Математички Билтен
Книга 3—4 (XXIX—XXX), 1979—1980, (45—48)
Скопје — Југославија

ON ZYGMUND'S THEOREM FOR Λ-BOUNDED VARIATION OF ORDER P.

Masaaki Shiba.

It is well known the absolute convergence of Fourier series of 2π-periodic function of bounded variation (BV) ———— that is, Zygmund's theorem ([4]).

PROPOSITION A. *If $f \in BV \cap C$, and*

(1) $\sum_1^\infty \omega_f^{1/2}(2\pi/n)\, n^{-1} < \infty,$

then, $\Sigma \rho_n < \infty$, where $\omega_f(\cdot)$ is the modulus of continuity of $f \in C$, $\rho_n = (a_n^2 + b_n^2)^{1/2}$ and $\{a_n\}$, $\{b_n\}$ are Fourier coefficients of f.

S. V. Bočkarev [1] shows that the condition (1) is the best possible, i. e.

PROPOSITION B. *Let the continuous function $\omega(h)$ satisfies the condition*

(2) $\sum_1^\infty \omega^{1/2}(1/n)/n = \infty\,,$

then there exists $f \in BV \cap C$ such that $\omega_f(h) = O(\omega(h))$ and $\Sigma\rho_n = \infty$.

D. Waterman [3] gives the condition of absolute convergence of Fourier reries of $f \in \Lambda - BV \cap C$, that is,

PROPOSITION C. $f \in \Lambda - BV \cap C$ *and*

(3) $\sum_1^\infty n^{-1}\, \lambda_n^{1/2}\, \omega_f^{1/2}(\pi/n) < \infty,$

then $\Sigma\, \rho_n < \infty$. If $f \in HBV \cap C$ and

Puting $N=2^{\nu}$

$$\frac{1}{2}\sum_{2^{\nu-1}+1}^{2^{\nu}} \rho_n^2 \leqslant (\pi/2^{\nu})\ \omega_f\ (\pi/2^{\nu-1})\ V^{(p)}(f)\ ([0,2\pi])\cdot$$

$$\left(\sum_{1}^{2^{\nu+1}} \lambda_k^{q/p}\right)^{1/p},$$

and so

$$\sum_{\nu=1}^{\infty}\sum_{2^{\nu-1}+1}^{2^{\nu}} \rho_n \leqslant \sum_{\nu=1}^{\infty} 2^{\nu/2}\left(\sum_{2^{\nu-1}+1}^{2^{\nu}} \rho_n^2\right)^{1/2}$$

$$=0\,(1)\sum_{1}^{\infty}\omega_f^{1/2}\ (\pi/2^{\nu-1})\left(\sum_{1}^{2^{\nu+1}} \lambda_k^{2/p}\right)^{1/2q}.$$

Convergence of the right hand of the above is equivalent to (5).

COROLLARY. *If* $f \in \{n^{\alpha}\} BV^{(p)} \cap C$ *and*

$$(6)\qquad \sum_{1}^{\infty}\omega_f^{1/2}\ (2\pi/n)\ n^{-[(1-\alpha)/p+1/2]} < \infty,$$

then $\sum \rho_n < \infty$, *where* $1 \leqslant p < \infty$ *and* $0 \leqslant \alpha \leqslant 1$.

PROOF OF COROLLARY. For $1 < p < \infty$ and $0 \leqq \alpha \leqq 1$, putting $\lambda_n = n^{\alpha}$, the condition (5) reduces (6) because of

$$\left(\sum_{1}^{2n} \lambda_k^{\varepsilon/p}\right)^{1/2q} = 0\ (n^{[(\alpha-1)/p+1/2]}).$$

For $p=1$, from the similar estimates of theorem, we have the result.

REMARK. In CROLLARY, for $p=1$ and $\alpha=1$ (6) is justly the same of (4) in PROPOSITION C.

REFERENCES

[1] S. V. Bočkarev, On a problem of Zygmund, Izv. Akad. Nauk SSSR, 37 (1973) 629-637.

[2] M. Shiba, On uniform covergence of function of Λ-bounded variation order p, (to appeare in Analysis Mathematica).

[3] D. Waterman, On convergence of Fourier seiries of function of generalized bounded variation, Studia Math., 44 (1972) 107-117.

[4] A. Zygmund, Trigonometric series, Cambridge Univ. Press, Cambridge (1959).

Summary

In this note, we have an extension of the absolute convergence criterion of Fourier series of a function of bunded variation . . . . that is, Zygmund's theorem.

This condition includes the conditions of Zygmund and also of D. Waterman [3].