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A BOUNDARY PROBLEM OF THE THIRD AND FOURTH ORDERS AS A PRODUCT OF BOUNDARY PROBLEMS OF THE SECOND ORDER

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Abstract

Conditions when a boundary problem of the third and fourth orders has as a solution a product of the solutions of boundary problems of the second order in the case of general homogeneous boundary conditions have been obtainned in this paper.

1. Introduction

Formation of linear differential equations the integrals of which are products of linear equations of the second order is presented in the paper [6].

The papers [2] and [7] consider the boundary problems of the fourth i.e. third order the solution of which of which is a third or second power of the solution of the boundary problem of the second order under special boundary conditions.

Boundary problems, the solution of which is a product of the solutions of boundary problems of the second order in the case of general linear homogeneous boundary conditions, are a subject of this paper.

1. Let the solutions of the two boundary problems of the second order be known:

$$u'' + p_1 u' + q_1 u = 0 \qquad (p_1,\, q_1\ -\ \text{const.}) \tag{1.1}$$

$$\alpha_{i_0}\, u_a + \alpha_{i_1}\, u'_a = -\left(\beta_{i_0}\, u_b + \beta_{i_1}\, u'_b\right)\,, \quad (i = 1,\, 2) \tag{1.2}$$

and

$$v'' + p_2 v' + q_2 v = 0\,, \qquad (p_2,\, q_2\ -\ \text{const.}) \tag{1.3}$$

$$\overline{\alpha}_{j_0}\, v_a + \overline{\alpha}_{j_1}\, v'_a = -\left(\overline{\beta}_{j_0}\, v_b + \overline{\beta}_{j_1}\, v'_b\right)\,, \quad (i = 1,\, 2) \tag{1.4}$$

where $u_c = u(c)$, $u'_c = u'(c)$ for $c = a, b$ and where the rank of the matrix of the coefficients $\alpha_{\mu\nu}$ and $\beta_{\mu\nu}$ i.e. $\overline{\alpha}_{\mu\nu}$ and $\overline{\beta}_{\mu\nu}$ ($\mu = 1, 2$; $\nu = 0, 1$) is two.

If we take p_i and q_i $(i = 1, 2)$ as parameters in the differential equation (1.1) and (1.3), then (1.1)–(1.2) and (1.3)–(1.4) are problems with eigen values.

1.1. Let u and v be respective solutions of the differential equations (1.1) and (1.3).

Theorem **1**. The function

$$y = uv \tag{1.5}$$

is the solution of the differential equation

$$\begin{aligned} y''' + 3(p_1 + p_2)y'' + [2\,(q_1 + 3q_2) + p_1\,(p_1 + 3p_2)]\,y' + \\ + [q_1\,(p_1 + 3p_2) + q_2\,(3p_1 + p_2)]\,y = 0 \end{aligned} \tag{1.6}$$

if

$$4(q_1 - q_2) + p_2^2 - p_1^2 = 0 \tag{1.7}$$

is satisfied for the coefficients of the differential equations (1.1) and (1.3).

Proof: Let the condition (1.7) be satisfied. Differentiating (1.5) twice we obtain

$$y' = u'\, v + u\, v'\,, \qquad y'' = u''\, v + 2u'\, v' + u\, v''\,. \tag{1.8}$$

In accordance with (1.1) and (1.3) we obtain

$$y'' + p_1\, y' + (q_1 + q_2)y + (p_2 - p_1)uv' = 2u'\, v'\,. \tag{1.9}$$

With the differentiation of (1.9) and the use of (1.1), (1.3), (1.8) and (1.9), the differential expression obtains the form of:

$$\begin{aligned} 2y''' + 3(p_1 + p_2)y'' + [2\,(q_1 + 3q_2) + p_1\,(p_1 + 3p_2)]\,y' + \\ + [q_1\,(p_1 + 3p_2) + q_2\,(3p_1 + p_2)]\,y + \\ + \left[4\,(q_1 - q_2) + p_2^2 - p_1^2\right]\,u\,v' = 0\,, \end{aligned} \tag{1.10}$$

while in accordance with (1.7), (1.6) is obtained.

