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GENERALIZED $(2m, m)-$RECTANGULAR BANDS OF TYPE $(3, 2)$

VALENTINA MIOVSKA AND DONČO DIMOVSKI

Abstract. Notions of generalized left-zero and right-zero $(2m, m)-$semigroups of type $(3, 2)$ are introduced, and so is the notion of generalized $(2m, m)-$rectangular band of type $(3, 2)$. Characterizations of generalized left-zero and right-zero $(2m, m)-$semigroups of type $(3, 2)$ and generalized $(2m, m)-$rectangular band of type $(3, 2)$ are given.

1. Introduction

First, we will introduce some notations which will be used further on:

1) The elements of Q^s, where Q^s denotes the s-th Cartesian power of Q, will be denoted by x_1^s.

2) The symbol x_i^j will denote the sequence $x_i, x_{i+1}, \dots, x_j$ when $i \le j$, and the empty sequence when $i > j$.

3) If $x_1 = x_2 = \cdots = x_s = x$, then x_1^s is denoted by the symbol $\overset{s}{x}$.

4) The set $\{1, 2, \dots, s\}$ will be denoted by $\mathbb{N}_s$.

Let $Q \neq \emptyset$ and n, m are positive integers. If $[\ ]$ is a mapping from Q^n into Q^m, then $[\ ]$ is called an $(n, m)-$operation. A pair $(Q; [\ ])$ where $[\ ]$ is an $(n, m)-$operation is said to be an (n, m) groupoid. Every $(n, m)-$operation on Q induces a sequence $[\ ]_1, [\ ]_2, \dots, [\ ]_m$ of $n-$ary operations on the set Q, such that

$$((\forall i \in \mathbb{N}_m) \quad [x_1^n]_i = y_i) \Leftrightarrow [x_1^n] = y_1^m.$$

Let $m \geq 2$, $k \geq 1$. An $(m+k, m)-$groupoid $(Q; [\ ])$ is called an $(m+k, m)-$semigroup if for each $i \in \{0, 1, 2, \dots, k\}$

$$\left[x_1^i \left[x_{i+1}^{i+m+k}\right] x_{i+m+k+1}^{m+2k}\right] = \left[\left[x_1^{m+k}\right] x_{m+k+1}^{m+2k}\right].$$

Let $(A; [\ ])$ be an $(m+k, m)-$groupoid, where $[\ ]$ is an $(m+k, m)-$operation defined by $\left[x_1^{m+k}\right] = x_1^m$. Then $(A; [\ ])$ is an $(m+k, m)-$semigroup and it is called a left-zero $(m+k, m)-$semigroup. Dually, a right-zero $(m+k, m)-$semigroup $(B; [\ ])$ is defined by the operation $\left[x_1^{m+k}\right] = x_{k+1}^{m+k}$.

The pair $(A \times B; [\ ])$ where $[\ ]$ is an $(m+k, m)-$operation on $A \times B$ defined by:

$$\left[x_1^{m+k}\right] = y_1^m \Leftrightarrow (x_i = (a_i, b_i), y_j = (a_j, b_{j+k}), i \in \mathbb{N}_{m+k}, j \in \mathbb{N}_m)$$

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is an $(m+k,m)-$semigroup and it is a direct product of a left-zero and a right-zero $(m+k,m)-$semigroup on A and B, respectively. Such an $(m+k,m)-$semigroup is called $(m+k,m)-$rectangular band.

2. Generalized left (right) zero $(2m,m)-$semigroups of type $(3,2)$

First, we will introduced the generalized left-zero $(2m,m)-$semigroups of type $(3,2)$.

Definition 2.1. *The $(2m,m)-$semigroup $(Q;[\,])$ in which the identities:*

(GLZ I) $\left[x_1^{2m}\right]_i = \left[y_1^{j-1}x_i y_{j+1}^{j+m-1}x_{i+m}y_{j+m+1}^{2m}\right]_j$, $i,j\in\mathbb{N}_m$

(GLZ II) $\left[x_1^{3m}\right] = \left[x_1^{2m}\right]$

hold, is called generalized left-zero $(2m,m)-$semigroup of type $(3,2)$.

