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TWO APPROACHES TO PROPER SHAPE THEORY

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Abstract

The first proper shape category is constructed in [1] by embedding a separable locally compact metric space in Hilbert cube without a point and by considering classes of proper fundamental nets as morphisms. In this paper we give a direct proof that this category is a subcategory of the proper shape category obtained by inverse systems of ANR's and proper maps. At the end are shown the main properties of the proper shape theory by use of inverse systems.

Introduction

At present time the mostly used approach to shape theory is by use of inverse systems and ANR expansions ([5]). There are many other approaches for which it is shown that they are equivalent.

In the proper shape theory the first approach ([1]) was by embedding a separable locally compact metric space in the Hilbert cube without a point, which coresponds to the original approach of Borsuk to shape theory.

In [2] are stated three equvalent approaches to proper shape: 1) by proper shapings, 2) using proper ANR expansions 3) using proper mutations.

Of special interest is the relation of the original approach of Ball and Sher in [1] and the approach by proper ANR expansions.

In [2], it is shown that the original proper shape category of Ball and Sher is a subcategory of the category obtained by proper ANR expansions approach. The proof is not direct and in fact this result is shown for proper mutations approach (instead of proper ANR approach) and then is used the equivalence of the approaches. For the same result in the more general case of proper n-shape we refer to [8].

In this paper we give a direct proof (Theorem 4). At the end are presented the main properties of the proper shape theory. Using the inverse system approach some of them are obvious.

I. Proper ANR systems

We repeat a few defenitions about proper maps and proper homotopy.

Let X, Y be topological spaces. A continuous map $f: X \to Y$ is *proper* if for any compact subset C of Y, $f^{-1}(C)$ is compact. Two proper maps $f_0, f_1: X \to Y$ are *properly homotopic* $(f_0 \overset{p}{\simeq} f_1)$ if there exists a proper map $F: X \times I \to Y$, such that $F(x, 0) = f_0(x)$ and $F(x, 1) = f_1(x)$.

The proper homotopy class of the proper map $f: X \to Y$ is denoted by $[f]_p$.

X, Y have *the same proper homotopy type* if there exist proper maps $f: X \to Y$ and $g: Y \to X$, such that $fg \overset{p}{\simeq} 1_Y$ and $gf \overset{p}{\simeq} 1_X$.

In the proper homotopy category PH, objects are all topological spaces, and morphisms are proper homotopy classes of proper maps.

All spaces considered here, will be metric and locally compact. ANR will mean an absolute neighbourhood retract for metric spaces. We repeat some known facts about ANR's and proper maps as:

Proposition 1. Let X be a locally compact ANR and $U \subseteq X$, U open. Then U is locally compact ANR.

Proposition 2. If X, Y are metric spaces, $f: X \to Y$ is a proper map and P is a closed subset of X then $f|_p: P \to Y$ is a proper map.

Definition 4: A proper ANR **expansion of a locally compact metric space** X consists of an inverse system $\boldsymbol{X} = \left(X_a, [f_{aa'}]_p, A\right)$ of locally compact ANR's and of a morphism $\boldsymbol{f}: X \to \boldsymbol{X}$ in pro–PH i.e. $\boldsymbol{f} = ([f_a]_p \mid a \in A)$ (a morphism $\boldsymbol{f}: X \to \boldsymbol{X}$ in pro–PH consists of proper maps $f_a: X \to X_a$ and $f_{aa'} f_{a'} \overset{p}{\simeq} f_a$, $a \leq a'$, $a, a' \in A$) such that

(i) If P is locally compact ANR and $h: X \to P$ is a proper map, then there exists $a \in A$ and a proper map $h_a: X_a \to P$ such that $h_a f_a \overset{p}{\simeq} h$.

(ii) Let P be locally compact ANR. For $a \in A$ let $h_a, h'_a: X_a \to P$ be proper maps such that $h_a f_a \overset{p}{\simeq} h'_a f_a$. Then there exists $a' \geq a$ such that $h_a f_{aa'} \overset{p}{\simeq} h'_a f_{aa'}$.

The following theorem is used in [2] (Theorem 3.2).

Theorem 1. Any locally compact metric spaces X can be embedded as a closed subset of a locally compact ANR P in such a way that there exists a cofinal set of closed ANR neighbourhoods of X in P.

Lemma 1. Let X, Y be locally compact metric spaces, P closed subset of X and $f: P \to Y$ a proper map. If $\widetilde{f}: X \to Y$ is an extension of $f: P \to Y$, then there exists a closed neighbourhood of T of P in X, such that $\widetilde{f}\,|_T: T \to Y$ is a proper map.

Proof. [[1], Lemma 3.2, p.168].

Lemma 2. Let X, Y be locally compact ANR's, P is a closed subset of X and $f, g: P \to Y$ are properly homotopic maps. If $\widetilde{f}, \widetilde{g}: X \to Y$ are extensions of f, g respectivaly, then there exists a closed neighbourhood T of P in X, such that $\widetilde{f}\,|_T \overset{p}{\simeq} \widetilde{g}\,|_T$.

Proof. Let P be a closed subset of X. Maps $f, g: P \to Y$ are properly homotopic i.e. there exists a proper homotopy $H: P \times I \to Y$, such that $H(x, 0) = f(x)$, $H(x, 1) = g(x)$.

Let $S = (X \times \{0\}) \cup (P \times I) \cup (X \times \{1\})$. S is closed subset of $X \times I$. We define a map $F: S \to Y$ with

$$F(x, t) = \begin{cases} \widetilde{f} \quad (x) & t = 0 \\ H \quad (x, t) & 0 \leq t \leq 1 \\ \tilde{g} \quad (x) & t = 1 \end{cases}$$

Because Y is an ANR, there exists an open neighbourhood U of S in $X \times I$ and an extension $\widetilde{F}: U \to Y$ of the proper map $F: S \to Y$.

U is a locally compact ANR, as an open subset of the space $X \times I$, which is a locally compact ANR. Since U, Y are locally compact spaces, S is a closed subset of of U and $\widetilde{F}$ is an extension of the proper map F, from Lemma 1 it follows that there exists a closed nighbourhood Q of S in $U \subseteq X \times I$, such that $\widetilde{F}\,|_Q: Q \to Y$ is a proper map.

We will choose a closed nighbourhood T of P in X such that $T \times I \subseteq Q$, in the following way:

Because U is a metric space, there exists an open nighbourhood V of S in U, such that $S \subseteq V \subseteq \overline{V} \subseteq Q$. Then from compactness of I we can find an open nighbourhood W of P in X, such that $W \times I \subseteq V$ i.e. $P \times I \subseteq W \times I \subseteq V$.

