

ПОЗИТИВНИ РАДИЈАЛНИ РЕШЕНИЈА НА ДАДЕНИ СЕМИЛИНЕАРНИ ЕЛИПТИЧНИ РАВЕНКИ СО ПРСТЕНЕСТИ ДОМЕНИ

Б. Врдољак

Р е з и м е

Во овој труд е добиено постоењето, однесувањето и апроксимацијата на позитивни радијални симетрични решенија од семилинеарни елиптични равенки (1) во прстенот Ω, каде што λ и F ги задоволуваат соодветните доволни услови.

University of Split
Faculty of Civil Engineering
Matice hrvatske 15
58000 Split
Croatia