Proposition 2.2. *The generalized left-zero $(2m,m)-$semigroup of type $(3,2)$ is a left-zero $(2m,m)-$semigroup if and only if $\left[\overset{2m}{x}\right] = \overset{m}{x}$, for each $x\in Q$.*

Proof. Cleary, if $(Q;[\,])$ is a left-zero $(2m,m)-$semigroup then $\left[\overset{2m}{x}\right] = \overset{m}{x}$, for each $x\in Q$ is satisfied in $(Q;[\,])$.

Conversely, let $(Q;[\,])$ be a generalized left-zero $(2m,m)-$semigroup of type $(3,2)$ in which $\left[\overset{2m}{x}\right] = \overset{m}{x}$, for each $x\in Q$ is satisfied. Then

$$\left[x_1^{2m}\right]_i \overset{(\mathrm{GLZ\,I})}{=} \left[\overset{m}{x_i}\overset{m}{x_{i+m}}\right]_1 = \left[\left[\overset{2m}{x_i}\right]\overset{m}{x_{i+m}}\right]_1 = \left[\overset{2m}{x_i}\overset{m}{x_{i+m}}\right]_1 \overset{(\mathrm{GLZ\,II})}{=} \left[\overset{2m}{x_i}\right]_1 = x_i,$$

and so $\left[x_1^{2m}\right] = x_1^m$, i.e. $(Q;[\,])$ is a left-zero $(2m,m)-$semigroup. □

In the sequel we will give a characterization of generalized left-zero $(2m,m)-$semigroups of type $(3,2)$, using the generalized left-zero semigroups of type $(3,2)$, where a generalized left-zero semigroups of type $(3,2)$ is a semigroup $(Q;*)$ that satisfies the identity $x*y*z = x*y$, for each $x,\ y,\ z\in Q$.

Theorem 2.3. $\mathbf{Q} = (Q;[\,])$ *is a generalized left-zero $(2m,m)-$semigroup of type $(3,2)$ if and only if a generalized left-zero semigroup of type $(3,2)$, $(Q;*)$, such that $\left[x_1^{2m}\right]_i = x_i * x_{i+m}$ exists.*

Proof. Let $\mathbf{Q} = (Q;[\,])$ be a generalized left-zero $(2m,m)-$semigroup of type $(3,2)$. For a fixed $a\in Q$, let $*$ be an operation defined on Q by $x*y = \left[x\overset{m-1}{a}y\overset{m-1}{a}\right]_1$.

A) Clearly, $(Q;*)$ is a groupoid.

B) Let $x,y,z\in Q$. Then:

$(x*y)*z = \left[\left[x\overset{m-1}{a}y\overset{m-1}{a}\right]_1\overset{m-1}{a}z\overset{m-1}{a}\right]_1$

$\overset{(\mathrm{GLZ\,I})}{=} \left[\left[x\overset{m-1}{a}y\overset{m-1}{a}\right]_1\ldots\left[x\overset{m-1}{a}y\overset{m-1}{a}\right]_{m-1}\left[x\overset{m-1}{a}y\overset{m-1}{a}\right]_m z\overset{m-1}{a}\right]_1$

$= \left[x\overset{m-1}{a}y\overset{m-1}{a}z\overset{m-1}{a}\right]_1$

$= \left[x\overset{m-1}{a}\left[y\overset{m-1}{a}z\overset{m-1}{a}\right]_1\ldots\left[y\overset{m-1}{a}z\overset{m-1}{a}\right]_{m-1}\left[y\overset{m-1}{a}z\overset{m-1}{a}\right]_m\right]_1$

$\overset{(GLZ\,I)}{=} \left[x \overset{m-1}{a} \left[y \overset{m-1}{a} z \overset{m-1}{a}\right]_1 \overset{m-1}{a}\right]_1 = x * (y * z).$

Hence, $(Q; *)$ is a semigroup.