Because X is a metric space, and P is a closed subset of X, there exists an open nighbourhood W' in X, such that $P \subseteq \overline{W}' \subseteq \overline{W}' \subseteq W$. If follows that $P \times I \subseteq W' \times I \subseteq \overline{W}' \times I \subseteq W \times I \subseteq V \subseteq Q$.

Finally, $T = \overline{W}'$ is a closed neighbourhood of P in X, such that $T \times I \subseteq Q$.

Let $\Phi = \widetilde{F}\,|_{T \times I}$. Since $T \times I$ is closed subset of Q it follows by Proposition 2, that $\Phi: T \times I \to Y$ is a proper map and $\Phi\,(x, 0) = \widetilde{f}(x)$, $\Phi\,(x, 1) = \widetilde{g}(x)$ for $x \in T$. We proved that $\widetilde{f}\,|_T \overset{p}{\simeq} \widetilde{g}\,|_T$. □

Let H be the Hilbert cube, $p \in H$ and $K = H \backslash \{p\}$. Since H is homogenous, the space K does not depend on the choice of p.

Theorem 2. Let X be a separable, locally compact metric space. Then there exists a proper ANR-expansion of X in K (i.e. an expansion in which all spaces are subsets of K and bonding maps are inclusions).

Proof. Let X be a separable, locally compact metric space. By Th. 1, X can be embedded as a closed subspace of the locally compact ANR space K and there exists a set of closed ANR neighbourhoods $\{X_a \mid a \in A\}$ of X in K, which is cofinal in the set of all neighbourhoods. The index set A is ordered by: $a \leq a'$ if and only if $X_{a'} \subseteq X_a$.

For $a \leq a'$, $a, a' \in A$, let $i_{aa'}: X_{a'} \to X_a$ and $i_a: X \to X_a$ be inclusions. We will show that the morphism in pro–PH, $\boldsymbol{i}: X \to \boldsymbol{X}$, $\boldsymbol{i} = ([i_a]_p \mid a \in A)$, is a proper ANR expansion of X in K.

Let $f: X \to P$ be a proper map. Because P is ANR, there exists an open neighbourhood U of X in K, and an extension $\widetilde{f}: U \to P$ of f. From Lemma 2 there exists a closed neighbourhood T of X in U, such that $\widetilde{f}\,|_T: T \to P$ is a proper map.

There exists X_a such that $X_a \subseteq T$. Then the map $f_a: X_a \to P$ defined by $f_a = (\widetilde{f}\,|_T)\,|_{X_a}: X_a \to P$ is a proper map and $f_a i_a = f$.

Let P be a locally compact ANR and let $f_a, f'_a: X_a \to P$ be proper maps such that $f_a i_a \overset{p}{\simeq} f'_a i_a$. There exists an open neighbourhood U of X in X_a and an extension $\widetilde{i_a}: U \to X_a$ of $i_a: X \to X_a$.

Now $f_a \widetilde{i_a}: U \to P$ and $f'_a \widetilde{i_a}: U \to P$ are extensions of $f_a i_a$ and $f'_a i_a$, respectively.

There exists $a' \in A$, $a' \geq a$, such that $X_{a'} \subseteq U$. Then

$$f_a \widetilde{i_a}\,|_{X_{a'}}: X_{a'} \to P \qquad \text{and} \qquad f'_a \widetilde{i_a}\,|_{X_{a'}}: X_{a'} \to P$$

are extensions of $f_a i_a$ and $f'_a i_a$. By Lemma 2 there exists a closed neighbourhood T of X in $X_{a'}$, such that $\left(f_a \widetilde{i_a}\,|_{X_{a'}}\right)\,|_T \ P\overset{p}{\simeq} \left(f'_a \widetilde{i_a}\,|_{X'_a}\right)\,|_T$. There exists $a'' \in A$, $a'' \geq a'$, such that $X_{a''} \subseteq T$. If $F: T \times I \to P$ is the proper homotopy connecting $\left(f_a \widetilde{i_a}\,|_{X_{a'}}\right)\,|_T$ and $\left(f'_a \widetilde{i_a}\,|_{X_{a'}}\right)\,|_T$, then $F\,|_{X_{a''} \times I}: X_{a''} \times I \to P$ is the proper homotopy connecting

$$\left(f_a \widetilde{i_a}\,|_{X_{a'}}\right)\,|_{X_{a''}} = f_a \widetilde{i_a}\,|_{X_{a''}} \qquad \text{and} \qquad \left(f'_a \widetilde{i_a}\,|_{X_{a'}}\right)\,|_{X_{a''}} = f'_a \widetilde{i_a}\,|_{X_{a''}} \ .$$

If follows that $f_a \widetilde{i_a}\,|_{X_{a'}} \overset{p}{\simeq} f'_a \widetilde{i_a}\,|_{X_{a''}}$ i.e. $f_a i_{aa''} \overset{p}{\simeq} f'_a i_{aa''}$. □

II. Proper shape category Sh^p

We will define a proper shape category Sh^p whose objects are all closed subsets of K in the standard way:

Theorem 3. Let $\boldsymbol{p}: X \to \boldsymbol{X}$, $\boldsymbol{X} = (X_a, [p_{aa'}]_p, A)$ be a proper ANR expansion of X and let $\boldsymbol{Y} = (Y_b [g_{bb'}]_p\,, B)$ be an inverse system of locally compact ANR spaces. If $\boldsymbol{g}: X \to \mathbf{Y}$ is a morphism in pro–PH there exists a unique morphism $\boldsymbol{F}: \boldsymbol{X} \to \boldsymbol{Y}$ in pro–PH, sush that $\boldsymbol{F}\,\boldsymbol{p} = \boldsymbol{g}$.

