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α-STRONG PRECOMPACTNESS IN FUZZY TOPOLOGICAL SPACES

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Abstract

A concept of an α-strong precompactness in fuzzy topological spaces is introduced and studied. Three countable properties of fuzzy topological spaces are introduced and investigated by the help of fuzzy strongly preopen sets.

Introduction

Zadeh introduced the concept of a fuzzy set in his classical paper [11]. Using this notion Chang [1] introduced the concept of a fuzzy topological space. Also compact fuzzy topological spaces were firstly introduced by Chang [1] who proved two results about such spaces. This approach towards the compactness in fuzzy topological spaces has serious limitations, for example, a fuzzy space with one point can fail to be compact. After that many authors tried to introduce the notion of compactness, but they lost the concept that the fuzzy topology generalizes ordinary topology. In 1978 Gantner and Steinage [4] proposed a new definition for an L-fuzzy topological space to be compact. Their α-compactness was defined only for the whole space. Mashhour and Abd El-Monsef [6] and Ganster [3] investigated the concepts of precompactness and pre-Lindelofness in ordinary topology. Dang, Behera and Nanda [2] introduced the concept of an α-compactness for supratopological spaces.

In the Section 3 we introduce the concept of an α-strong precompactness which is strictly stronger then the concept of an α-compactness. Also, we give an extension of this notion to arbitrary fuzzy sets.

The first attempt to define some countable properties for fuzzy topological spaces was done by Wong [10]. Following the lines of Gantner's α-compactness Malghan and Benchalli [7] defined countable compactness and Lindelof property.

In the Section 4 by the help of the fuzzy strongly preopen sets introduced in [5] we shalldefine three countable properties of fuzzy topological spaces.

1. α-Strong precompactness

Throughout this paper by (X, τ) or simply by X we will denote a fuzzy topological space (fts) due to Chang [1].

The author in [5], defined a fuzzy set A of an fts X to be a fuzzy strongly preopen if $A \leq \mathrm{int}(\mathrm{pcl}A)$. The family of all fuzzy strongly preopen sets of an (X, τ) will be denoted by FSPO(τ). The complement of a fuzzy strongly preopen set A of an fts X is called fuzzy strongly preclosed set, denoted by A^c. For a fuzzy set A of an fts X, the intersection of all fuzzy strongly preclosed sets containing A is called the fuzzy strong preclosure of A, denoted by spcl A.

For a fuzzy set A of a set X, the support of A, denoted by supp A, is the subset of X given by $\mathrm{supp}\, A = \{x \in X : A(x) > 0\}$ [8].

Definition 1.1. *Let A be a fuzzy set of an fts (X, τ) and $\alpha \in [0, 1]$. A collection $\mathcal{U} \subseteq FSPO(\tau)$ is called a fuzzy strongly preopen α-shading (resp. α^*-shading) of A if, for each $x \in \mathrm{supp}(A)$, there exists $U \in \mathcal{U}$ such that $U(x) > \alpha$ (resp. $U(x) \geq \alpha$). A subcollection $\mathcal{V}$ of a fuzzy strongly preopen α-shading (res. α^*-shading) $\mathcal{U}$ of A that is also a fuzzy strongly preopen α-shading (resp. α^*-shading) is called a fuzzy strongly preopen α-subshading of $\mathcal{U}$.*

Definition 1.2. *A fuzzy set A of an fts X is called α-strongly precompact (resp. α^*-strongly precompact) if each fuzzy strongly preopen α-shading (resp. α^*-shading) of A has a finite fuzzy strongly preopen α-subshading (resp. α^*-subshading).*

If A is the whole space, then we say that the fts X is α-strongly precompact (resp. α^*-strongly precompact).

Definition 1.3. *A fuzzy set A of an fts X is called countably α-strongly precompact (resp. α^*-strongly precompact) if each countable fuzzy strongly preopen α-shading (resp. α^*-shading) of A has a finite fuzzy strongly preopen α-subshading.*

If A is the whole space, then we say that the fts X is countably α-strongly precompact (resp. α^*-strongly precompact).