Theorem **2**. The function $y = uv$ is a solution of the differential equation

$$y^{\mathrm{IV}}+2(p_1+p_2)y'''+\left[2\,(q_1+q_2)+(p_1+p_2)^2+p_1p_2\right]y''+$$
$$+(p_1+p_2)\left[2(q_1+q_2)+p_1p_2\right]y'+$$
$$+\left[(q_1-q_2)^2+(p_1+p_2)(p_1q_2+p_2q_1)\right]y=0 \qquad (1.11)$$

if

$$4q_1-4q_2+p_2^2-p_1^2\neq 0\,. \qquad (1.12)$$

Proof: Let the condition (1.12) be satisfied. The linear homogeneous differential equation (1.11) is obtaind by the differentiation of (1.11) and regarding (1.3), (1.5), (1.9) and (1.10).

If the equations

$$\begin{aligned} r^2+p_1r+q_1&=0\,,\\ r^2+p_2r+q_2&=0 \end{aligned} \qquad (1.13)$$

are characteristic equations of the differential equations (1.1) and (1.3) respectively, it follows from the the condition (1.7):

$$p_1^2-4q_1=p_2^2-4q_2\,, \qquad (1.14)$$

i.e. discriminants of the characteristic equations (1.13) are equal.

Consequance **1**. A differential equation, having as solution a product of the solutions of the differential equations (1.1) and (1.3), is of the third order, if the discriminants of the characteristic equations (1.13) are equal to each other. The differential equation is of the fourth order, if the discriminants of the characteristic equations (1.3) differ from each other.

1.2. Let u_1, u_2 and v_1, v_2 be linearly independent particular integrals corresponding to the differential equations (1.1) and (1.3).

Theorem **3**. The functions

$$y_1=u_1\,v_1\,,\qquad y_2=u_1\,v_2\,,\qquad y_3=u_2\,v_1\,,\qquad y_4=u_2\,v_2 \qquad (1.15)$$

are linearly independent praticular integrals of the differential equation (1.11) if the (1.12) is satisfied.

Proof: Let $W_1=W(u_1,u_2)$ and $W_2=W(v_1,v_2)$ be Wronskian determinants for the particular integrals u_1, u_2 and v_1, v_2 respektively. The Wronskian determinant for the system of functions (1.15), having been figured and (1.12) having been taken into account, is different from zero, i.e.

$$W=W_1^2\,W_2^2\left[4(q_2-q_1)+p_1^2-p_2^2\right]\neq 0\,,$$

which means that the functions (1.15) are linearly independent, i.e. y_1, y_2, y_3 and y_4 are linearly independent particular integrals of the linear homogeneous differential equations (1.11).

Consequence **1**. If

$$4(q_2 - q_1) + p_1^2 - p_2^2 = 0 \tag{1.7}$$

then the functions y_1, y_2, y_3 and y_4 are linearly independent.

Consequence **2**. Three functions among the functions (1.15) are always linearly independent.

Truly, let us suppose that every triple of the functions (1.15) are linearly independent i.e. the minors of the third order which correspond to the elements of the fourth order of the Wronskian determinant

$$W(y_1, y_2, y_3, y_4) = \begin{vmatrix} u_1v_1 & u_1v_2 & u_2v_1 & u_2v_2 \\ (u_1v_1)' & (u_1v_2)' & (u_2v_1)' & (u_2v_2)' \\ (u_1v_1)'' & (u_1v_2)'' & (u_2v_1)'' & (u_2v_2)'' \\ (u_1v_1)''' & (u_1v_2)''' & (u_2v_1)''' & (u_2v_2)''' \end{vmatrix} \tag{1.16}$$

all of them equal zero, i.e.