C) Because:

$x * y * z = \left[x \overset{m-1}{a} y \overset{m-1}{a} z \overset{m-1}{a}\right]_1 \overset{(GLZ\,II)}{=} \left[x \overset{m-1}{a} y \overset{m-1}{a}\right]_1 = x * y,$

$(Q; *)$ is a generalized left-zero semigroups of type $(3, 2)$.

D) Moreover:

$\left[x_1^{2m}\right]_i \overset{(GLZ\,I)}{=} \left[x_i \overset{m-1}{a} x_{i+m} \overset{m-1}{a}\right]_1 = x_i * x_{i+m},$

for $x_1^{2m} \in Q^{2m}$, $i \in \mathbb{N}_m$.

Conversely, suppose that a generalized left-zero semigroup of type $(3, 2)$, $(Q; *)$, exists. We define a $(2m, m)-$operation on Q by:

$$(\forall x_1^{2m} \in Q^{2m})\,(\forall i \in \mathbb{N}_m)\,\left[x_1^{2m}\right]_i = x_i * x_{i+m}.$$

E) Clearly, $(Q; [\,])$ is a $(2m, m)-$groupoid.

F) Let $x_1^{3m} \in Q^{3m}$, $i \in \mathbb{N}_m$. Then:

$\left[\left[x_1^{2m}\right] x_{2m+1}^{3m}\right]_i = \left[x_1^{2m}\right]_i * x_{2m+i} = (x_i * x_{i+m}) * x_{2m+i} = x_i * x_{i+m}.$

We will prove that $\left[x_1^j \left[x_{j+1}^{j+2m}\right] x_{j+2m+1}^{3m}\right]_i = x_i * x_{i+m}$, for $j \in \mathbb{N}_m$.

F1) If $i \le j$ then $i + m \le j + m$. Let $i + m = j + \lambda$, where $\lambda \in \mathbb{N}_m$. Then:

$\left[x_1^j \left[x_{j+1}^{j+2m}\right] x_{j+2m+1}^{3m}\right]_i = x_i * \left[x_{j+1}^{j+2m}\right]_\lambda = x_i * (x_{j+\lambda} * x_{j+\lambda+m}) = x_i * x_{j+\lambda}$
$= x_i * x_{i+m}.$

F2) If $j < i$ then $j < m$. Let $j + r = m$, $j + \lambda = i$, where $\lambda \in \mathbb{N}_r$ and $i + m > j + m$. Then:

$\left[x_1^j \left[x_{j+1}^{j+2m}\right] x_{j+2m+1}^{3m}\right]_i = \left[x_{j+1}^{j+2m}\right]_\lambda * x_{j+2m+\lambda} = (x_{j+\lambda} * x_{j+\lambda+m}) * x_{j+2m+\lambda}$
$= x_{j+\lambda} * x_{j+\lambda+m} = x_i * x_{i+m}.$

Hence, $(Q; [\,])$ is a $(2m, m)-$semigroup.

G) Let $x_1^{2m}, y_1^{2m} \in Q^{2m}$ and $x_i = y_j, x_{i+m} = y_{j+m}$ for some $i, j \in \mathbb{N}_m$. Then:

$\left[x_1^{2m}\right]_i = x_i * x_{i+m} = y_j * y_{j+m} = \left[y_1^{2m}\right]_j = \left[y_1^{j-1} x_i y_{j+1}^{j+m-1} x_{i+m} y_{j+m+1}^{2m}\right]_j.$

Thus, (GLZ I) is satisfied in $(Q; [\,])$.

