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SUBCATEGORY OF METRIC COMPACTA IN THE STRONG SHAPE CATEGORY

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Abstract. In the previous paper of the authors, for the first time is presented an intrinsic definition of the strong shape category. The morphisms of this category are homotopy classes of coherent proximate nets. In this paper it is given the proof that the subcategory with objects compact metric spaces, is isomorphic with the strong shape category of metric compacta previously constructed by Shekutkovski, where morphisms are homotopy classes of strong proximate sequences.

Shape theory is a tool for investigation of spaces that not behave well locally. Strong shape is a stronger version of the theory [4]. In the paper [3] is presented an intrinsic approach to strong shape for metric compacta and in the paper [4] for the first time is presented an intrinsic definition of the strong shape category for more general spaces. We denote this strong shape category by SSh^{∞} . The objects of this category are all strong paracompacta. Below there are given the definitions that are used to obtain the mentioned category: the definition of coherent proximate net (*CPN*) $f^{\infty} : X \to Y$, indexed by special subsets of all star-finite coverings of Y and the definition of coherent homotopy between two coherent proximate nets. Morphisms in this category are the homotopy classes of *CPN*. This paper is a part of Andonovik's Ph.D thesis written under supervision of N. Shekutkovski.

We repeat the constructtion of strong shape category presented in [4].

Definition 1: Let X, Y, be spaces, and $\mathcal{V}$ be a covering of Y. The function $f : X \to Y$ is $\mathcal{V}$-continuous, if for any $x \in X$, there exists a neighborhood U of x, such that $f(U) \subseteq V$, for some member $V \in \mathcal{V}$. (The family of all U, form a covering $\mathcal{U}$ of X. Shortly, we say that $f : X \to Y$ is $\mathcal{V}$-continuous, if there exists $\mathcal{U}$ such that $f(\mathcal{U}) \prec \mathcal{V}$.)

Definition 2: Two $\mathcal{V}$ - continuous functions $f, g : X \to Y$ are $\mathcal{V}$-homotopic, if there exists a function $F : X \times I \to Y$ such that:

1) $F : X \times I \to Y$ is st$\mathcal{V}$ - continuous;

2) There exists a neighborhood $N = [0, \varepsilon) \cup (1-\varepsilon, 1]$ of $\{0, 1\}$ in $[0, 1]$ such that $F|_{X \times N}$ is $\mathcal{V}$ - continuous

3)$F(x, 0) = f(x)$, $F(x, 1) = g(x)$.

The relation of homotopy between $\mathcal{V}$ - continuous functions is an equivalence relation.

Let $Cov_*(Y) = \{\mathcal{U} | \mathcal{U}$ is a covering of Y, $\mathcal{U}$ is star $-$ finite$\}$. Since $Cov_*(Y)$ is not a cofinite directed set , we construct the following set:

$$Cov_*^{\max}(Y) = \{a | a \subseteq Cov_*(Y),\ |a| < \infty,\ \exists \max a\}.$$

This set is ordered by inclusions i.e. $a \leq b$ if and onlt if $a \subseteq b$.

Definition 3: A ***coherent proximate net*** $\underline{f} : X \to Y$, (CPN), is a set of functions

$$\underline{f} = \{f_{\underline{a}} | \forall \underline{a} = (a_0, a_1, ..., a_n),\ a_n > ... > a_0,\ a_i \in Cov_*^{\max}(Y),\ i = 0, ..., n\}.$$

such that for each $f_{\underline{a}} : \Delta^n \times X \to Y$ the following conditions are satisfied:

$$f_{\underline{a}} : \Delta^n \times X \to Y \text{ is } st^n \max a_0 - \text{continuous} \tag{1}$$

(2) There exists a neighbourhood $N$of $\partial\Delta^n$, such that $f_{\underline{a}}|_{N \times X}$ is $st^{n-1} \max a_0$-continuous.