$$\begin{aligned} W_1W_2\left[(p_1 - p_2)u_2v_2 + 2(u_2'\,v_2 - u_2v_2')\right] &= 0\,, \\ W_1W_2\left[(p_1 - p_2)u_2v_1 + 2(u_2'\,v_1 - u_2v_1')\right] &= 0\,, \\ W_1W_2\left[(p_1 - p_2)u_1v_2 + 2(u_1'\,v_2 - u_1v_2')\right] &= 0\,, \\ W_1W_2\left[(p_1 - p_2)u_1v_1 + 2(u_1'\,v_1 - u_1v_1')\right] &= 0\,. \end{aligned} \tag{1.17}$$

It is obtained from the first two equations of the system of equations (1.17)

$$\frac{v_1'}{v_1} = \frac{v_2'}{v_2} = 0 \qquad \text{i.e} \qquad W(v_1, v_2) = 0$$

which is contrary to the supposition that $W(v_1, v_2) \neq 0$.

By analogy, from the two last equations of the system of equations (1.17), we come to the conclusion that $W(u_1, u_2,) = 0$, which is contrary to the supposition that $W(u_1, u_2) \neq 0$.

Thus, we have proved that at least one minor of the third order is different from zero i.e. three of the particular integrals y_i $(i = \overline{1, 4})$ are always linearly independent.

2. Let us determine the boundary conditions.

2.1 We will obtain the boundary conditions according to y if we from following products from the boundary conditions (1.2) and (1.4)

$$\begin{aligned} &(\alpha_{i_0}u_a + \alpha_{i_1}u_a')\,(\overline{\alpha}_{j_0}v_a + \overline{\alpha}_{j_1}v_a') = \\ &= (\beta_{i_0}u_b + \beta_{i_1}u_b')\,(\overline{\beta}_{j_0}v_b + \overline{\beta}_{j_1}v_b') \qquad (i, j = 1, 2)\,. \end{aligned} \tag{2.1}$$

Regarding (1.5), (1.6), (1.8) and (1.9) they are of the type

$$\begin{aligned} &\gamma_{k_0} y_a + \gamma_{k_1} y_a' + \gamma_{k_2} y_a'' + \gamma_{k_3} y_a''' = \\ &= \delta_{k_0} y_b + \delta_{k_1} y_b' + \delta_{k_2} y_b'' + \delta_{k_3} y_b''' , \qquad (k = \overline{0, 3}) \end{aligned} \tag{2.2}$$

where

$$\begin{aligned} \gamma_{k_0} =& 2\left[4(q_1 - q_2) + p_2^2 - p_1^2\right] \alpha_{i_0} \overline{\alpha}_{j_0} + \\ &+ 2\left[2(q_1^2 - q_2^2) - (p_2 - p_1)(p_1 q_2 + p_2 q_1)\right] \alpha_{i_1} \overline{\alpha}_{j_1} + \\ &+ 2\left[p_1(q_1 + 3q_2) + p_2(3q_1 + q_2)\right] (\alpha_{i_1} \overline{\alpha}_{j_o} - \alpha_{i_0} \overline{\alpha}_{j_1}) , \\ \gamma_{k_1} =& 2\left[p_1(3q_1 + q_2) - p_2(q_1 + 3q_2) + p_1 p_2(p_1 - p_2)\right] \alpha_{i_1} \overline{\alpha}_{j_1} + \\ &+ 2\left[2(3q_1 + q_2) + p_2(3p_1 + p_2)\right] \alpha_{i_1} \overline{\alpha}_{j_0} - \\ &- 2\left[2(q_1 + 3q_2) + p_1(p_1 + 3p_2)\right] \alpha_{i_0} \overline{\alpha}_{j_1} \\ \gamma_{k_2} =& 2\left[2(q_1 - q_2) + p_1^2 - p_2^2\right] \alpha_{i_1} \overline{\alpha}_{j_1} + \\ &+ 6(p_1 + p_2) (\alpha_{i_1} \overline{\alpha}_{i_0} - \alpha_{i_0} \overline{\alpha}_{j_1}) , \\ \gamma_{k_3} =& 4 (\alpha_{i1} \overline{\alpha}_{j_0} - \alpha_{i_0} \overline{\alpha}_{j_1}) + 2(p_1 - p_2) \alpha_{i_1} \overline{\alpha}_{j_1} , \end{aligned} \tag{2.3}$$

while $\delta_{k_l} (l = \overline{0, 3})$ are obtained if α_{i_l} and $\overline{\alpha}_{j_l}$ in the coefficients γ_{k_l} are replaced by β_{i_l} and $\overline{\beta}_{j_l}$ respectively, supposing that (1.12) is satisfied.