H) Let $x_1^{3m} \in Q^{3m}$, $i \in \mathbb{N}_m$. Then:

$\left[x_1^{3m}\right]_i = x_i * x_{i+m} = \left[x_1^{2m}\right]_i,$

for each $i \in \mathbb{N}_m$. So, (GLZ II) is satisfied in $(Q; [\,])$. Hence, $(Q; [\,])$ is a generalized left-zero $(2m, m)-$semigroup of type $(3, 2)$. □

For the corresponding right cases we have:

Definition 2.4. *The $(2m, m)-$semigroup $(Q; [\,])$ in which the identities:*

(GRZ I) $\left[x_1^{2m}\right]_i = \left[y_1^{j-1} x_i y_{j+1}^{j+m-1} x_{i+m} y_{j+m+1}^{2m}\right]_j$, $i, j \in \mathbb{N}_m$

(GRZ II) $\left[x_1^{3m}\right] = \left[x_{m+1}^{3m}\right]$

hold, is called generalized right-zero $(2m, m)-$semigroup of type $(3, 2)$.

Proposition 2.5. *The generalized righ-zero $(2m, m)-$semigroup of type $(3, 2)$ is a right-zero $(2m, m)-$semigroup if and only if $\left[\overset{2m}{x}\right] = \overset{m}{x}$, for each $x \in Q$.*

Theorem 2.6. $\mathbf{Q} = (Q; [\,])$ *is a generalized right-zero* $(2m, m)-$*semigroup of type* $(3, 2)$ *if and only if a generalized right-zero semigroup of type* $(3, 2)$ *such that* $\left[x_1^{2m}\right]_i = x_i * x_{i+m}$ *exists.*

3. Generalized $(2m, m)-$rectangular bands of type $(3, 2)$

Definition 3.1. *The* $(2m, m)-$*semigroup* $(Q; [\,])$ *in which the identities:*

(G I) $\left[x_1^{2m}\right]_i = \left[y_1^{j-1} x_i y_{j+1}^{j+m-1} x_{i+m} y_{j+m+1}^{2m}\right]_j$, $i, j \in \mathbb{N}_m$

(G II) $\left[x_1^{5m}\right] = \left[x_1^{2m} x_{3m+1}^{5m}\right]$

(G III) $\left[\overset{2m}{\left[x_1^{2m}\right]_i}\right] = \overset{m}{\left[x_1^{2m}\right]_i}$, $i \in \mathbb{N}_m$

hold, is called generalized $(2m, m)-$*rectangular band of type* $(3, 2)$.

Proposition 3.2. *Let* $(L; [\,]^l)$ *be a generalized left-zero and* $(R; [\,]^r)$ *a generalized right-zero* $(2m, m)-$*semigroup of type* $(3, 2)$. *Then* $(Q; [\,])$, *where* $Q = L \times R$, *is a generalized* $(2m, m)-$*rectangular band of type* $(3, 2)$.

Proof. Let $(L; [\,]^l)$ be a generalized left -zero, $(R; [\,]^r)$ a generalized right-zero $(2m, m)-$semigroup of type $(3, 2)$ and $Q = L \times R$. Then $(Q; [\,])$ is a $(2m, m)-$semigroup.

Let $(x_\alpha, y_\alpha), (u_\alpha, v_\alpha) \in Q, \alpha \in \mathbb{N}_{2m}$ and $(x_i, y_i) = (u_j, v_j), (x_{i+m}, y_{i+m}) = (u_{j+m}, v_{j+m})$ for some $i, j \in \mathbb{N}_m$. Then:

$$[(x_1, y_1) \ldots (x_{2m}, y_{2m})]_i = \left(\left[x_1^{2m}\right]_i^l, \left[y_1^{2m}\right]_i^r\right) = \left(\left[u_1^{2m}\right]_j^l, \left[v_1^{2m}\right]_j^r\right)$$
$$= [(u_1, v_1) \ldots (u_{j-1}, v_{j-1})(x_i, y_i) \ldots (u_{j+m-1}, v_{j+m-1})(x_{i+m}, y_{i+m}) \ldots (u_{2m}, v_{2m})].$$

So, (G I) is satisfied in $(Q; [\,])$.