(3) The condition of coherence holds:

$$f_{\underline{a}}(t_1, t_2, ..., t_n, x) = \begin{cases} f_{a_1 ... a_n}(t_2, ..., t_n, x),\ t_1 = 1 \\ f_{a_0 ... \hat{a}_i ... a_n}(t_1, ..., \hat{t}_i, ..., t_n, x),\ t_i = t_{i+1} \\ f_{a_0 ... a_{n-1}}(t_1, ..., t_{n-1}, x),\ t_n = 0 \end{cases}$$

The coherent proximate net we will shortly denote by $\underline{f} = (f_{\underline{a}})$.

Definition 4: Let $\underline{f}, \underline{g} : X \to Y$be CPNs. For $\underline{f}$ and $\underline{g}$ we say that are ***homotopic*** (denotation: $\underline{f} \approx \underline{g}$), if there exists a CPN $\underline{H} : I \times X \to Y$, where $\underline{H} = (H_{\underline{a}})$, so that $H_{\underline{a}} : \Delta^n \times I \times X \to Y$ is $st^{n+1} \max a_0 -$ continuous,

$$H_{\underline{a}}(\underline{t}, 0, x) = f_{\underline{a}}(\underline{t}, x),$$
$$H_{\underline{a}}(\underline{t}, 1, x) = g_{\underline{a}}(\underline{t}, x),\ \underline{t} \in \Delta^n,$$

and $\exists W$, a neighbourhood of $\partial(\Delta^n \times I)$, so that $H_{\underline{a}}|_{W \times X}$ is $st^n \max a_0 -$continuous.

Now we give the definitions of strong proximate sequence (CPS) and homotopy of strong proximate sequences, that are used to obtain the category $SSh^2(Cpt)$. In that category the objects are all compact metric spaces Cpt, and morphisms are the classes of strong proximate sequences.

Let $X, Y \in Cpt$ and let $\mathcal{V}_1 \succ \mathcal{V}_2 \succ \mathcal{V}_3 \succ ...$ be a sequence of finite coverings, that is cofinal in the family of all finite coverings of Y.

Definition 5: The sequence of pairs of functions$(f_n, f_{n,n+1}) : X \to Y$ is called ***strong proximate sequence*** from X to Y, if $f_n : X \to Y$ is $\mathcal{V}_n$-continuous, and $f_{n,n+1} : I \times X \to Y$ is a homotopy between $\mathcal{V}_n$ - continuous functions f_n and f_{n+1}.

We say that $(f_n, f_{n,n+1})$ is a strong proximate sequence over $(\mathcal{V}_n)$.

Definition 6: Let $(f_n, f_{n,n+1})$ and $(f'_n, f'_{n,n+1})$ be strong proximate sequences. We say that $(f_n, f_{n,n+1})$ and $(f'_n, f'_{n,n+1})$ are ***homotopic***, if there is a strong proximate sequence $(F_n, F_{n,n+1}) : I \times X \to Y$,over $(\mathcal{V}_n)$ so that:

1) f'_n and f_n are homotopic by a homotopy F_n as $\mathcal{V}_n$ - continuous functions, and

2) $f_{n,n+1} : I \times X \to Y$ and $f'_{n,n+1} : I \times X \to Y$ are homotopic as $st\mathcal{V}_n$ – continuous functions by a homotopy $F_{n,n+1} : I \times I \times X \to Y$ (picture 1) and:

$$F_{n,n+1}(t,0,x) = F_n(t,x)$$
$$F_{n,n+1}(t,1,x) = F_{n+1}(t,x)$$

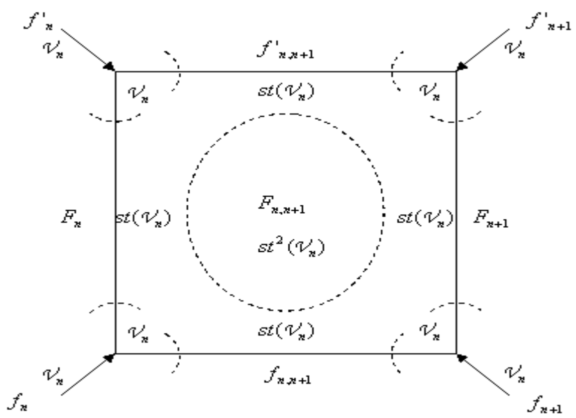

FIGURE 1

We will show the following theorem:

Theorem 1. *Let Cpt be the family of compact metric spaces. The category of strong shape with objects Cpt and morphisms classes of coherent proximate nets ([4]) is isomorphic to the category of strong shape with objects Cpt and morphisms classes of strong proximate sequences ([3]).*

Proof. Let $X, Y \in Cpt$ and let $\mathcal{V}_1 \succ \mathcal{V}_2 \succ \mathcal{V}_3 \succ ...$ be a sequence of finite coverings, that is cofinal in the family of all finite coverings of Y.

Let $\underline{\mathcal{V}}$ be the set of all finite (ordered) subsets of the sequence of coverings $\mathcal{V}_1 \succ \mathcal{V}_2 \succ \mathcal{V}_3 \succ ....$ Each of these sets is of the following type: $\mathcal{V}_{n_1} \succ \mathcal{V}_{n_2} \succ ... \succ \mathcal{V}_{n_k}$, i.e. each of these sets is uniquely determined by a finite increasing sequence of natural numbers $(n_1, n_2, ..., n_k)$, including both $(n, n+1)$ and (n).

By $SSh^{\infty}(Cpt)$ we denote the category of strong shape with objects Cpt and morphisms classes of coherent proximate nets, and by $SSh^{2}(Cpt)$ we denote the category of strong shape with objects Cpt and morphisms classes of strong proximate sequences.

We will define a functor $\Phi : SSh^{\infty}(Cpt) \to SSh^{2}(Cpt)$.

Consider a coherent proximate net $\underline{f} = f^{\infty}$ in $SSh^{\infty}(Cpt)$. It cosists of functions $f_{\underline{a_1}\underline{a_2}...\underline{a_n}}$, where and $\underline{a_1} < \underline{a_2} < ... < \underline{a_n}$.

We define:

$$f_0 = \Phi(f_{(\mathcal{V}_0)})$$
$$f_1 = \Phi(f_{(\mathcal{V}_0,\mathcal{V}_1)})$$
$$f_2 = \Phi(f_{(\mathcal{V}_0,\mathcal{V}_1,\mathcal{V}_2)})$$
$$\dots$$

Generally, $f_n = \Phi(f_{(\mathcal{V}_0,\mathcal{V}_1,\mathcal{V}_2,\dots,\mathcal{V}_n)})$.

$$f_{0,1} = \Phi(f_{(\mathcal{V}_0),(\mathcal{V}_0,\mathcal{V}_1)})$$
$$f_{1,2} = \Phi(f_{(\mathcal{V}_0,\mathcal{V}_1),(\mathcal{V}_0,\mathcal{V}_1,\mathcal{V}_2)})$$
$$f_{2,3} = \Phi(f_{(\mathcal{V}_0,\mathcal{V}_1,\mathcal{V}_2),(\mathcal{V}_0,\mathcal{V}_1,\mathcal{V}_2,\mathcal{V}_3)})$$

Generally, $f_{n,n+1} = \Phi(f_{(\mathcal{V}_0,\mathcal{V}_1,\mathcal{V}_2,\dots,\mathcal{V}_n),(\mathcal{V}_0,\mathcal{V}_1,\mathcal{V}_2,\dots,\mathcal{V}_{n+1})})$.

We define $\Phi(f^\infty) = (f_n, f_{n,n+1})$, where $(f_n, f_{n,n+1})$ is a strong proximate sequence in $SSh^2(Cpt)$.

Let g^∞ consists of $g_{\underline{b_1 b_2}\dots\underline{b_n}}$, where $\underline{b_i} \in \underline{\mathcal{W}}$, and $\underline{\mathcal{W}}$ is the set of all finite (ordered) subsets of the sequence of coverings of Z that have a maximal element. Let $\mathcal{V}_n$ be a covering of Y, so that $W_n \succ g(\mathcal{V}_n)$.