Note: In the coefficients γ_{k_l} and δ_{k_l}

when	$i = j = 1$	we take	$k = 0$;
when	$i = 2, \quad j = 1$	we take	$k = 1$;
when	$i = 1, \quad j = 2$	we take	$k = 2$;
when	$i = j = 2$	we take	$k = 3$.

2.2. If the boundary conditions (1.2) and (1.4) are Sturm's i.e. of the type

$$\begin{aligned} \alpha_{10}\, u_a + \alpha_{11}\, u_a' &= 0 , \\ \beta_{20}\, u_b' + \beta_{21}\, u_b' &= 0 , \end{aligned} \tag{2.4}$$

and

$$\begin{aligned} \overline{\alpha}_{10}\, v_a + \overline{\alpha}_{11}\, v_a' &= 0 , \\ \overline{\beta}_{20}\, v_b + \overline{\beta}_{21}\, v_b' &= 0 , \end{aligned} \tag{2.5}$$

then it is easy to provide the boundary conditions for the fourth order problem in the point of $x = a$, while namely

$$
\begin{aligned}
\alpha_{11}\, y_a' + \alpha_{10}\, y_a - \alpha_{11}\, u_a\, v_a' &= 0\,, \\
\alpha_{11}\, y_a'' + p_1\alpha_{11}\, y_a' + (q_1+q_2)\alpha_{11}\, y_a + & \\
+\big[2\alpha_{10} + (p_2-p_1)\alpha_{11}\big]\, u_a\, v_a' &= 0 \\
\overline{\alpha}_{10} y_a + \overline{\alpha}_{11}\, u_a\, v_a' &= 0 \\
\overline{\alpha}_{11} y_a'' + \big(p_1\overline{\alpha}_{11} + \overline{\alpha}_{10}\big) y_a' + (q_1+q_2)\overline{\alpha}_{11}\, y_a + & \\
+\big[(p_2-p_1)\overline{\alpha}_{11} - \overline{\alpha}_{10}\big]\, u_a\, v_a' &= 0
\end{aligned} \tag{2.6}
$$

where $u_a v_a'$ is replased by (1.10). Accordingly, the boundary conditions concerning the problem in the point of $x = b$ are obtained.

Eight boundary conditions are obtained, four in each point. These quadruples of boundary conditions are dependent, but three for each point are linearly independet. These six conditions determine $\binom{6}{4}$ quadruples of boundary conditions, which with the equation (1.11) determine $\binom{6}{4} = 15$ the fourth order boundary problems, the solution of which is a product of the solutions of the problems (1.1)–(2.4) and (1.3)–(2.5).

Note: Boundary conditions which are not grouped two in each point will emerge among the quadruple boundary conditions, but there are also such boundary conditions, in which one boundary condition is at one end (a or b), while the remaining three are at the other end of the interval (b or a), and it is not common conditions not in equal numbers at the two ends of the interval in case of the problems with eigen values with a differential equation of an even order to be considered.

3 Let us see the manifestation of the results obtained,.

The differential equation (1.1) is reduced to the differential equation of G. Cimmino [1]

$$y^{\mathrm{IV}} + (\lambda+1)m^2 y'' + \lambda m^4 y = 0 \qquad (m+0) \tag{3.1}$$

if

$$p_1 + p_2 = 0 \tag{3.2}$$

$$2(q_1+q_2) + (p_1+p_2)^2 + p_1 p_2 = (\lambda+1)m^2\,, \tag{3.3}$$

$$(p_1+p_2)\big[2(q_1+q_2) + p_1 p_2\big] = 0 \tag{3.4}$$

$$(q_1-q_2)^2 + (p_1+p_2)(p_1 q_2 + p_2 q_1) = \lambda m^4\,. \tag{3.5}$$

Il follows from the conditions (3.4) and (3.2) that also the expression

$$2(q_1+q_2)+p_1p_2$$

can be zero, but it does not have to.