Let $(x_\alpha, y_\alpha), \in Q, \alpha \in \mathbb{N}_{5m}$. Then:

$$[(x_1, y_1) \ldots (x_{5m}, y_{5m})]_i = \left(\left[x_1^{5m}\right]_i^l, \left[y_1^{5m}\right]_i^r\right)$$
$$= \left(\left[\left[x_1^{3m}\right]^l x_{3m+1}^{5m}\right]_i^l, \left[y_1^{2m}\left[y_{2m+1}^{5m}\right]^r\right]_i^r\right) = \left(\left[\left[x_1^{3m}\right]^l x_{3m+1}^{4m}\right]_i^l, \left[y_{m+1}^{2m}\left[y_{2m+1}^{5m}\right]^r\right]_i^r\right)$$
$$= \left(\left[\left[x_1^{2m}\right]^l x_{2m+1}^{4m}\right]_i^l, \left[y_{m+1}^{3m}\left[y_{3m+1}^{5m}\right]^r\right]_i^r\right) = \left(\left[\left[x_1^{2m}\right]^l x_{2m+1}^{3m}\right]_i^l, \left[y_{2m+1}^{3m}\left[y_{3m+1}^{5m}\right]^r\right]_i^r\right)$$
$$= \left(\left[x_1^{3m}\right]_i^l, \left[y_{2m+1}^{5m}\right]_i^r\right) = \left(\left[x_1^{2m}\right]_i^l, \left[y_{3m+1}^{5m}\right]_i^r\right) = \left(\left[x_1^{2m} x_{3m+1}^{4m}\right]_i^l, \left[y_{m+1}^{2m} y_{3m+1}^{5m}\right]_i^r\right)$$
$$= \left(\left[x_1^{2m} x_{3m+1}^{5m}\right]_i^l, \left[y_1^{2m} y_{3m+1}^{5m}\right]_i^r\right)$$
$$= [(x_1, y_1) \ldots (x_{2m}, y_{2m})(x_{3m+1}, y_{3m+1}) \ldots (x_{5m}, y_{5m})]_i.$$

So, (G II) is satisfied in $(Q; [\,])$.

Let $(x_\alpha, y_\alpha), \in Q, \alpha \in \mathbb{N}_{2m}$. Then:

$$\left[\overset{2m}{[(x_1, y_1) \ldots (x_{2m}, y_{2m})]_i}\right]_j = \left[\overset{2m}{\left(\left[x_1^{2m}\right]_i^l, \left[y_1^{2m}\right]_i^r\right)}\right]_j = \left(\left[\overset{2m}{\left[x_1^{2m}\right]_i^l}\right]_j^l, \left[\overset{2m}{\left[y_1^{2m}\right]_i^r}\right]_j^r\right)$$
$$= \left(\left[\left[x_1^{2m}\right]_i^l \overset{m-1}{a} \left[x_1^{2m}\right]_i^l \overset{m-1}{a}\right]_1^l, \left[\left[y_1^{2m}\right]_i^r \overset{m-1}{a} \left[y_1^{2m}\right]_i^r \overset{m-1}{a}\right]_1^r\right)$$
$$= \left(\left[\left[x_i \overset{m-1}{a} x_{i+m} \overset{m-1}{a}\right]_1^l \overset{m-1}{a} \left[x_1^{2m}\right]_i^l \overset{m-1}{a}\right]_1^l, \left[\left[y_1^{2m}\right]_i^r \overset{m-1}{a} \left[y_i \overset{m-1}{a} y_{i+m} \overset{m-1}{a}\right]_1^r \overset{m-1}{a}\right]_1^r\right)$$

$$= \Bigg(\Big[\big[x_i \overset{m-1}{a} x_{i+m} \overset{m-1}{a} \big]_1^l \ldots \big[x_i \overset{m-1}{a} x_{i+m} \overset{m-1}{a} \big]_m^l \big[x_1^{2m} \big]_i^l \overset{m-1}{a} \Big]_1^l ,$$