A fuzzy point x_α of an fts X is called a fuzzy set of X defined by:

$$x_\alpha(y) = \begin{cases} \alpha, & \text{for } y = x; \\ 0, & \text{otherwise} \end{cases} \qquad 0 < \alpha \leq 1.$$

The fuzzy point x_α is said to have support x and value α. Then x_α is said to be in a fuzzy set A of X or A contains x_α, denoted by $x_\alpha \in A$, if and only if $\alpha \leq A(x)$ [9].

Let (X, τ) be an fts and $\alpha \in [0,1]$. According to [4], a collection $\mathcal{U} \subseteq \tau$ is called an α-shading (resp. α^*-shading) of X if for each $x \in X$, there exists $U \in \mathcal{U}$ with $U(x) > \alpha$ (resp. $U(x) \geq a$). A subcollection $\mathcal{V}$ of an α-shading (resp. α^*-shading) $\mathcal{U}$ of X that is also an α-shading (resp. α^*-shading) is called an α-subshading (resp. α^*-subshading) of $\mathcal{U}$. An fts X is called α-compact (resp. α^*-compact) if each α-shading (resp. α^*-shading) of X has a finite α-subshading (resp. α^*-subshading). An fts X is called countably α-compact (resp. α^*-compact) if each countable α-shading (resp. α^*-shading) of X has a finite α-subshading (α^*-subshading).

Remark 1.1. Immediately from the definitions above and the Theorem 3.2 [5] it follows that,

1) every α-strongly precompact (resp. α^*-strongly precompact) fuzzy set of an fts X is countably α-strongly precompact (resp. α^*-strongly precompact);

2) every fuzzy point of an fts X is an α-strongly precompact (resp. α^*-strongly precompact) fuzzy set;

3) every fuzzy set of an fts X is 1-strongly precompact and 0^*-strongly precompact;

4) every α-strongly precompact fts is α-compact, hence every countable α-strongly precompact fts is countably α-compact.

Let X be a set and $\alpha \in [0,1]$. A collection $\mathcal{U}$ of fuzzy sets of X is called α-centered (resp. α^*-centered) if for all $C_1, \ldots, C_n \in \mathcal{U}$, there exists $x \in X$ such that $C_k(x) \geq 1-\alpha$ (resp. $C_k(x) > 1-\alpha$), for all $k = 1, \ldots, n$ [4].

Theorem 1.1. *An fts X is α-strongly precompact (resp. α^*-strongly precompact) if and only if, for every α-centered (resp. α^*-centered) collection $\mathcal{F}$ of fuzzy strongly preclosed sets of X, there exists $x \in X$ such that $F(x) \geq 1-\alpha$ (resp. $F(x) > 1-\alpha$), for all $F \in \mathcal{F}$.*

Proof. Let $\mathcal{F}$ be a centered collection of fuzzy strongly preclosed sets of X such that, for each $x \in X$, there exists $F \in \mathcal{F}$ with $F(x) < 1-\alpha$. Then $\mathcal{U} = \{F^c : F \in \mathcal{F}\}$ is a fuzzy strongly preopen α-shading of X that does not have a finite fuzzy strongly preopen α-subshading. In fact, if $F_1^c, \ldots, F_n^c \in \mathcal{U}$, then, since $\mathcal{F}$ is α-centered, there exists $x \in X$ such that $F_k(x) \geq 1-\alpha$ for all $k = 1, \ldots, n$. Hence $F_k^c(x) \leq \alpha$ for all $k = 1, \ldots, n$.