Let $(h^\infty) = (g^\infty)(f^\infty)$. Next we show that:

$$\Phi(g^\infty f^\infty) = (f_n, f_{n+1})\Phi(g^\infty)\Phi(f^\infty).$$

Step 0.
In $SSh^\infty(Cpt)$ we have $h_{(\mathcal{W}_0,W_1,\dots,\mathcal{W}_n)} = g_{(\mathcal{W}_0,W_1,\dots,W_n)} f_{(\mathcal{V}_0,V_1,\dots,V_n)}$. In $SSh^2(Cpt)$ we have $h_n = g_n f_n$.

Step 1.
In $SSh^\infty(Cpt)$, according to the composition formula ([4]), we have:

$$h_{(\mathcal{W}_0,\mathcal{W}_1,\dots,\mathcal{W}_n),(\mathcal{W}_0,\mathcal{W}_1,\dots,\mathcal{W}_{n+1})}(t,x) =$$
$$= \begin{cases} g_{(\mathcal{W}_0,\mathcal{W}_1,\dots,\mathcal{W}_n)} f_{(\mathcal{V}_0,\mathcal{V}_1,\mathcal{V}_2,\dots,\mathcal{V}_n),(\mathcal{V}_0,\mathcal{V}_1,\mathcal{V}_2,\dots,\mathcal{V}_{n+1})}(2t,x), \ t \le \frac{1}{2} \\ g_{(\mathcal{W}_0,\mathcal{W}_1,\dots,\mathcal{W}_n),(\mathcal{W}_0,\mathcal{W}_1,\dots,\mathcal{W}_{n+1})}(2t-1, f_{(\mathcal{V}_0,\mathcal{V}_1,\mathcal{V}_2,\dots,\mathcal{V}_{n+1})}(x)), \ t \ge \frac{1}{2} \end{cases}$$

In $SSh^2(Cpt)$, we have:

$$h_{n,n+1}(t,x) = \begin{cases} g_n f_{n,n+1}(2t,x), \ t \le \frac{1}{2} \\ g_{n,n+1}(2t-1, f_{n+1}(x)), \ t \ge \frac{1}{2} \end{cases}$$

It follows that it holds $\Phi(g^\infty f^\infty) = (f_n, f_{n+1})\Phi(g^\infty)\Phi(f^\infty)$.

It is clear that for the identity h0lds $\Phi(1^\infty) = (1_n, 1_{n+1})$.

Next we show that Φ is surjection.

Let the strong proximate sequence $(f_n, f_{n,n+1})$ be given. We will construct a coherent proximate net (CPN) f^∞, such that $\Phi(f^\infty) = (f_n, f_{n,n+1})$.

We define:

$$f_{(\mathcal{V}_0)} = f_0$$
$$f_{(\mathcal{V}_0,\mathcal{V}_1)} = f_1$$
$$f_{(\mathcal{V}_0,\mathcal{V}_1,\mathcal{V}_2)} = f_2$$
$$\dots$$

Generally, $f_{(\mathcal{V}_0,\mathcal{V}_1,\mathcal{V}_2,\dots,\mathcal{V}_n)} = f_n$.

$$f_{(\mathcal{V}_0),(\mathcal{V}_0,\mathcal{V}_1)} = f_{0,1}$$
$$f_{(\mathcal{V}_0,\mathcal{V}_1),(\mathcal{V}_0,\mathcal{V}_1,\mathcal{V}_2)} = f_{1,2}$$
$$f_{(\mathcal{V}_0,\mathcal{V}_1,\mathcal{V}_2),(\mathcal{V}_0,\mathcal{V}_1,\mathcal{V}_2,\mathcal{V}_3)} = f_{2,3}$$
$$\dots$$

Generally, $f_{(\mathcal{V}_0,\mathcal{V}_1,\mathcal{V}_2,\dots,\mathcal{V}_n),(\mathcal{V}_0,\mathcal{V}_1,\mathcal{V}_2,\dots,\mathcal{V}_{n+1})} = f_{n,n+1}$.

By the previous, the functions that have up to two members in the finite sequence of the index are defined.