$$\Big[\big[y_1^{2m} \big]_i^r \overset{m-1}{a} \big[y_i \overset{m-1}{a} y_{i+m} \overset{m-1}{a} \big]_1^r \ldots \big[y_i \overset{m-1}{a} y_{i+m} \overset{m-1}{a} \big]_m^r \Big]_1^r \Bigg)$$

$$= \Big(\big[x_i \overset{m-1}{a} x_{i+m} \overset{m-1}{a} \big[x_1^{2m} \big]_i^l \overset{m-1}{a} \big]_1^l , \big[\big[y_1^{2m} \big]_i^r \overset{m-1}{a} y_i \overset{m-1}{a} y_{i+m} \overset{m-1}{a} \big]_1^r \Big)$$

$$= \Big(\big[x_i \overset{m-1}{a} x_{i+m} \overset{m-1}{a} \big]_1^l , \big[y_i \overset{m-1}{a} y_{i+m} \overset{m-1}{a} \big]_1^r \Big)$$

$$= \Big(\big[x_1^{2m} \big]_i^l , \big[y_1^{2m} \big]_1^r \Big) = \big[(x_1, y_1) \ldots (x_{2m}, y_{2m}) \big]_i .$$

So, (G III) is satisfied in $(Q; [\,])$. Hence, $(Q; [\,])$ is a generalized $(2m, m)-$ rectangular band of type $(3, 2)$. □

In the sequel we will give a characterization of generalized $(2m, m)-$rectangular bands of type $(3, 2)$, using the generalized rectangular bands of type $(3, 2)$, where a generalized rectangular bands of type $(3, 2)$ is a semigroup $(Q; *)$ that satisfies the identities:

$x * y * z * u * v = x * y * u * v$ and

$x * y * x * y = x * y$,

for each $x, y, z, u, v \in Q$.

Theorem 3.3. $\mathbf{Q} = (Q; [\,])$ *is a generalized* $(2m, m)-$*rectangular band of type* $(3, 2)$ *if and only if a generalized rectangular band of type* $(3, 2)$, $(Q; *)$, *such that* $\big[x_1^{2m}\big]_i = x_i * x_{i+m}$ *exists.*

Proof. Let $\mathbf{Q} = (Q; [\,])$ be a generalized $(2m, m)-$rectangular band of type $(3, 2)$. For a fixed $a \in Q$, let $*$ be an operation defined on Q by $x * y = \big[x \overset{m-1}{a} y \overset{m-1}{a} \big]_1$.

A) Clearly, $(Q; *)$ is a groupoid.

B) Let $x, y, z \in Q$. Then:

$$(x * y) * z = \Big[\big[x \overset{m-1}{a} y \overset{m-1}{a} \big]_1 \overset{m-1}{a} z \overset{m-1}{a} \Big]_1$$

$$\overset{(\mathrm{G\,I})}{=} \Big[\big[x \overset{m-1}{a} y \overset{m-1}{a} \big]_1 \ldots \big[x \overset{m-1}{a} y \overset{m-1}{a} \big]_{m-1} \big[x \overset{m-1}{a} y \overset{m-1}{a} \big]_m z \overset{m-1}{a} \Big]_1$$

$$= \big[x \overset{m-1}{a} y \overset{m-1}{a} z \overset{m-1}{a} \big]_1$$

$$= \Big[x \overset{m-1}{a} \big[y \overset{m-1}{a} z \overset{m-1}{a} \big]_1 \ldots \big[y \overset{m-1}{a} z \overset{m-1}{a} \big]_{m-1} \big[y \overset{m-1}{a} z \overset{m-1}{a} \big]_m \Big]_1$$

$$\overset{(\mathrm{G\,I})}{=} \Big[x \overset{m-1}{a} \big[y \overset{m-1}{a} z \overset{m-1}{a} \big]_1 \overset{m-1}{a} \Big]_1 = x * (y * z).$$

Hence, $(Q; *)$ is a semigroup.