Conversely, suppose that $\mathcal{U}$ is a fuzzy strongly preopen α-shading of X that does not have a finite fuzzy strongly preopen α-subshading. Then $\mathcal{F} = \{U^c : U \in \mathcal{U}\}$ is a collection of fuzzy strongly preclosed sets of X. The collection $\mathcal{F}$ is α-centered because if $U_1^c, \dots, U_n^c \in \mathcal{F}$, then there exists $x \in X$ such that $U_k(x) \leq \alpha$ for all $k = 1, \dots, n$, and hence $U_k^c(x) \geq 1 - \alpha$ for all $k = 1, \dots, n$. But for each $x \in X$, there is $U \in \mathcal{U}$ such that $U(x) > \alpha$, hence $U^c \in \mathcal{F}$ and $U^c(x) < 1 - \alpha$.

The proof for an α^*-strong precompactness is similar. □

Corollary 1.2. *An fts X is α-strongly precompact (resp. α^*-strongly precompact) if and only if, for every α-centered (resp. α^*-centered) collection $\mathcal{F}$ of fuzzy sets of X, there exists $x \in X$ such that $spcl C(x) \geq 1 - \alpha$* (spcl $(x) > 1 - \alpha$), *for all $C \in \mathcal{F}$.* □

Theorem 1.3. *Let X be an fts. If A_1 is an α-strongly precompact (resp. α^*-strongly precompact) fuzzy set, then for each fuzzy strongly preclosed set A_2 of X, $A_1 \wedge A_2$ is α-strongly precompact (resp. α^*-strongly precompact).*

Proof. Let $\mathcal{V} = \{V_i : i \in I\}$ be a fuzzy strongly preopen α-shading of $A_1 \wedge A_2$ We claim that $\{V_i : i \in I\} \cup \{A_2\}^c$ is a fuzzy strongly preopen α-shading of A_1. If $x \in \operatorname{supp} A_1$, then $x \in \operatorname{supp}(A_1 \wedge A_2)$ or $A_2(x) = 0$. If $x \in \operatorname{supp}(A_1 \wedge A_2)$, then there exists $V_i \in \mathcal{V}$ such that $V_i(x) > \alpha$. If $A_2(x) = 0$, then $A_2^c(x) = 1 > \alpha$. According to the assumption there exists a finite fuzzy strongly preopen α-subshading $\{V_i : i = 1, \dots, n\} \cup \{A_2^c\}$. Then $\{V_i : i = 1, \dots, n\}$ is a finite fuzzy strongly preopen α-subshading of $\mathcal{V}$.

The case when A_1 is an α^*-strongly precompact fuzzy set can be proved in a similar manner. □

Corollary 1.4. *Let X be an fts. If A is an α-strongly precompact (resp. α^*-strongly precompact) fuzzy set, then each fuzzy strongly preclosed set contained in A is α-strongly precompact (resp. α^*-strongly precompact) as well.* □

Theorem 1.5. *Let X be an fts. If A_1 and A_2 are α-strongly precompact (resp. α^*-strongly precompact) fuzzy sets, then $A_1 \vee A_2$ is α-strongly precompact (resp. α^*-strongly precompact) as well.*

Proof. Let $\mathcal{V} = \{V_i : i \in I\}$ be a fuzzy strongly preopen α-shading of $A_1 \vee A_2$. Then $\mathcal{V}$ is a fuzzy strongly preopen α-shading of A_1 and A_2. Since A_1 and A are α-strongly precompact there are finite subcollections I_1 and I_2 of I, such that $\{V_i : i \in I_j\}$ is a fuzzy strongly preopen α-shading of A_j, $j = 1, 2$. For $x \in \operatorname{supp}(A_1 \vee A_2) = \operatorname{supp} A_1 \vee \operatorname{supp} A_2$, there exists $V_m, m \in I_1 \cup I_2$, such that $V_m(x) > \alpha$. Hence $\{V_i : i \in I_1 \cup I_2\}$ is a finite fuzzy strongly preopen α-subshading of $\mathcal{V}$.

The case when A_1 and A_2 are α^*-strongly precompact fuzzy sets can be proved in a similar manner. □

Corollary 1.6. *Let X be an fts. Every fuzzy set A with a finite support is α-strongly precompact (resp. α^*-strongly precompact).* □

A mapping $f:(X,\tau_1) \to (Y,\tau_2)$ from an fts X into an fts Y is called fuzzy strong irresolute precontinuous (fuzzy strong precontinuous) if $f^{-1}(B) \in$FSPO(τ_1), for each $B \in$FSPO(τ_2) (for each $B \in \tau_2$) [5].