1°. Let

$$2(q_1+q_2)+p_1p_2=0\,. \tag{3.6}$$

With regard (3.4), the expression (3.6) is

$$2(q_1+q_2)=p_1^2$$

and then it follows from (3.3) that

$$(\lambda+1)m^2=0$$

from where $\lambda=-1$ and the differential equation (3.1) obtains a from of

$$y^{\mathrm{IV}}-m^4y=0\,. \tag{3.7}$$

The condition (3.5) points out that $\lambda=-1$ is not possible, since if $\lambda=-1$, then it follows from (3.5) that

$$(q_1-q_2)^2=-m^4\,.$$

Accordingly, in the case of the condition (3.6) the differential equation (3.1) has no solution of the from we are searching for.

2°. Let

$$2(q_1+q_2)+p_1p_2\neq 0\,. \tag{3.8}$$

Then the conditions from (3.2) to (3.5) are reduced to the conditions

$$p_1+p_2=0$$

$$2(q_1+q_2)+p_1p_2=(\lambda+1)m^2$$

$$(q_1-q_2)^2=\lambda m^4$$

from where it follows that $\lambda>0$.

Let $\lambda=k^4$, then from $p_1=-p_2=p$ it follows that

$$2(q_1+q_2)-p^2=(k^4+1)m^2$$

$$(q_1-q_2)^2=k^4m^4$$

i.e.

$$q_1+q_2=\frac{1}{2}\left[p^2+m^2\left(1+k^2\right)\right]$$

$$q_1-q_2=\pm k^2m^2\,.$$

a) If $q_1-q_2=k^2m^2$, then

$$q_1=\frac{1}{4}\left[p^2+m^2\left(k^2+1\right)^2\right]\,,\qquad q_2=\frac{1}{4}\left[p^2+m^2\left(k^2-1\right)^2\right]\,.$$

b) If $q_1 - q_2 = -k^2 m^2$, then

$$q_1 = \frac{1}{4}\left[p^2 + m^2\left(k^2 - 1\right)^2\right], \qquad q_2 = \frac{1}{4}\left[p^2 + m^2\left(k^2 + 1\right)^2\right].$$

i.e. we have obtained the pairs of differential equations, respectively

$$u'' + pu' + \frac{1}{4}\left[p^2 + m^2\left(k^2 + 1\right)^2\right] u = 0 \tag{3.9}$$

(I)

$$v'' - pv' + \frac{1}{4}\left[p^2 + m^2\left(k^2 - 1\right)^2\right] v = 0 \tag{3.10}$$

i.e.

$$u'' + pu' + \frac{1}{4}\left[p^2 + m^2\left(k^2 - 1\right)^2\right] u = 0, \tag{3.11}$$

(II)

$$v'' - pv' + \frac{1}{4}\left[p^2 + m^2\left(k^2 + 1\right)^2\right] v = 0. \tag{3.12}$$

The discriminants of each of the characteristic equations of the differential equations (I) and (II) respectively are:

$$\begin{array}{ll} \mathrm{I}_1 & -m^2(k^2+1)^2 = -D_1^2, \\ \mathrm{I}_2 & -m^2(k^2-1)^2 = -D_2^2, \\ \mathrm{II}_1 & -m^2(k^2-1)^2 = -D_2^2, \\ \mathrm{II}_2 & -m^2(k^2+1)^2 = -D_1^2. \end{array}$$

They differ from each other. Consequently, the differential equation (3.1) has a solution which is a product of the differential equations (3.9) and (3.10) i.e. (3.11) and (3.12).