C) Because:

$$x * y * z * u * v = \big[x \overset{m-1}{a} y \overset{m-1}{a} z \overset{m-1}{a} u \overset{m-1}{a} v \overset{m-1}{a} \big]_1 \overset{(\mathrm{G\,II})}{=} \big[x \overset{m-1}{a} y \overset{m-1}{a} u \overset{m-1}{a} v \overset{m-1}{a} \big]_1$$

$= x * y * u * v$

and

$$x * y * x * y = \big[x \overset{m-1}{a} y \overset{m-1}{a} x \overset{m-1}{a} y \overset{m-1}{a} \big]_1 = \Big[\big[x \overset{m-1}{a} y \overset{m-1}{a} \big] x \overset{m-1}{a} y \overset{m-1}{a} \Big]_1$$

$\overset{(\mathrm{G\,I})}{=} \left[\left[x \overset{m-1}{a} y \overset{m-1}{a}\right]_1 \overset{m-1}{a} x \overset{m-1}{a} y \overset{m-1}{a}\right]_1 = \left[\left[x \overset{m-1}{a} y \overset{m-1}{a}\right]_1 \overset{m-1}{a} \left[x \overset{m-1}{a} y \overset{m-1}{a}\right]\right]_1$

$\overset{(\mathrm{G\,I})}{=} \left[\left[x \overset{m-1}{a} y \overset{m-1}{a}\right]_1 \overset{m-1}{a} \left[x \overset{m-1}{a} y \overset{m-1}{a}\right]_1 \overset{m-1}{a}\right]_1 = \left[\overset{2m}{\left[x \overset{m-1}{a} y \overset{m-1}{a}\right]_1}\right]_1$

$\overset{(\mathrm{G\,III})}{=} \left[x \overset{m-1}{a} y \overset{m-1}{a}\right]_1 = x * y,$

$(Q; *)$ is a generalized rectangular band of type $(3, 2)$.

D) Moreover:

$\left[x_1^{2m}\right]_i \overset{(\mathrm{G\,I})}{=} \left[x_i \overset{m-1}{a} x_{i+m} \overset{m-1}{a}\right]_1 = x_i * x_{i+m},$

for $x_1^{2m} \in Q^{2m}$, $i \in \mathbb{N}_m$.

Conversely, suppose that a generalized rectangular band of type $(3, 2)$, $(Q; *)$, exists. We define a $(2m, m)-$operation on Q by:

$$(\forall x_1^{2m} \in Q^{2m})\, (\forall i \in \mathbb{N}_m)\, \left[x_1^{2m}\right]_i = x_i * x_{i+m}.$$

E) Clearly, $(Q; [\,])$ is a $(2m, m)-$groupoid.

F) Let $x_1^{3m} \in Q^{3m}$, $i \in \mathbb{N}_m$. Then:

$\left[\left[x_1^{2m}\right] x_{2m+1}^{3m}\right]_i = \left[x_1^{2m}\right]_i * x_{2m+i} = (x_i * x_{i+m}) * x_{2m+i} = x_i * x_{i+m} * x_{i+2m}.$

We will prove that $\left[x_1^j \left[x_{j+1}^{j+2m}\right] x_{j+2m+1}^{3m}\right]_i = x_i * x_{i+m} * x_{i+2m}$, $j \in \mathbb{N}_m$.

a) Let $i \leq j$. Then $i + m \leq j + m$. Now, let $i + m = j + \lambda$, where $\lambda \in \mathbb{N}_m$. Then:

$\left[x_1^j \left[x_{j+1}^{j+2m}\right] x_{j+2m+1}^{3m}\right]_i = x_i * \left[x_{j+1}^{j+2m}\right]_\lambda = x_i * (x_{j+\lambda} * x_{j+\lambda+m}) = x_i * x_{i+m} * x_{i+2m}.$

b) Let $j < i$. Then $j < m$. Now, let $j + r = m$, $j + \lambda = i$, where $\lambda \in \mathbb{N}_r$. Because $i + m > j + m$, so