Theorem 1.7. *Let $f: X \to Y$ be a fuzzy strong irresolute precontinuous mapping from an fts X into an fts Y. If A is an α-strongly precompact (resp. α^*-strongly precompact) fuzzy set of X, then $f(A)$ is an α-strongly precompact (resp. α^*-strongly precompact) fuzzy set of Y.*

Proof. Let $\mathcal{U} = \{U_i : i \in I\}$ be a fuzzy strongly preopen α-shading of $f(A)$. Then $f^{-1}(\mathcal{U}) = \{f^{-1}(U_i) : U_i \in \mathcal{U}\}$ is a collection of fuzzy strongly preopen set of X. We claim that $f^{-1}(\mathcal{U})$ is a fuzzy strongly preopen α-shading of A. Let $x \in \operatorname{supp}(A)$. Then $f(x) \in f(\operatorname{supp} A) = \operatorname{supp} f(A)$, hence there exists $U_m \in \mathcal{U}$ such that $U_m(f(x)) > \alpha$ which implies that $f^{-1}(U_m)(x) > \alpha$. Thus $f^{-1}(\mathcal{U})$ is a fuzzy strongly preopen α-shading of A. According to the assumption $f^{-1}(\mathcal{U})$ has a finite fuzzy strongly preopen α-subshading $\{f^{-1}(U_i) : i = 1, \dots, n\}$. For $y \in \operatorname{supp} f(A)$, $y = f(x)$ for some $x \in \operatorname{supp} A$ and hence there exists U_m such that $f^{-1}(U_m)(x) > \alpha$ which implies $U_m(f(x)) = U_m(y) > \alpha$. Then $\mathcal{U}$ has a finite fuzzy strongly preopen α-subshading $\{U : i = 1, \dots, n\}$, hence $f(A)$ is an α-strongly precompact set of Y.

The case when A is an α^*-strongly precompact fuzzy set can be proved in a similar manner. □

Theorem 1.8. *Let $f: X \to Y$ be a fuzzy strong precontinuous mapping from an fts X onto an fts Y. If X is α-strongly precompact (resp. α^*-strongly precompact) then Y is α-compact (resp. α^*-compact).* □

2. Countable properties

Definition 2.1. *Let X be an fts. A collection $\mathcal{B}$ of fuzzy strongly preopen sets of X is called a base for fuzzy strongly preopen sets of X if and only if each fuzzy strongly preopen set can be expressed as a union of some members of $\mathcal{B}$.*

Theorem 2.1. *Let X be an fts with a countable base for fuzzy strongly preopen sets. Then a fuzzy set A is α-strongly precompact (resp. α^*-strongly precompact) if and only if it is countably α-strongly precompact (resp. α^*-strongly precompact).*

Proof. In the Remark 1.1 it is mentioned, that if A is α-strongly precompact, then A is countably α-strongly precompact.

Conversely, let $\mathcal{V} = \{V_i : i \in I\}$ be a fuzzy strongly preopen α-shading of A. According to the assumption X has a countable base for fuzzy strongly preopen sets, $\{B_m\}_{m\in\mathbf{N}}$. Then $V_i = V_{k=1}^{n} B_{i_k}$, where n may be infinite. Let $\mathcal{B}_0 = \{B_{i_k} : i \in I,\, k = 1,\dots n\}$. For $x \in \operatorname{supp} A$, there exists $V_i \in \mathcal{V}$ such that $V_i\ (x) > \alpha$. Hence there exists $k_0 \in \{1,\dots,n\}$ such that $B_{i'_{k_0}}(x) > \alpha$. We conclude that there exists $B_{i'_{k_0}} \in \mathcal{B}_0$ such that $B_{i'_{k_0}} x) > \alpha$. Thus $\mathcal{B}_0$ is a countable fuzzy strongly preopen α-shading of A. By the countable α-strong precompactness of A there exists a finite fuzzy strongly preopen α-subshading, $\mathcal{B}_1 \subseteq \mathcal{B}_0$. Each member of $B \in \mathcal{B}_1$ satisfies $B \le V_i$ for some $i \in I$. In fact if $B \in \mathcal{B}_1$ then $B = B_{i'_{k}}$ for some $i' \in I$ and $k' \in \{1,\dots,n\}$ because $\mathcal{B}_1 \subseteq \mathcal{B}_0$ and $B = B_{i'_{k'}} \le \vee_{k=1}^{n} B_{i'_k} = V_{i'}$.