Proof. Let $\boldsymbol{g} = ([g_b]_p \mid b \in B)$, $\boldsymbol{g}: X \to \boldsymbol{Y}$ be a morphism in pro–PH. For $b \in B$, $g_b: X \to Y_b$ is a proper map, and because $\boldsymbol{p}: X \to \boldsymbol{X}$ is proper ANR expansion there exists $F(b) \in A$ and a morphism $F_b: X_{F(b)} \to Y_b$ such that

$$F_b\, p_{F(b)} \overset{p}{\simeq} g_b\,, \tag{1}$$

To show that the pair $([F_b]_p, F)$, is a morphism in inv-PH ([5]). Let $b \le b'$. There exists $a \in A$ such that $a \ge F(b), a \ge F(b')$. Let $h_1, h_2: X_a \to Y_b$ be the morphisms: $h_1 = F_b p_{F(b)a}$ and $h_2 = q_{bb'}\, F_{b'}\, p_{F(b')a}$. We have

$$h_1 p_a = F_b\, p_{F(b)a}\, p_a \overset{p}{\simeq} F_b\, p_{F(b)} \overset{p}{\simeq} g_b \tag{2}$$

$$h_2 p_a = q_{bb'}\, F_{b'}\, p_{F(b')a}\, p_a \overset{p}{\simeq} q_{bb'}\, F_{b'}\, p_{F(b')} \overset{p}{\simeq} q_{bb'}\, g_{b'} \overset{p}{\simeq} g_b \tag{3}$$

i.e. $h_1 p_a \overset{p}{\simeq} h_2 p_a$.

From the condition ii) for expansion there is $a' \ge a$ such that $h_1 p_{aa'} \overset{p}{\simeq} h_2 p_{aa'}$. It follows $F_b\, p_{F(b)a}\, p_{aa'} \overset{p}{\simeq} q_{bb'}\, F_{b'}\, p_{F(b')a}\, p_{aa'}$, i.e. $F_b\, p_{F(b)a'} \overset{p}{\simeq} q_{bb'}\, F_{b'}\, p_{F(b')a'}$ which means that the following diagram commutes up to proper homotopy

$$\begin{array}{ccc} & X_{a'} & \\ p_{F(b)a'} \swarrow & & \searrow p_{F(b')a'} \\ X_{F(b)} & & X_{F(b')} \\ F_b \downarrow & & \downarrow F_{b'} \\ Y_b & \overset{q_{bb'}}{\leftarrow} & Y'_b \end{array}$$

We proved that $([F_b]_p,\, F)$ is a morphism of inverse systems (i.e. a morphism in inv-PH).

If $\boldsymbol{F}: \boldsymbol{X} \to \boldsymbol{Y}$ is the morphism in pro–PH, defined as the equivalence of $([F_b]_p,\, F)$, then $\boldsymbol{F}\,\boldsymbol{p} = \boldsymbol{g}$.

To show the uniqueness of the morphism $\boldsymbol{F}: \boldsymbol{X} \to \boldsymbol{Y}$, suposse that $\boldsymbol{F}': \boldsymbol{X} \to \boldsymbol{Y}$ is another morphism in pro–PH such that

$$\boldsymbol{F}'\,\boldsymbol{p} = \boldsymbol{g} \tag{4}$$

Let $\boldsymbol{F}'$ be the equivalence class of $([F'_b]_p,\, F')$. From (4) it follows

$$F'_b\, p_{F'(b)} \overset{p}{\simeq} g_b \qquad \text{for every} \qquad b \in B \tag{5}$$

From (1) and (5) it follows $F'_b\, p_{F'(b)} \overset{p}{\simeq} F_b\, p_{F(b)}$. There exists $a \in A$, such that $a \geq F(b), a \geq F'(b)$. Then $F'_b\, p_{F'(b)a} p_a \overset{p}{\simeq} F_b\, p_{F(b)a}\, p_a$. From the condition ii) from the definition of expansion, it follows that there exists $a' \in A$, such that $F'_b\, p_{F'(b)a}\, p_{aa'} \overset{p}{\simeq} F_b\, p_{F(b)a}\, p_{aa'}$, i.e. $F'_b\, p_{F'(b)a'} \overset{p}{\simeq} F_b\, p_{F(b)a'}$.

It follows that $([F_b]_p, F)$ and $([F'_b]_p, F')$ are in the same equivalence class i.e. $\boldsymbol{F} = \boldsymbol{F}'$.

Theorem 4. Let $\boldsymbol{p}: X \to \boldsymbol{X}$ and $\boldsymbol{p}': X \to \boldsymbol{X}'$ be proper ANR expansion. Then there is a unique isomorphism in pro–PH, $\boldsymbol{i}: \boldsymbol{X} \to \boldsymbol{X}'$, such that $\boldsymbol{ip} = \boldsymbol{p}'$.

Proof. Because $\boldsymbol{p}: X \to \boldsymbol{X}$ is a proper ANR expansion from Theorem 3, there exists a unique morphism $\boldsymbol{i}: \boldsymbol{X} \to \boldsymbol{X}'$ in pro–PH such that $\boldsymbol{ip} = \boldsymbol{p}'$. On the other side, from $\boldsymbol{p}': X \to \boldsymbol{X}'$ is proper ANR expansion there exists a unique morphism $\boldsymbol{j}: \boldsymbol{X}' \to \boldsymbol{X}$ in pro–PH such that $\boldsymbol{j\,p}' = \boldsymbol{p}$. Then $\boldsymbol{jip} = \boldsymbol{jp}' = \boldsymbol{p}$. It follows $\boldsymbol{ji} = 1_{\boldsymbol{X}}$. In the same way $\boldsymbol{ij} = 1_{\boldsymbol{X}'}$ i.e. $\boldsymbol{i}$ is isomorphism in pro–PH. □

Let $X\,Y$ be closed subsets of K. Then, from Theorem 2 there exist proper ANR expansions of X and Y, in K. Let $\boldsymbol{i}: X \to \boldsymbol{X}$, $\boldsymbol{X} = (X_a, [i_{aa'}]_p, A)$ and $\boldsymbol{i}': X \to \boldsymbol{X}'$, $\boldsymbol{X}' = \left(X_{a_1}, [i_{a_1 a'_1}]_p, A_1\right)$ be two proper ANR expansions of X in K, and $\boldsymbol{i}_1: Y \to \boldsymbol{Y}$, $\boldsymbol{Y} = (Y_b, [i_{bb'}]_p, B)$ and $\boldsymbol{i}'_1: Y \to \boldsymbol{Y}'$, $\boldsymbol{Y}' = \left(Y_{b_1}, [i_{b_1 b'_1}]_p, B_1\right)$ be two proper ANR expansions of Y in K. Because of Theorem 4 there exist unique isomorphisms $\boldsymbol{j}: \boldsymbol{X} \to \boldsymbol{X}'$ and $\boldsymbol{j}': \boldsymbol{Y} \to \boldsymbol{Y}'$ in pro–PH such that $\boldsymbol{j\,i} = \boldsymbol{i}'$ and $\boldsymbol{j}'\,\boldsymbol{i}_1 = \boldsymbol{i}'_1$.