$\left[x_1^j \left[x_{j+1}^{j+2m}\right] x_{j+2m+1}^{3m}\right]_i = \left[x_{j+1}^{j+2m}\right]_\lambda * x_{j+2m+\lambda} = (x_{j+\lambda} * x_{j+\lambda+m}) * x_{j+2m+\lambda}$
$= x_i * x_{i+m} * x_{i+2m}.$

Hence, for all $i, j \in \mathbb{N}_m$, $\left[\left[x_1^{2m}\right] x_{2m+1}^{3m}\right]_i = \left[x_1^j \left[x_{j+1}^{j+2m}\right] x_{j+2m+1}^{3m}\right]_i$, i.e. $(Q; [\,])$ is a $(2m, m)-$semigroup.

G) Let $x_1^{2m}, y_1^{2m} \in Q^{2m}$ and, for some $i, j \in \mathbb{N}_m$, $x_i = y_j$, $x_{i+m} = y_{j+m}$. Then:

$\left[x_1^{2m}\right]_i = x_i * x_{i+m} = y_j * y_{j+m} = \left[y_1^{2m}\right]_j = \left[y_1^{j-1} x_i y_{j+1}^{j+m-1} x_{i+m} y_{j+m+1}^{2m}\right]_j.$

So, (G I) is satisfied in $(Q; [\,])$.

H) Let $x_1^{5m} \in Q^{5m}$, $i \in \mathbb{N}_m$. Then:

$\left[x_1^{5m}\right]_i = \left[\left[x_1^{3m}\right] x_{3m+1}^{5m}\right]_i = (x_i * x_{i+m} * x_{i+2m}) * x_{i+3m} * x_{i+4m}$
$= x_i * x_{i+m} * x_{i+3m} * x_{i+4m} = \left[\left[x_1^{2m} x_{3m+1}^{4m}\right] x_{4m+1}^{5m}\right]_i = \left[x_1^{2m} x_{3m+1}^{5m}\right]_i$, for all $i \in \mathbb{N}_m$.

So, (G II) is satisfied in $(Q; [\,])$.

J) Let $x_1^{2m} \in Q^{2m}$ and $i, j \in \mathbb{N}_m$. Then:

$\left[\overset{2m}{\left[x_1^{2m}\right]_i}\right]_j = \left[x_1^{2m}\right]_i * \left[x_1^{2m}\right]_i = (x_i * x_{i+m}) * (x_i * x_{i+m}) = x_i * x_{i+m} = \left[x_1^{2m}\right]_i,$

for all $j \in \mathbb{N}_m$. So, (G III) is satisfied in $(Q; [\,])$.

Hence, $\mathbf{Q} = (Q; [\,])$ is a generalized $(2m, m)-$rectangular band of type $(3, 2)$. □

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ОБОПШТЕНИ $(2m, m)$ – **ПРАВОАГОЛНИ ЛЕНТИ ОД ТИП** $(3, 2)$

Валентина Миовска и Дончо Димовски

Р е з и м е

Во овој труд, се воведени поимите за обопштените лево-нулти и десно-нулти $(2m, m)-$ полугрупи од тип $(3, 2)$, како и поимот за обопштени $(2m, m)-$ правоаголни ленти од тип $(3, 2)$. Дадени се карактеризации за обопштените лево-нулти и десно-нулти $(2m, m)-$ полугрупи од тип $(3, 2)$ и обопштените $(2m, m)-$ правоаголни ленти од тип $(3, 2)$.

"St. Cyril and Methodius University",
Faculty of Natural Sciences and Mathematics,
Institute of Mathematics, P.O. Box 162,
Skopje, Macedonia
E-mail address: miovska@pmf.ukim.mk
E-mail address: donco@pmf.ukim.mk