LINEARIZATION OF A PRODUCT OF Q-APPELL POLYNOMIALS

Математички Билтен, 15 (XLI), 1991, 33-34

Let

$$p_n(x) = (x+1)(x+q)\dots(x+q^{n-1}),\quad n=1,2,\dots \qquad (1)$$
$$p_o(x) = 1$$

The purpose of this Note is to find the coefficients a(k,m,n) defined by [1]

$$p_n(x)p_m(x) = \sum_{k=o}^{m+n} a(k,m,n)p_k(x). \qquad (2)$$

From

$$p_m(x) = \sum_{k=o}^{n-1} a(k,m,n)\frac{p_k(x)}{p_n(x)} + \sum_{k=o}^{m} a(n+k,m,n)\frac{p_{n+k}(x)}{p_n(x)}$$

it follows

$$a(k,m,n) = 0, \text{ for } k < n,$$
$$a(m+n,m,n) = 1.$$

It is known that

$$\frac{p_{n+k}(x)}{p_n(x)} = \sum_{r=o}^{k} s(r,k,n)x^{k-r}$$

where s(r,k,n), r=1,2,...,m, n=1,2,... are symmetrical fundamental functions and that

$$s(r,k,n) = q^{rn}s(r,k,o).$$

Then

$$\sum_{k=o}^{m} s(k,m,o)x^{m-k} = \sum_{k=o}^{m} a(n+k,m,n)\sum_{r=o}^{k} s(r,k,o)q^{rn}x^{k-r}$$

from where we obtain

$$\sum_{r=1}^{k} s(k-r,m-r,n)a(m+n-r,m,n) = s(k,m,o)(1-q^{kn}),\quad k=1,2,\dots,m.$$

Because

$$s(r,k,n) = \begin{bmatrix}k\\r\end{bmatrix}q^{nr+r(r-1)/2}$$

we find

$$\sum_{r=1}^{k} \begin{bmatrix}m-r\\k-r\end{bmatrix}q^{n(k-r)+(k-r)(k-r-1)/2}a(m+n-r,m,n) = \begin{bmatrix}m\\k\end{bmatrix}(1-q^{kn}), \qquad (3)$$

k=1,2,...,m

with

$$\begin{bmatrix}r\\k\end{bmatrix} = \frac{(q)_k}{(q)_r(q)_{k-r}},\quad (q)_k = (1-q)(1-q^2)\dots(1-q^k).$$

R E F E R E N C E S

[1] Problem S7(1979,222), American Mathematical Monthly, Vol. 86

ЛИНЕАРИЗАЦИЈА НА ПРОДУКТ ОД Q-АПЕЛ ПОЛИНОМИ

Р е з и м е

За полиномите (1) се определува продуктот (2), чии коефициенти се дадени со (3).