Let $\mathcal{U} = \{V_i : B \le V_i,\, B \in \mathcal{B}_1\}$. Then $\mathcal{U}$ is a finite α-subshading of $\mathcal{V}$. For $x \in \operatorname{supp} A$, *since*$\mathcal{B}_1$ is a fuzzy strongly preopen α-shading of A, there exists $B \in \mathcal{B}_1$, such that $B(x) > \alpha$. Since $B \in \mathcal{B}_1$, there exists $V_i \in \mathcal{U}$ such that $B \le V_i$. Thus $V_i(x) > \alpha$.

The case when A is α^*-strongly precompact, can be proof in a similar manner. □

A mapping $f\colon (X,\tau_1)w \to (Y,\tau_2)$ from an fts X into an fts Y is called fuzzy strongly irresolute preopen if $f(A) \in \mathrm{FSPO}(\tau_2)$, for each $A \in \mathrm{FSPO}(\tau_1)$ [5].

Theorem 2.2. *Let $f\colon X \to Y$ be a fuzzy strong irresolute precontinuous and fuzzy strongly preopen irresolute mapping from an fts X onto an fts Y. If X has a countable base for fuzzy strongly preopen sets, then Y has a countable base for fuzzy strongly preopen sets as well.*

Proof. Let $\mathcal{B}$ be a counatble base for fuzzy strongly preopen sets of X. For $B \in \mathcal{B}$, $f(B)$ is a fuzzy strongly preopen set of Y, since f is fuzzy strongly irresolute preopen. The collection $\{f(B) : B \in \mathcal{B}\}$ forms a countable base for fuzzy strongly preopen sets of Y. To show that, let V be a fuzzy strongly preopen set of Y. Then $f^{-1}(V)$ is a fuzzy strongly preopen set of X. Thus $f^{-1}(V) = \vee_{i=I} B_i$. It follows that

$$V = ff^{-1}(V) = f(\vee_{i=I} B) = \vee_{i=I} f(B). \quad \square$$

Definition 2.2. *An fts X is called strongly preseparable if and only if there exists a countable sequence of fuzzy points $\{p\}_{i\in\mathbf{N}}$, such that for every fuzzy strongly preopen set $A \ne 0_x$, there exists $p_i \in A$.*

Theorem 2.3. *If an fts X has a countable base for fuzzy strongly preopen sets, then it is strongly preseparable.*

Proof. Let $\mathcal{B} = \{B\}_{i\in\mathbf{N}}$, be a countable base for fuzzy strongly preopen sets. For $B \ne 0_x$, there exists $x_i \in X$ such that $B_i(x_i) > 0$. We define fuzzy points p_i as follows:

$$p_i(x) = \begin{cases} 1/2B_i(x_i)\,, & \text{for } x = x_i \\ 0\,, & \text{otherwise.} \end{cases}$$

Clearly $p_i \in B_i$. The countable sequence $\{p_i\}_{i \in \mathbf{N}}$ is the required sequence for X to be strongly preseparable, because every fuzzy strongly preopen set A of X contains a member of $\mathcal{B}$, $B_i \leq A$. Consequently, $p_i \in A$. □

Theorem 2.4. *Let $f: X \to Y$ be a fuzzy strong irresolute precontinuous mapping from an fts X onto an fts Y. If X is strongly preseparable, then Y is strongly preseparable as well.*