Next we define the functions that have three members in the finite sequence of the index, and that are of a type

$$f_{(\mathcal{V}_0,\mathcal{V}_1,\mathcal{V}_2,\dots,\mathcal{V}_n),(\mathcal{V}_0,\mathcal{V}_1,\mathcal{V}_2,\dots,\mathcal{V}_n,\mathcal{V}_{n+1}),(\mathcal{V}_0,\mathcal{V}_1,\mathcal{V}_2,\dots,\mathcal{V}_{n+1},\mathcal{V}_{n+2})}.$$

As the simplest example of a such function, we observe the function

$$f_{(\mathcal{V}_0),(\mathcal{V}_0,\mathcal{V}_1),(\mathcal{V}_0,\mathcal{V}_1,\mathcal{V}_2)} : \Delta^2 \times X \to Y,$$

where $\Delta^2 = \{(t_1, t_2) | 1 \geq t_1 \geq t_2 \geq 0\}$ is the non-standard simplex. We define it as follows:

Above there are already defined the functions with two successive members in the finite sequence of the index, i.e., the functions $f_{(\mathcal{V}_0),(\mathcal{V}_0,\mathcal{V}_1)}$ and $f_{(\mathcal{V}_0,\mathcal{V}_1),(\mathcal{V}_0,\mathcal{V}_1,\mathcal{V}_2)}$ are defined (picture 2).

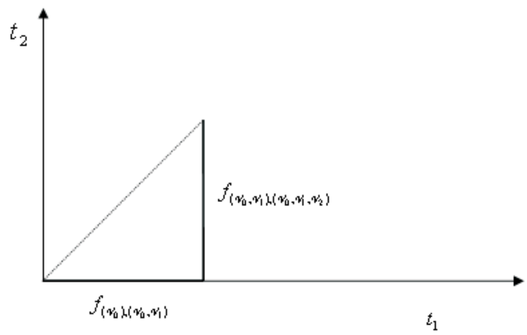

FIGURE 2

The function $f_{(\mathcal{V}_0),(\mathcal{V}_0,\mathcal{V}_1)}$ is defined on the edge $t_2 = 0$ at the simplex Δ^2, and the function $f_{(\mathcal{V}_0,\mathcal{V}_1),(\mathcal{V}_0,\mathcal{V}_1,\mathcal{V}_2)}$ is defined on the edge $t_1 = 1$ in Δ^2.

We denote by L^2 the part of $\partial\Delta^2$ where $t_2 = 0$ or $t_1 = 1$. We define a function $f'_{(\mathcal{V}_0),(\mathcal{V}_0,\mathcal{V}_1),(\mathcal{V}_0,\mathcal{V}_1,\mathcal{V}_2)} : L^2 \times X \to Y$ in the following way:

$$f'_{(\mathcal{V}_0),(\mathcal{V}_0,\mathcal{V}_1),(\mathcal{V}_0,\mathcal{V}_1,\mathcal{V}_2)}(\underline{t}, x) = \begin{cases} f_{(\mathcal{V}_0),(\mathcal{V}_0,\mathcal{V}_1)}(\underline{t}, x), \ t_2 = 0 \\ f_{(\mathcal{V}_0,\mathcal{V}_1),(\mathcal{V}_0,\mathcal{V}_1,\mathcal{V}_2)}(\underline{t}, x), \ t_1 = 1 \end{cases}$$

There exists a continuous retraction $r^2 : \Delta^2 \to L^2$. Now we may define the function $f_{(\mathcal{V}_0),(\mathcal{V}_0,\mathcal{V}_1),(\mathcal{V}_0,\mathcal{V}_1,\mathcal{V}_2)} : \Delta^2 \times X \to Y$ as follows:

$$f_{(\mathcal{V}_0),(\mathcal{V}_0,\mathcal{V}_1),(\mathcal{V}_0,\mathcal{V}_1,\mathcal{V}_2)}(\underline{t}, x) = f'_{(\mathcal{V}_0),(\mathcal{V}_0,\mathcal{V}_1),(\mathcal{V}_0,\mathcal{V}_1,\mathcal{V}_2)}(r^2(\underline{t}), x).$$