Let $\boldsymbol{F}: \boldsymbol{X} \to \boldsymbol{Y}$ and $\boldsymbol{F}': \boldsymbol{X}' \to \boldsymbol{Y}'$ be morphisms in pro–PH. We define an equivalence relation "$\sim$" between triads by:

$$(\boldsymbol{i}, \boldsymbol{i}_1, \boldsymbol{F}) \sim (\boldsymbol{i}', \boldsymbol{i}'_1, \boldsymbol{F}') \qquad \text{if and only if} \qquad \boldsymbol{j}'\,\boldsymbol{F} = \boldsymbol{F}'\,\boldsymbol{j}\,. \tag{6}$$

The proper shape morphisms are the equivalence classes of this relation, i.e. $\underline{\boldsymbol{F}} = [(\boldsymbol{i}, \boldsymbol{i}_1, \boldsymbol{F})]$. These are morphisms of the category Sh^p.

We define the composition of morphisms $\underline{\boldsymbol{F}}: X \to Y$, $\underline{\boldsymbol{F}} = [(\boldsymbol{i}, \boldsymbol{i}_1, \boldsymbol{F})]$ and $\underline{\boldsymbol{G}}: Y \to Z$, $\underline{\boldsymbol{G}} = [(\boldsymbol{i}_1, \boldsymbol{i}_2, \boldsymbol{G})]$ in Sh^p. We define $\underline{\boldsymbol{GF}}: X \to Z$, as the equivalence class of the triad $(\boldsymbol{i}, \boldsymbol{i}_2, \boldsymbol{GF})$.

III. Proper shape morphisms defined by proper fundamental nets

In this part we repeat the original definition of proper shape given by Ball and Sher in [1].

Let $Y_0, Y_1, Z, (Y_0 \subseteq Z, Y_1 \subseteq Z)$, be subsets of the topological space Y. The proper maps $f_0: X \to Y_0$, $f_1: X \to Y_1$ are *properly homotopic in* Z (this is denoted by $f_0 \overset{p}{\simeq} f_1$ in Z) if there is a proper map $F: X \times I \to Z$, such that $F(x, 0) = f_0(x)$, $F(x, 1) = f_1(x)$, for every $x \in X$. This is an equivalence relation.

Definition. Let X, Y be closed subsets of K, let A be a directed set and let $\underline{f} = \{f_a \mid a \in A\}$ be a family of maps fom K to K. *A proper fundamental net from* X to Y (in (K, K)) is the triad $(\underline{f}, X, Y)$, with the following property: for every closed neighbourhood Q of Y in K, there exists a closed neighbourhood P of X in K and an index $a_0 \in A$, such that

$$f_a \,|_P \overset{p}{\simeq} f_{a_0} \,|_P \quad \text{in} \quad Q\,,$$

for all indices $a \geq a_0$, (sometimes we denote a proper fundamental net by $\underline{f}\colon X \to Y$ or only by $\underline{f}$).

Definition. Two proper fundamental nets $(\underline{f}, X, Y)$,

$$\underline{f} = \{f_a \mid a \in A\} \quad \text{and} \quad (\underline{f}', X, Y)\,, \quad \underline{f}' = \{f'_{a'} \mid a' \in A'\}$$

are *properly homotopic* $\left(\underline{f} \overset{p}{\simeq} \underline{f}'\right)$ if for any closed neighbourhood Q of Y in K, there exist a closed neighbourhood P of X in K and indices $a_0 \in A$ and $a'_0 \in A'$, such that

$$f_a \,|_P \overset{p}{\simeq} f'_{a'} \,|_P \quad \text{in} \quad Q\,,$$

for every $a \geq a_0$ and for every $a' \geq a'_0$.

This is an equivalence relation. The *proper fundamental class* is the equivalence class of the proper fundamental net $\underline{f}\colon X \to Y$. It is denoted by $[\underline{f}]$ or $[(\underline{f}, X, Y)]$.

Let $(\underline{f}, X, Y)$, $\underline{f} = \{f_a \mid a \in A\}$ and $(\underline{g}, X, Y)$, $\underline{g} = \{g_b \mid b \in B\}$ be two proper fundamental nets. We define composition $\underline{g}\,\underline{f}$ of proper fundamental nets as proper fundamental net $(\underline{h}, X, Y)$,

$$\underline{h} = \{g_b f_a \mid (a, b) \in A \times B\}\,.$$

Then the composition of two proper fundamental classes is defined in the usual way i.e.

$$[\underline{g}]\;[\underline{f}] = [\underline{g}\,\underline{f}]$$

The identical proper fundamental net on X, is the degenerated net consisting only of the function 1_K, the identity function on K.

In this way is defined another proper shape category Sh^p_{BS} whose objects are all closed subsets of K, and morphisms are the proper fundamental classes.

Relation between the categories Sh^p and Sh^p_{BS}

Theorem 4. There is a functor $\Phi: Sh^p_{BS} \to Sh^p$ defined by $\Phi(X) = X$ on objects of Sh^p_{BS} and such that Φ is injective on the set of morphisms of Sh^p_{BS}.

Proof: 1) The construction of Φ

We will define a functor $\Phi: Sh^p_{BS} \to Sh^p$. On objects it is defined by $\Phi(X) = X$.

Let X, Y be closed subsets of K. In order to associate to a morphism $[(\underline{f}, X, Y)]$ in Sh^p_{BS} a morphism in Sh^p, we choose a representative of this class, i.e. a fundamental net $(\underline{f}, X, Y)$, $\underline{f} = \{f_\lambda \mid \lambda \in \Lambda\}$. To this fundamental net we associate a triad $(\boldsymbol{i}, \boldsymbol{j}, \boldsymbol{F})$ in the following way:

Let $\boldsymbol{i}: X \to \boldsymbol{X}$, $\boldsymbol{i} = \left([i_a]_p \mid a \in A\right)$, $\boldsymbol{X} = \left(X_a, [i_{aa'}]_p, A\right)$, and $\boldsymbol{j}: Y \to \boldsymbol{Y}$, $\boldsymbol{j} = \left([j_b]_p \mid b \in B\right)$, $\boldsymbol{Y} = \left(Y_b, [j_{bb'}]_p, B\right)$, be proper ANR expansion in K, of X and Y respectively.

For $b \in B$, there exists a closed neighbourhood P of X in K, and an index $\lambda_0 \in \Lambda$ such that

$$f_\lambda |_P \overset{p}{\simeq} f_{\lambda_0} |_P \quad \text{in} \quad Y_b \tag{14}$$

for $\lambda \geq \lambda_0$.