The particular integrals correspondings to the pairs of differential equations (I) and (II) are:

$$u_1 = e^{-px/2}\cos(D_1 x/2), \qquad u_2 = e^{-px/2}\sin(D_1 x/2);$$

$$v_1 = e^{px/2}\cos(D_2 x/2), \qquad v_2 = e^{px/2}\sin(D_2 x/2);$$

$$u_1 = e^{-px/2}\cos(D_2 x/2), \qquad u_2 = e^{-px/2}\sin(D_2 x/2);$$

$$v_1 = e^{px/2}\cos(D_1 x/2), \qquad v_2 = e^{px/2}\sin(D_1 x/2).$$

The particular integrals of the differential equation (3.1) are:

$$y_1 = u_1 v_1 = \overline{u}_1\,\overline{v}_1 = \cos(D_1 x/2)\cos(D_2 x/2)\,,$$

$$y_2 = u_1 v_2 = \overline{u}_2\,\overline{v}_1 = \cos(D_1 x/2)\sin(D_2 x/2)\,,$$

$$y_3 = u_2 v_1 = \overline{u}_1\,\overline{v}_2 = \sin(D_1 x/2)\cos(D_2 x/2)\,,$$

$$y_4 = u_2 v_2 = \overline{u}_2\,\overline{v}_2 = \sin(D_1 x/2)\sin(D_2 x/2)\,.$$

It is easy to check that each of them is a linear combination of the functions:

$$\cos mk^2x\,, \qquad \cos mx \qquad \sin mk^2x\,, \qquad \sin mx$$

which are linearly and they are the particular integrals obtained in [3], with the difference that k^2 here, is k in [3].

To the problem with eigen values with a differential equation (3.1) and boundary conditions

$$y(0) = y'(0) = y\left(\frac{2\pi}{m}\right) = y'\left(\frac{2\pi}{m}\right) = 0 \tag{3.13}$$

the problems with eigen values of the second order with a differential equation (3.9) and boundary conditions

$$u(0) = u'\left(\frac{2\pi}{m}\right) = 0 \tag{3.14}$$

and problem with a differential equation (3.10) and boundary conditions

$$v(0) = v'\left(\frac{2\pi}{m}\right) = 0\,, \tag{3.15}$$

are related.

It follows from the coefficients in the case of boundary conditions (3.14) and (3.15) that

$$A = B = \overline{A} = \overline{B} = 0$$

and

$$A_{00} = \overline{A}_{00} = 1, \quad A_{01} = \overline{A}_{01} = 0, \quad A_{10} = \overline{A}_{10} = 0, \quad A_{11} = \overline{A}_{11} = 0$$

i.e. both the boundary problems of the second order have the same transcendent equation for obtaining eigen values, [4],

$$\sin\frac{m(k^2+1)}{2}\frac{2\pi}{m} = 0\,, \qquad \sin(k^2+1)\pi = 0. \tag{3.16}$$

The eigen values are

$$\lambda_{n-1} = k^4 = (n-1)^2 \quad \text{i.e.} \quad \lambda_n = n^2\,, \qquad n \in N, \tag{3.17}$$

whole the expressions of the eigen values, [4], are correspondingly

$$u_n = e^{-px/2} \cdot 2sin\, \frac{m(k^2+1)}{2}\, x$$

$$v_n = e^{px/2} \cdot 2sin\, \frac{m(k^2-1)}{2}\, x\,.$$

The expression of the eigen values for the problem with eigen values (3.1)–(3.13) is:

$$y_n = u_n v_n = 4 \sin\, \frac{m}{2}\,(k^2+1)x \cdot \sin\, \frac{m}{2}(k^2-1)x$$

i.e.

$$y_n = 2(\cos\, mx - \cos\, mk^2 x) = 2(\cos\, mx - \cos\, mnx)\,. \tag{3.18}$$

The transcendent equation (3.16) for obtaining the eigen values (3.17) are obtained in [3, 1].