Proof. Let $\{p_i\}_{i \in \mathbf{N}}$ be a countable sequence of fuzzy points such that for each fuzzy strongly preopen set $A \neq 0_x$ there exists $p_i \in A$. The collection $\{f(p_i)\}_{i \in \mathbf{N}}$ forms a countable sequence of fuzzy points in Y. Let B be a fuzzy strongly preopen set of Y. Then $f^{-1}(B)$ is a fuzzy strongly preopen set of X, and hence there exists a fuzzy point p_i, such that $p_i \in f^{-1}(B)$. Consequently, $f(p_i) \in B$. Thus Y is strongly preseparable. □

An fts (X, τ) is called separable if and only if there exists a countable sequence of fuzzy points $\{p_i\}_{i \in \mathbf{N}}$, such that for every fuzzy open set $A \neq 0_x$, there exists $p_i \in A$ [10].

Theorem 2.5. *Let $f: X \to Y$ be a fuzzy strong precontinuous mapping from an fts X onto an fts Y. If X is strongly preseparable, then Y is separable.* □

Definition 2.3. *Let $\alpha \in \{0, 1\}$. A fuzzy set A of an fts X is called α-strongly pre-Lindèlof (resp. α^*-strongly pre-Lindelof) if each fuzzy strongly preopen α-shading (resp. α^*-shading) of A has a countable fuzzy strongly preopen α-subshading (resp. α^*-subshading).*

If A is the whole space, then we say that the fts X is α-strongly pre-Lindelof (resp. α^*-strongly pre-Lindelof).

Remark 2.1. From the definition above we may conclude that,

1) every α-strongly precompact (resp. α^*-strongly precompact) fuzzy set of an fts X is α-strongly pre-Lindelof (resp. α^*-strongly pre-Lindelof);

2) every α-strongly pre-Lindelof (resp. α^*-strongly pre-Lindelof) fts is α-Lindelof (resp. α^*-Lindelof);

3) every fuzzy set of an fts X is 1-strongly pre-Lindeof and 0-strongly pre-Lindelof.

Theorem 2.6. *Let $\alpha \in [0, 1]$. If an fts X has a countable base for fuzzy strongly preopen sets, then X is α-strongly pre-Lindelof (resp. α^*-strongly pre-Lindelof).*

Proof. The proof is simlar to the proof of the Theorem 2.1. □

Theorem 2.7. *Let A be an α-strongly pre-Lindelof (resp. α^*-strongly pre-Lindelof) fuzzy set of an fts X. Then A is countably α-strongly precompact (resp. α^*-strongly precompact) if and only if A is α-strongly precompact (resp. α^*-strongly precompact).*

Proof. In the Remark 3.1 it is mentioned, that if A is α-strongly precompact, then A is countably α-strongly precompact.

Conversely, let $\mathcal{U}$ be a fuzzy strongly preopen α-shading of A. Since A is α-strongly pre-Lindelof, there exists a countable fuzzy strongly preopen α-subshading $\mathcal{U}_1$ of $\mathcal{U}$. From the countable α-strong precompactness of A, there is a finite fuzzy strongly preopen α-subshading $\mathcal{U}_2$ of $\mathcal{U}_1$. Thus $\mathcal{U}_2$ is a finite fuzzy strongly preopen α-subshading of $\mathcal{U}$. Hence A is an α-strongly precompact fuzzy set of X.