Because $\{X_a \mid a \in A\}$ is cofinal in the set of all neighbourhoods there exists $a \in A$, such that $X_a \subseteq P$, and form (14) we obtain,

$$f_\lambda |_{X_a} \overset{p}{\simeq} f_{\lambda_0} |_{X_a} \quad \text{in} \quad Y_b\,,$$

for $\lambda \geq \lambda_0$. In this way, by putting $F(b) = a$, it is defined a function $F: B \to A$. We put

$$F_b = f_{\lambda_0} |_{X_{F(b)}} : X_{F(b)} \to Y_b\,.$$

Note that if for $\lambda, \lambda \geq \lambda_0$ we put $F'_b = f_\lambda |_{X_{F(b)}} : X_{F(b)} \to Y_b$ then $F'_b \overset{p}{\simeq} F_b$ i.e. $F'_b \in [F_b]_p$. This shows that a choice of any $\lambda \geq \lambda_0$, instead of λ_0, produces the same class of proper homotopy $[F_b]_p$.

We now show that the pair $\left([F_b]_p, F\right)$ is a morphism in inv-PH ([5]).

Let $b \leq b'$, i.e. $Y_{b'} \subseteq Y_b$. Then $F_b = f_{\lambda_0} |_{X_{F(b)}} : X_{F(b)} \to Y_b$ and $F'_b = f_{\lambda_1} |_{X_{F(b')}} : X_{F(b')} \to Y_b$.

There exists $a \in A$, such that $a \geq F(b)$, $F(b')$. Because X_a is a closed subset of $X_{F(b)}$,

$$f_{\lambda_0} |_{X_a} \overset{p}{\simeq} f_\lambda |_{X_a} \quad \text{in} \quad Y_b \tag{15}$$

for $\lambda \geq \lambda_0$, and since X_a is a closed subset of $X_{F(b')}$,

$$f_{\lambda_1}\,|_{X_a} \overset{p}{\simeq} f_\lambda\,|_{X_a} \qquad \text{in} \qquad Y_{b'}\,, \tag{16}$$

for $\lambda' \geq \lambda_1$.

There exists $\lambda_2 \in \Lambda$, such that $\lambda_2 \geq \lambda_0, \lambda_1$. From (15) and (16) we have $f_{\lambda_0}\,|_{X_a} \overset{p}{\simeq} f_{\lambda_2}\,|_{X_a}$ in Y_b and $f_{\lambda_1}\,|_{X_a} \overset{p}{\simeq} f_{\lambda_2}\,|_{X_a}$ in $Y_{b'}$. It follows

$$j_{bb'}\, f_{\lambda_1}\,|_{X_a} \overset{p}{\simeq} f_{\lambda_0}\,|_{X_a} \qquad \text{in} \qquad Y_b\,,$$

i.e.

$$j_{bb'}\, F_{b'}\, i_{F(b')\,a} \overset{p}{\simeq} F_b\, i_{F(b)\,a}\,,$$

i.e. the following diagram commutes up to proper homotopy:

$$\begin{array}{ccc}
 & X_a & \\
{}^{i_{F(b)a}}\swarrow & & \searrow{}^{i_{F(b')a}} \\
X_{F(b)} & & X_{F(b')} \\
{}^{F_b}\downarrow & & \downarrow{}^{F'_b} \\
Y_b & \overset{j_{bb'}}{\longleftarrow} & Y_{b'}
\end{array}$$

We proved that $\left([F_b]_p\,,\,F\right)$ is a morphism in inv-PH. Let $\boldsymbol{F}$ be the morphism in pro–PH defined by the equivalence class of $\left([F_b]_p\,,\,F\right)$.

To the proper fundamental net $\left(\underline{f},\, X, Y\right)$, $\underline{f} = \{f_\lambda \mid \lambda \in \Lambda\}$ we associate the triad $(\boldsymbol{i},\, \boldsymbol{j},\, \boldsymbol{F})$.

Finally, we define $\Phi\colon Sh^p_{BS} \to Sh^p$, by associating with a proper shape morphism in Sh^p_{BS}, $\left[\left(\underline{f},\, X,\, Y\right)\right]$, the proper shape morphism in Sh^p, $\underline{F}\colon X \to Y$ such that $\underline{F} = [(\boldsymbol{i},\, \boldsymbol{j},\, \boldsymbol{F})]$.

2) Φ is well defined

To prove that Φ is well defined, let $\underline{f},\, \underline{f}' \in \left[\left(\underline{f},\, X,\, Y\right)\right]$, i.e. $\underline{f} \overset{p}{\simeq} \underline{f}'$. Let $\boldsymbol{i}'\colon X \to \boldsymbol{X}'$, $\boldsymbol{i}' = \left\{[i_c]_p \mid c \in C\right\}$, $i_c\colon X \to X_c$, $\boldsymbol{X}' = \left(X_c,\, [i_{cc'}]_p\,,\, C\right)$ and let $\boldsymbol{j}'\colon Y \to \boldsymbol{Y}'$, $\boldsymbol{j}' = \left\{[j_d]_p \mid d \in D\right\}$, $j_d\colon Y \to Y_d$, $\boldsymbol{Y}' = \left(Y_d,\, [j_{dd'}]_p\,,\, D\right)$ be another proper ANR expansions of X and Y in K, respectively.

From Th. 2, there is a unique isomorphism in pro–PH, $\boldsymbol{k}\colon \boldsymbol{X} \to \boldsymbol{X}'$, such that $\boldsymbol{k}\boldsymbol{i} = \boldsymbol{i}'$. This isomorphism can be defined in the following way:

for $c \in C$, there exists $k(c) \in A$, such that $k(c) \geq c$, i.e. $X_c \supseteq X_{k(c)}$. Then, $\boldsymbol{k}: \boldsymbol{X} \to \boldsymbol{X}'$ is defined as the equivalence class of the morphism in inv-PH, $\left(\left[i_{ck(c)}\right]_p, k\right)$.

In the same way we can define the unique morphism $\boldsymbol{l}: \boldsymbol{Y} \to \boldsymbol{Y}'$ which has the property $\boldsymbol{l}\boldsymbol{j} = \boldsymbol{j}'$.