The boundary conditions (3.14) and (3.15) regarding (2.6) determine the conditions

$$y_a = y'_a = y'''_a = y_b = y'_b = y'''_b = 0\,. \tag{3.19}$$

The problems with eigen values with a differential equation (3.1) and a quadruple of boundary conditions of the conditions (3.19) have the same solution an same eigen values as the problem with eigen values (3.1)–(3.13). They are $\binom{6}{4} = 15$ problems with eigen values which have been solved in this way. Their boundary conditions are:

1°. (0, 1; 0, 1) i.e. $y(0) = y'(0) = y\left(\frac{2\pi}{m}\right) = y'\left(\frac{2\pi}{m}\right) = 0$

2°. (0, 1; 0, 3) **3°.** (0, 3; 0, 1)

4°. (0, 1; 1, 3) **5°.** (1, 3; 0, 1)

6°. (0, 3; 0, 3)

7°. (0, 3; 1, 3) **8°.** (1, 3; 0, 3)

9°. (1, 3; 1, 3)

10°. (0; 0, 1, 3) **11°.** (0, 1, 3; 0)

12°. (1; 0, 1, 3) **13°.** (0, 1, 3; 1)

14°. (3; 0, 1, 3) **15°.** (0, 1, 3; 3).

The boundary conditions of **1°** come from the problem with which we begin;

The problems with boundary condituons from **1°** to **9°** are solved in [3] and they are each with two boundary conditions at each of bouth the end points of the interval.

The results in the case of the boundary problems of **3°**, **5°** and **8°** are obtained from the results obtained in the case of the boundary conditions of **2°**, **4°** and **7°**, if the left and of the interval is replaced by the right one and viceversa.

The problems with eigen values of **10°** to **15°** are with one boundary condition at one end of the interval and three boundary conditions at the other end of the interval, which is of interest, since the number of the conditions is not the same at the two ends at the interval.

References

[1] Cimmino, G.: *Autosoluzioni e autovalori nelle equazioni differenziali lineari ordinarie autoaquinte di ordine superiore*, Math. Zeit., **32**, 4–58, 1930

[2] Георгиевска, С., С.: *Сведување контурен проблем од четврти ред на контурен проблем од втор ред*, Математички Билтен, Скопје, **11–12**, 29–38, 1987–1988

[3] Георгиевска, С., С.: *За еден Штурм–Лиувилов проблем од* IV *ред*, Editions speciales, sec. Equations differentialles, Skopje, liv. 3(20), 49-54, 1977

[4] Georgievska, S., S.: *Reduction of a boundary third order problem to a boundary second order problem*, Математички Билтен, Скопје, **18**, 87–92, 1994

[5] Камке, Э.: *Спровочник по обыкновенным дифференциалньниым уравнени м, Наука, Москва, 1971*

[6] Mitrinović, D., S. et Vasić, P., M.: *Complements au traite de Kamke* XI, Matematički vesnik, Beograd, I(**16**), 181–185, 1964

[7] Шапкарев, И., А.: *За еден контурен проблем од трет ред*, Годишен Зборник на Еелектротехнички факултет, Скопје, **6–7**, 11–13, 1990

КОНТУРЕН ПРОБЛЕМ ОД ТРЕТИ И ЧЕТВРТИ РЕД КАКО ПРОИЗВОД НА КОНТУРНИ ПРОБЛЕМИ ОД ВТОР РЕД

Слободанка С. Георгиевска

Р е з и м е

Во трудот е добиено дека линеарен хомоген контурен проблем од трети ред со диференцијална равенка (1.10) има решение производ од решенијата на линеарните хомогени контурни проблеми од втор ред (1.1)–(1.2) и (1.3)–(1.4) ако важи (1.7), а контурен проблем од четврти ред со диференцијална равенка (1.11) и контурни услови (2.2) има решение производ од наведените контурни проблеми од втор ред ако важи (1.12), т.е. дискриминантите на карактеристичните равенки (1.13) на диференцијалните равенки (1.1) и (1.3) се еднакви меѓу себе односно се различни меѓу себе.

Ако контурните услови за проблемите од втор ред се Штурмови тогаш соодветните контурни услови за проблемот од четврти ред се шест, со што се добива една класа од контурни проблеми (петнаесет) што имаат исто решение, и контурните делови се со по два во секоја крајна точка или со различен број на контурни услови во крајните точки.

Тврдењата се демонстрирани на еден пример.

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