In order to define the function with three members in the finite sequence of the index $f_{(\mathcal{V}_0,\mathcal{V}_1,\mathcal{V}_2,...,\mathcal{V}_n),(\mathcal{V}_0,\mathcal{V}_1,\mathcal{V}_2,...,\mathcal{V}_n,\mathcal{V}_{n+1}),(\mathcal{V}_0,\mathcal{V}_1,\mathcal{V}_2,...,\mathcal{V}_{n+1},\mathcal{V}_{n+2})}$ generally, we apply the same method. Namely, the functions with two members in the finite sequence of the index, the function $f_{(\mathcal{V}_0,\mathcal{V}_1,\mathcal{V}_2,...,\mathcal{V}_n),(\mathcal{V}_0,\mathcal{V}_1,\mathcal{V}_2,...,\mathcal{V}_n,\mathcal{V}_{n+1})}$ and the function $f_{(\mathcal{V}_0,\mathcal{V}_1,\mathcal{V}_2,...,\mathcal{V}_n,\mathcal{V}_{n+1}),(\mathcal{V}_0,\mathcal{V}_1,\mathcal{V}_2,...,\mathcal{V}_{n+1},\mathcal{V}_{n+2})}$ are already defined.

In the same way as above we define the set L^2, and the function

$$f'_{(\mathcal{V}_0,\mathcal{V}_1,\mathcal{V}_2,...,\mathcal{V}_n),(\mathcal{V}_0,\mathcal{V}_1,\mathcal{V}_2,...,\mathcal{V}_n,\mathcal{V}_{n+1}),(\mathcal{V}_0,\mathcal{V}_1,\mathcal{V}_2,...,\mathcal{V}_{n+1},\mathcal{V}_{n+2})} : L^2 \times X \to Y.$$

Then we define the function
$f_{(\mathcal{V}_0,\mathcal{V}_1,\mathcal{V}_2,...,\mathcal{V}_n),(\mathcal{V}_0,\mathcal{V}_1,\mathcal{V}_2,...,\mathcal{V}_n,\mathcal{V}_{n+1}),(\mathcal{V}_0,\mathcal{V}_1,\mathcal{V}_2,...,\mathcal{V}_{n+1},\mathcal{V}_{n+2})} : \Delta^2 \times X \to Y$ by:

$$f_{(\mathcal{V}_0,...,\mathcal{V}_n),(\mathcal{V}_0,...,\mathcal{V}_{n+1}),(\mathcal{V}_0,...,\mathcal{V}_{n+2})}(\underline{t}, x) = f'_{(\mathcal{V}_0,...,\mathcal{V}_n),(\mathcal{V}_0,...,\mathcal{V}_{n+1}),(\mathcal{V}_0,...,\mathcal{V}_{n+2})}(r^2(\underline{t}), x).$$

By that we have defined the functions with three members in the finite sequence of the index. Let us assume that the functions with up to $n-1$ members in the finite sequence of the index are defined.

Generally, for the function $f_{(\mathcal{V}_0,...,\mathcal{V}_i),(\mathcal{V}_0,...,\mathcal{V}_{i+1}),...,(\mathcal{V}_0,...,\mathcal{V}_{i+n})}$ with n members in the finite sequence of the index, there are defined $f_{(\mathcal{V}_0,...,\mathcal{V}_i),(\mathcal{V}_0,...,\mathcal{V}_{i+1}),...,(\mathcal{V}_0,...,\mathcal{V}_{i+n-1})}$ and $f_{(\mathcal{V}_0,...,\mathcal{V}_{i+1}),...,(\mathcal{V}_0,...,\mathcal{V}_{i+n})}$ because of the inductive assumption.

We define $L^n = \{\underline{t} \in \partial\Delta^n | t_1 = 1 \text{ or } t_n = 0\}$.