Now, let $d \in D$. Since $\underline{f} \overset{p}{\simeq} \underline{f}'$, for $Y_{l(d)}$ there exist a closed neighbourhood P of X in K and indices $\lambda_1 \in \Lambda$ and $\delta_1 \in \Delta$, such that for $\lambda \geq \lambda_1$ and for $\delta \geq \delta_1$,

$$f_\lambda \,|_P \overset{p}{\simeq} f'_\delta \,|_P \quad \text{in} \quad Y_{l(d)}\,.$$

It follows that

$$f_\lambda \,|_X \overset{p}{\simeq} f'_\delta \,|_X \quad \text{in} \quad Y_{l(d)}\,. \tag{17}$$

Let $\boldsymbol{F}: \boldsymbol{X} \to \boldsymbol{Y}$ be the morphism in pro–PH associated with $\underline{f}$, and let $\boldsymbol{F}': \boldsymbol{X}' \to \boldsymbol{Y}'$ be the morphism in pro–PH associated with $\underline{f}' = \{f'_\delta \mid \delta \in \Delta\}$.

By the definition of the morphism $\boldsymbol{F}': \boldsymbol{X}' \to \boldsymbol{Y}'$ it follows: for $d \in D$ and $F'(d)$, there is an index δ_0, such that

$$f'_\delta \,|_{X_{F'(d)}} \overset{p}{\simeq} f'_{\delta_0} \,|_{X_{F'(d)}} \quad \text{in} \quad Y_d\,,$$

for $\delta \geq \delta_0$. (By the remark in the definition it is possible to choose δ_0, so that $\delta_0 \geq \delta_1$ and to choose $F'_d = f'_{\delta_0}: X_{F'(d)} \to Y_d$.)

In the same way, by the definition of the morphism $\boldsymbol{F}: \boldsymbol{X} \to \boldsymbol{Y}$, it follows: for $l(d) \in B$, one can choose $F\big(l(d)\big)$ and an index λ_0, such that

$$f_\lambda \,|_{X_{F(l(d))}} \overset{p}{\simeq} f_{\lambda_0} \,|_{X_{F(l(d))}} \quad \text{in} \quad Y_{l(d)}$$

for $\lambda \geq \lambda_0$. (By the remark in the definition one can choose λ_0 so that $\lambda_0 \geq \lambda_1$ and to choose $F_d = f_{\lambda_0}: X_{F(l(d))} \to Y_{l(d)}$.)

It follows that

$$f'_\delta \,|_X \overset{p}{\simeq} f'_{\delta_0} \,|_X \quad \text{in} \quad Y_d \tag{18}$$

for $\delta \geq \delta_0$, and also

$$f_\lambda \,|_X \overset{p}{\simeq} f_{\lambda_0} \,|_X \quad \text{in} \quad Y_{l(d)}\,. \tag{19}$$

for $\lambda \geq \lambda_0$.

From (17) we have

$$f'_{\delta_0} \,|_X \overset{p}{\simeq} f_{\lambda_0} \,|_X \quad \text{in} \quad Y_{l(d)} \subseteq Y_d\,,$$

i.e.

$$F'_d\,|_X \overset{p}{\simeq} j_{dl(d)}\,F_{l(d)}\,|_X \quad \text{in} \quad Y_d\,.$$

It follows that

$$F'_d\, i_{F'(d)\,k\left(F'(d)\right)}\, i_{k\left(F'(d)\right)} \overset{p}{\simeq} j_{dl(d)}\,F_{l(d)}\, i_{F\left(l(d)\right)}\,,$$

i.e. $\boldsymbol{F'k\,i} = \boldsymbol{l\,F\,i}$. From Theorem 2 it follows that $\boldsymbol{F'k} = \boldsymbol{l\,F}$.
Hence, we have shown that

$$(\boldsymbol{i},\,\boldsymbol{j},\,\boldsymbol{F}) \sim (\boldsymbol{i}',\,\boldsymbol{j}',\,\boldsymbol{F}')\,,$$

i.e. that Φ is well defined.

3) Φ is a functor

Let X, Y, Z be closed subsetes of K and let

$$\boldsymbol{i}\colon X \to \boldsymbol{X},\, i = \left([i_a]_p \mid a \in A\right),\, \boldsymbol{X} = \left(X_a,\,[i_{aa'}]_p\,,\,A\right);$$
$$\boldsymbol{j}\colon Y \to \boldsymbol{Y},\, \boldsymbol{j} = \left([j_b]_p \mid b \in B\right),\, \boldsymbol{Y} = \left(Y_b,\,[j_{bb'}]_p\,,\,B\right)$$
$$\boldsymbol{m}\colon Z \to \boldsymbol{Z},\, \boldsymbol{m} = \left([m_c]_p \mid c \in C\right),\, \boldsymbol{Z} = \left(Z_c,\,[m_{cc'}]_p\,,\,C\right)$$

be proper ANR expansions in K of X, Y and Z, respectively. Let $[(\underline{f}, X, Y)]$ and $\left[(\underline{g}, Y, Z)\right]$ be proper shape morphism in Sh^p_{BS}. To the representatives of the equivalence classes $\left(\underline{f}, X, Y\right) \in \left[\left(\underline{f}, X, Y\right)\right]$ and $\left(\underline{g}, Y, Z\right) \in \left[\left(\underline{g}, Y, Z\right)\right]$, in the same way as above we associate triads $(\boldsymbol{i}, \boldsymbol{j}, \boldsymbol{F})$ and $(\boldsymbol{j}, \boldsymbol{m}, \boldsymbol{G})$ (here $\boldsymbol{F}$, $\boldsymbol{G}$ are morphisms in pro–PH, i.e. equivalence classes of morphisms $\left([F_b]_p\,,\,F\right)$ and $\left([G_c]_p\,,\,G\right)$, respectively). The composition $\boldsymbol{GF}$ in pro–PH is defined as the equivalence class of the morphism $\left(\left[G_c\,F_{G(c)}\right]_p\,,\,FG\right)$, where $FG\colon C \to A$, and

$$G_c\,F_{G(c)}\colon X_{FG(c)} \to Z_c,\; c \in C\,.$$

Let $c \in C$. Since $\left(\underline{g}, Y, Z\right)$, $\underline{g} = \{g_\delta \mid \delta \in \Delta\}$ is a proper fundamental net, for the closed neighbourhood Z_c of Z in K there exist a closed neighbourhood $Y_{G(c)}$ of Y in K and an index $\delta_1 \in \Delta$, such that for $\delta \geq \delta_1$,

$$g_\delta\,|_{Y_{G(c)}} \overset{p}{\simeq} g_{\delta_1}\,|_{Y_{G(c)}} \quad \text{in} \quad Z_c\,,$$

i.e. there exists a proper homotopy $\widetilde{G}\colon Y_{G(c)} \times I \to Z_c$, such that

$$\widetilde{G}\,(y,\,0) = g_\delta\,|_{Y_{G(c)}}\,(y) \quad \text{and} \quad \widetilde{G}\,(y,\,1) = g_{\delta_1}\,|_{Y_{G(c)}}\,(y)\,.$$