The case when A is α^*-strongly pre-Lindelof can be prove in a similar manner. □

Theorem 2.8. *Let X be an fts. If A is an α-strongly pre-Lindelof (resp. α^*-strongly pre-Lindelof) fuzzy set, then for each fuzzy strongly preclosed set A of X, $A \wedge A$ is α-strongly pre-Lindelof (resp. α^*-strongly pre-Lindelof).*

Proof. It is similar to the proof of the Theorem 1.3. □

Corollary 2.9. *Let X be an fts. If A is an α-strongly pre-Lindelof (resp. α^*-strongly pre-Lindelof) fuzzy set, then each fuzzy strongly preclosed set contained in A is α-strongly pre-Lindelof (resp. α^*-strongly pre-Lindelof) as well.* □

Theorem 2.10. *Let X be an fts. If A and A are α-strongly pre-Lindelof (resp. α^*-strongly pre-Lindelof) fuzzy sets, then $A \vee A$ is α-strongly pre-Lindelof (resp. α^*-strongly pre-Lindelof) as well.*

Proof: It is similar to the proof of the Theorem 1.5. □

Remark 2.2. Among the three types of countable properties, namely, the existence of a countable base for fuzzy strongly preopen sets, an α-strong pre-Lindelofness and a fuzzy strong preseparablity, the first is stronger.

Theorem 2.11. *Let $f: X \to Y$ be a fuzzy strong irresolute precontinuous mapping from an fts X into an fts Y. If A is an α-strongly pre-Lindelof (resp. α^*-strongly pre-Lindelof) fuzzy set of X, then $f(A)$ is an α-strongly pre-Lindelof (resp. α^*-strongly pre-Lindelof) fuzzy set of Y.*

Proof. *It is similar to the proof of the Theorem* 1.7. □

Let $\alpha \in [0,1]$. An fts X is called α-Lindelof (resp. α^*-Lindelof) if each α-shading (resp. α^*-shading) of X has a countable α-subshading (resp. α^*-subshading) [7].

Theorem 2.12. *Let $f: X \to Y$ be a fuzzy strong precontinuous mapping from an fts X onto an fts Y. If X is α-strongly pre-Lindelof (resp. α^*-strongly pre-Lindelof), then Y is α-Lindelof (resp. α^*-Lindelof).* □

References

[1] C.L. Chang: *Fuzzy topological spaces*, J. Math. Anal. Appl. **24**, 182-190 (1968).

[2] S. Dang, A. Behera and S. Nanda : *Some results on fuzzy supratopological spaces*, Fuzzy Sets and Systems **62**, 333-339 (1994).

[3] M. Ganster: *A note on strongly Lindelof spaces*, Soochow J. Math. **15**, 99-104 (1989).

[4] T.E. Gantner and R.C. Steinage: *Compactness in Fuzzy Topological Spaces*, J. Math. Anal. Appl. **62**, 547-562 (1978).

[5] B. Krsteska: *Fuzzy strongly preopen sets and fuzzy strong precontinuity*, Mat. Vesnik **50**, 111-123 (1998).

[6] A.S. Mashhour, M.E. Abd El-Monsef, I.A. Hasanein and T. Noiri, Strongly compact spaces, Delta J. Sci. **8**, 30-46 (1984).

[7] S.R. Malghan and S.S. Benchalli: *On fuzzy topological spaces*, Glasnik Mat. **16**, 313-325 (1981).

[8] S.R. Malghan and S.S. Benchalli: *Open Maps, Closed Maps and Local Compactness in Fuzzy Topoogical Spaces*, J. Math. Anal. Appl. **99**, 338-349 (1984).

[9] Pu P.-Ming and Liu Y.-Ming: *Fuzzy topology I. Neighborhood structure of a point and Moore Smith convergence*, J. Math. Anal. Appl. **76**, 571-599 (1980).

[10] C.K. Wong: *Fuzzy Points and Local Properties of Fuzzy topology*, J. Math. Anal. Appl. **40**, 316-328 (1974).

[11] L. A. Zadeh: *Fuzzy sets*, Information and Control **8**, 338-353 (1965).

α-ЈАКА ПРЕКОМПАКТНОСТ ВО ФАЗИ ТОПОЛОШКИ ПРОСТОР

Билјана Крстеска

Р е з и м е

Во ова работа е воведен поим за α-јако прекомпактно фази множество во фази тополошки простор. Во продолжение со помош на фази јако преотворените множества се дефинирани три преброиви својства на фази тополошките простори.

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