ON A PROPERTY OF GENERALISED GEGENBAUER POLYNOMIALS

Прилози МАНУ, Оддел. за мат.-тех. науки, IX/1, 1977, 5-7

This is an extension of a relation obtained by Ainsworth and Lin [1] for Legendre polynomials. We establish the following result

$$(n+k\nu)\, C_n^{\nu}(x,k) = k\nu \sum_{r=o}^{n} T_r(x,k)\, C_{n-r}^{\nu}(x,k),$$

where $C_n^{\nu}(x,k)$ and $T_n(x,k)$ are generalised Gegenbauer and generalised Tchebycheff polynomials, respectively.

A generalisation of the Gegenbauer polynomials is given by means of the generating relation [2]

$$(1) \qquad (1-kxt+t^k)^{-\nu} = \sum_{n=o}^{\infty} C_n^{\nu}(x,k)\, t^n.$$

Similarly, we give

$$(1-(k-1)\,xt)\,(1-kxt+t^k)^{-1} = \sum_{n=o}^{\infty} T_n(x,k)\, t^n.$$

Let

$$F = (1-kxt+t^k)^{-\nu}.$$

Then

$$(x-t^{k-1})\,\frac{\partial F}{\partial x} - t\,\frac{\partial F}{\partial x} = 0,$$

so that we obtain

$$(2) \qquad k\nu t\,(x-t^{k-1})\,(1-kxt+t^k)^{-\nu-1} = \sum_{n=o}^{\infty} nC_n^{\nu}(x,k)\, t^n.$$

The use of (1) and (2) leads to

$$\sum_{n=o}^{\infty} (n+k\nu)\, C_n^{\nu}(x,k)\, t^n = k\nu\,(1-(k-1)\,xt)\,(1-kxt+t^k)^{-\nu-1}.$$

Since

$$(1-(k-1)\,xt)\,(1-kxt+t^k)^{-\nu-1} = \sum_{n=o}^{\infty} \sum_{r=o}^{n} T_r(x,k)\, C_{n-r}^{\nu}(x,k)\, t^n$$

we have

$$(n+k\nu)\, C_n^{\nu}(x,k) = k\nu \sum_{r=o}^{n} T_r(x,k)\, C_{n-r}^{\nu}(x,k).$$

In particular, if $k=3$ for the Humbert polynomials [3] $h_n(x)$ we obtain

$$(n+3\nu)\, h_n(x) = 3\nu \sum_{r=o}^{n} T_r(x, 3)\, h_{n-r}(x),$$

and if $\nu = 1$, we have

$$(n+k)\, U_n(x, k) = k \sum_{r=o}^{n} T_r(x, k)\, U_{n-r}(x, k),$$

where $U_n(x, k) = C_n^1(x, k)$ are the generalised Tchebycheff polynomials of the second kind.

REFERENCES

1. *Ainsworth, D. R.*; *Lin, C. K.*, An Interesting Property of Legendre Polynomials, Franklin Inst. 298, 71—72 (1974).

2. *Barrucand P.*, Sur une formule generale de recurrence et quelquesunes de ces applications, C. R. Acad. Sci. Paris, Vol. 264, Ser. A., 1967, 792.

3. *Rainville, E. D.*, Special Functions, New York, 1960.

РЕЗИМЕ

ЕДНО СВОЈСТВО НА ОБОПШТЕНИТЕ ПОЛИНОМИ НА GEGENBAUER

Се покажува дека за обопштените полиноми на Gegenbauer определени со релацијата

$$(1 - kxt + t^k)^{-\nu} = \sum_{n=o}^{\infty} C_n^{\nu}(x, k)\, t^n,$$

важи релацијата

$$(n + k\nu)\, C_n^{\nu}(x, k) = k\nu \sum_{r=o}^{n} T_r(x, k)\, C_{n-r}^{\nu}(x, k),$$

каде што $T_n(x, k)$ се обопштени полиноми на Чебишев.

Посебно ако е $k = 2$ и $\nu = \frac{1}{2}$ ја добиваме релацијата дадена од Ainsworth и Lin за полиномите на Legendre.

За $k = 3$ добиваме слична релација за полиномите на Humbert.