Since $(\underline{f}, X, Y)$, $\underline{f} = \{f_\lambda \mid \lambda \in \Lambda\}$ is a proper fundamental net, for the closed neighbourhood $Y_{G(c)}$ of Y, there exists a closed neighbourhood $X_{FG(c)}$ of X in K and an index $\lambda_1 \in \Lambda$, such that for $\lambda \geq \lambda_1$,

$$f_\lambda \,|_{X_{FG(c)}} \overset{p}{\simeq} f_{\lambda_1} \,|_{X_{FG(c)}} \quad \text{in} \quad Y_{G(c)} \, ,$$

i.e. there exists a proper homotopy $\tilde{F}: X_{FG(c)} \times I \rightarrow Y_{G(c)}$, such that $\tilde{F}(x, 0) = f_\lambda \,|_{X_{FG(c)}} (x)$ and $\tilde{F}(x, 1) = f_\lambda \,|_{X_{FG(c)}} (x)$.

Then $(g_\delta \,|_{Y_{G(c)}} \circ \tilde{F}: X_{FG(c)} \times I \rightarrow Z_c$ is a proper homotopy which connects $g_\delta f_\lambda \,|_{X_{FG(c)}}$ and $g_\delta f_{\lambda_1} \,|_{X_{FG(c)}}$, i.e.

$$g_\delta f_\lambda \,|_{X_{FG(c)}} \overset{p}{\simeq} g_\delta f_{\lambda_1} \,|_{X_{FG(c)}} \quad \text{in} \quad Z_c \, , \tag{20}$$

and $\widetilde{G}\left(f_{\lambda_1} \,|_{X_{FG(c)}} (x), t\right)$ is a proper homotopy in Z_c, connecting $g_\delta f_{\lambda_1} \,|_{X_{FG(c)}}$ and $g_{\delta_1} f_{\lambda_1} \,|_{X_{FG(c)}}$, i.e.

$$g_\delta f_{\lambda_1} \,|_{X_{FG(c)}} \overset{p}{\simeq} g_{\delta_1} f_{\lambda_1} \,|_{X_{FG(c)}} \quad \text{in} \quad Z_c \, . \tag{21}$$

By (20), (21) we have for $(\lambda, \delta) \geq (\lambda_1, \delta_1)$,

$$g_\delta f_{\lambda_1} \,|_{X_{FG(c)}} \overset{p}{\simeq} g_{\delta_1} f_{\lambda_1} \,|_{X_{FG(c)}} \quad \text{in} \quad Z_c \, .$$

We have thus proved that for every $c \in C$, for the closed neighbourhood Z_c of Z, there exists an index $FG(c) \in A$, i.e. a closed neighbourhood $X_{FG(c)}$ of X and there exists a pair of indices (λ_1, δ_1), such that for $(\lambda, \delta) \geq (\lambda_1, \delta_1)$,

$$g_\delta f_\lambda \,|_{X_{FG(c)}} \overset{p}{\simeq} g_{\delta_1} f_{\lambda_1} \,|_{X_{FG(c)}} \quad \text{in} \quad Z_c \, .$$

We put $H = FG$, and $H_c = g_{\delta_1} f_{\lambda_1} \,|_{X_{FG(c)}}$. Then, by the above definition, the equivalence class of $\left([H_c]_p \, , \, H\right)$, is the morphism H in pro–PH associated with the proper fundamental net $\underline{gf} = \{g_\delta f_\lambda \mid (\lambda, \delta) \in \Lambda \times \Delta\}$.

Since

$$H_c = g_{\delta_1} f_{\lambda_1} \,|_{X_{FG(c)}} = g_{\delta_1} \,|_{Y_{G(c)}} \; f_{\lambda_1} \,|_{X_{FG(c)}} = G_c F_{G(c)} \, ,$$

we have $\left([H_c]_p \, , \, H\right) = \left([G_c]_p \, , \, G\right) \left(\left[F_{G(c)}\right]_p \, , \, F\right)$ in inv–PH, and it follows that $\boldsymbol{H} = \boldsymbol{GF}$ in pro–PH.

Then $\underline{H}: X \to Z$ is a proper shape morphism in Sh^p, such that

$$\underline{H} = [(i, m, H)] = [(i, m, GF)] \,. \tag{22}$$

From $\underline{GF} = [(i, m, GF)]$ and (22) we conclude that

$$\underline{H} = \underline{G}\underline{\mathbf{F}} \text{ in } Sh^p \,. \tag{23}$$

It follows that

$$\Phi \left([(\underline{f}, X, Y)] \; [(\underline{g}, Y, Z)] \right) = \Phi \left([(\underline{f}, X, Y)] \right) \Phi \left([(\underline{g}, Y, Z)] \right) \,.$$

Now, to the proper fundamental net $(\underline{1}, X, X)$, $\underline{1} = \{1_k\}$ we associate the triad $(i, i, \mathbf{1}_X)$.

$$i: X \to \boldsymbol{X}, \qquad i = \left([i_a]_p \mid a \in A \right), \qquad \boldsymbol{X} = \left(X_a, [i_{aa'}]_p, A \right)$$

is a proper ANR expansion of X in K, and $\mathbf{1}_X$ is the equivalence class of the morphism $\left([1_{X_a}]_p, 1_A \right)$. Then $\Phi \left([(\underline{1}, X, X)] \right) = 1_{\Phi(X)} = \mathbf{1}_X$.

We have thus proved that Φ is functor.

Φ is injective

Let $i: X \to \boldsymbol{X}$, $i = \left([i_a]_p \mid a \in A \right)$, $\boldsymbol{X} = \left(X_a, [i_{aa'}]_p, A \right)$ and let $i': X \to \boldsymbol{X}'$, $i' = \left([i_b]_p \mid b \in B \right)$, $\boldsymbol{X}' = \left(X_b, [i_{bb'}]_p, B \right)$ be two proper ANR expansions of X in K, and

$$j: Y \to \boldsymbol{Y}, \quad j = \left([j_c]_p \mid c \in C \right), \quad \boldsymbol{Y} = \left(Y_c, [j_{cc'}]_p, C \right)$$

and

$$j': Y \to \boldsymbol{Y}', \quad i' = \left([j_d]_p \mid d \in D \right), \quad \boldsymbol{Y}' = \left(Y_d, [j_{dd'}]_p, D \right)$$

be two proper ANR expansions of Y in K.

Let $(\underline{f}, X, Y)$, $\underline{f} = \{f_\lambda \mid \lambda \in \Lambda\}$ and $(\underline{g}, X, Y)$, $\underline{g} = \{g_\delta \mid \delta \in \Delta\}$ be two proper fundamental nets and let

$$\Phi \left([(\underline{f}, X, Y)] \right) = \underline{F} \qquad \text{and} \qquad \Phi \left([(\underline{g}, X, Y)] \right) = \underline{G} \,.$$

Let $i, j, F)$ be a representative of $\underline{F}$, and (i', j', G) be a representative of $\underline{G}$. Suposse that $\underline{F} = \underline{G}$. It follows that $(i, j, F) \sim (i', j', G)$, i.e.

$$l\,F = G\,k \,, \tag{24}$$

where $\boldsymbol{k}: \boldsymbol{X} \to \boldsymbol{X}'$ and $\boldsymbol{l}: \boldsymbol{Y} \to \boldsymbol{Y}'$ are isomorphisms in pro–PH, defined by the $\left(\left[i_{bk(b)}\right]_p, k\right)$ and $\left(\left[j_{dl(d)}\right]_p, l\right)$, morphisms in inv-PH, respectively.

In order to prove that Φ is injective, we have to show that $\underline{f} \overset{p}{\simeq} \underline{g}$.

Let $d \in D$ and Y_d be a closed neighbourhood of Y in K. Then for the map $j_{dl(d)} F_{l(d)}: X_{F\left(l(d)\right)} \to Y_d$ we have

$$j_{dl(d)} F_{l(d)} \overset{p}{\simeq} j_{dl(d)} f_\lambda \,|X_{F\left(l(d)\right)} \quad \text{in } Y_d \tag{25}$$

for $\lambda \geq \lambda_0$, and also for the map $G_d\, i_{G(d)k\left(G(d)\right)}: X_{k\left(G(d)\right)} \to Y_d$, we have

$$G_d\, i_{G(d)k\left(G(d)\right)} \overset{p}{\simeq} g_\delta\, i_{G(d)k\left(G(d)\right)} \,|X_{k\left(G(d)\right)} \quad \text{in } Y_d \tag{26}$$

for $\delta \geq \delta_0$.

By (24), for $d \in D$ there exists $a \in A$, such that $a \geq F\big(l(d)\big), k\big(G(d)\big)$ and there exists a closed neighbourhood X_a of X such that:

$$j_{dl(d)} F_{l(d)} \,|X_a \overset{p}{\simeq} G_d\, i_{G(d)k\left(G(d)\right)} \,|X_a \quad \text{in } Y_d\,. \tag{27}$$

On the other side from (25) and (26), it follows that:

$$j_{dl(d)} F_{l(d)} \,|X_a \overset{p}{\simeq} j_{dl(d)} f_\lambda \,|X_a \quad \text{in } Y_d\,. \tag{28}$$

for $\lambda \geq \lambda_0$, and

$$i_{G(d)k\left(G(d)\right)} G_d \,|X_a \overset{p}{\simeq} g_\delta\, i_{G(d)k\left(G(d)\right)} \,|X_a \quad \text{in } Y_d \tag{29}$$

for $\delta \geq \delta_0$.

From (27), (28) and (29) it follows that for $\lambda \geq \lambda_0$ and $\delta \geq \delta_0$

$$j_{dl(d)} f_\lambda \,|X_a \overset{p}{\simeq} g_\delta, i_{G(d)k\left(G(d)\right)} \,|X_a \quad \text{in } Y_d\,,$$

i.e. $\underline{f} \overset{p}{\simeq} g$. It follows that $\big[(\underline{f}, X, Y)\big] = \big[(\underline{g}, X, Y)\big]$, i.e. Φ is injective. □

The question whether $\Phi: Sh^p_{BS} \to Sh^p$ is an isomorphism remains open.

By the previous theorem the image under the functor $\Phi: Sh^p_{BS} \to Sh^p$ of an isomorphism $\big[(\underline{f}, X, Y)\big]$ in category Sh^p_{BS}, is an isomorphism in the category Sh^p. Therefore we obtain the following theorem.

Theorem 5. If X, Y are subsets of K, then $Sh^p X = Sh^p_{BS} Y$ implies $Sh^p_{BS} X = Sh^p Y$. □

From this theorem and from some known facts about inverse systems, the following fundamental properties of the proper shape category Sh^p (first stated in [1] and [2]) are easily proved.

If X, Y are locally compact metric spaces and $X \overset{p}{\simeq} Y$, then from [1], Theorem 3.10, $Sh^p_{BS} X = Sh^p_{BS} Y$, and from Theorem 5, it follows $Sh^p X = Sh^p Y$.

In the other direction we have:

Theorem 6. If X, Y are locally compact metric ANR's and $Sh^p X = Sh^p Y$ then $X \overset{p}{\simeq} Y$.

Proof. We can associate with X and Y a trivial ANR expansions consisting only of the spaces X and Y respectively. □

Let X, Y be compact metric spaces. We can associate with these spaces ANR expansions $X = (X_n, [i_{n,n+1}])$ and $Y = (Y_n, [i_{n,n+1}])$. which are inverse sequences of compact sets. Since maps between compact sets are always proper, the proper shape theory for compact sets is the same as the usual shape thory i.e. we have the following theorem.

Theorem 7. If X, Y are compact metric spaces, then $Sh^p X = Sh^p Y$ if and only if $Sh\, X = Sh\, Y$. □

Finally if X, Y are locally compact metric spaces, X is compact and Y is not then X has an ANR expansion which is an inverse sequence and Y does not. It follows that $Sh^p X$ differs from $Sh^p Y$.

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ДВА ПРИСТАПИ ВО ТЕОРИЈАТА НА ПРАВ ОБЛИК

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Резиме

Во денешно време, најмногу користен пристап во теоријата на облик е пристапот со инверзни системи на АНР експанзии. Оргиналниот пристап во теоријата на прав облик е со сместување на сепарабилен локално компактен метрички простор во Хилбертов куб без една точка [1]. Морфизми на правиот облик помеѓу два простора се класите на еквиваленција на правите фундаментални мрежи.

Во овој труд даваме директен доказ дека оригиналната категоријата на прав облик од [1] е поткатегорија на категоријата добиена со инверзни системи и прави АНР експанзии. На крајот се покажани неколку основни теореми во теоријата на прав облик за пристапот со инверзни системи и прави АНР експанзии.

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