Then we define the function $f'_{(\mathcal{V}_0,...,\mathcal{V}_i),(\mathcal{V}_0,...,\mathcal{V}_{i+1}),...,(\mathcal{V}_0,...,\mathcal{V}_{i+n})} : L^n \times X \to Y$ in the following way:

$$f'_{(\mathcal{V}_0,...,\mathcal{V}_i),(\mathcal{V}_0,...,\mathcal{V}_{i+1}),...,(\mathcal{V}_0,...,\mathcal{V}_{i+n})}(\underline{t}, x) = \begin{cases} f_{(\mathcal{V}_0,...,\mathcal{V}_i),(\mathcal{V}_0,...,\mathcal{V}_{i+1}),...,(\mathcal{V}_0,...,\mathcal{V}_{i+n-1})}(\underline{t}, x), \ t_n = 0 \\ f_{(\mathcal{V}_0,...,\mathcal{V}_{i+1}),...,(\mathcal{V}_0,...,\mathcal{V}_{i+n})}(\underline{t}, x), \ t_1 = 1 \end{cases}$$

There there exists a retraction $r^n : \Delta^n \to L^n$. Finally, we define the function $f_{(\mathcal{V}_0,...,\mathcal{V}_i),(\mathcal{V}_0,...,\mathcal{V}_{i+1}),...,(\mathcal{V}_0,...,\mathcal{V}_{i+n})}$ by:

$$f_{(\mathcal{V}_0,...,\mathcal{V}_i),(\mathcal{V}_0,...,\mathcal{V}_{i+1}),...,(\mathcal{V}_0,...,\mathcal{V}_{i+n})}(\underline{t}, x) = f'_{(\mathcal{V}_0,...,\mathcal{V}_i),(\mathcal{V}_0,...,\mathcal{V}_{i+1}),...,(\mathcal{V}_0,...,\mathcal{V}_{i+n})}(r^n(\underline{t}), x).$$

In such a way we obtain all functions of a type $f_{(\mathcal{V}_0,...,\mathcal{V}_i),(\mathcal{V}_0,...,\mathcal{V}_{i+1}),...,(\mathcal{V}_0,...,\mathcal{V}_{i+n})}$.
We will not lose of generality if we observe
$f_{(\mathcal{V}_0),(\mathcal{V}_0,\mathcal{V}_1),(\mathcal{V}_0,\mathcal{V}_1,\mathcal{V}_2),(\mathcal{V}_0,\mathcal{V}_1,\mathcal{V}_2,\mathcal{V}_3),...,(\mathcal{V}_0,\mathcal{V}_1,\mathcal{V}_2,...,\mathcal{V}_n)}$. Each function whose finite sequence of the index has one member less is already defined, because the coherence condition holds.

$$f_{(\mathcal{V}_0),(\mathcal{V}_0,\mathcal{V}_1),(\mathcal{V}_0,\mathcal{V}_1,\mathcal{V}_2),(\mathcal{V}_0,\mathcal{V}_1,\mathcal{V}_2,\mathcal{V}_3),...,(\mathcal{V}_0,\mathcal{V}_1,\mathcal{V}_2,...,\mathcal{V}_n)}(t_1, t_2, ..., t_n, x) =$$
$$= \begin{cases} f_{(\mathcal{V}_0,\mathcal{V}_1),(\mathcal{V}_0,\mathcal{V}_1,\mathcal{V}_2),(\mathcal{V}_0,\mathcal{V}_1,\mathcal{V}_2,\mathcal{V}_3),...,(\mathcal{V}_0,\mathcal{V}_1,\mathcal{V}_2,...,\mathcal{V}_n)}(t_2, ..., t_n, x), \ \ t_1 = 1 \\ f_{(\mathcal{V}_0),(\mathcal{V}_0,\mathcal{V}_1),(\mathcal{V}_0,\mathcal{V}_1,\mathcal{V}_2),...,<(\mathcal{V}_0,\mathcal{V}_1,\mathcal{V}_2,...,\mathcal{V}_i)>,...,(\mathcal{V}_0,\mathcal{V}_1,\mathcal{V}_2,...,\mathcal{V}_n)}(t_1, ..., \hat{t}_i, ..., t_n, x), t_i = t_{i+1} \\ f_{(\mathcal{V}_0),(\mathcal{V}_0,\mathcal{V}_1),(\mathcal{V}_0,\mathcal{V}_1,\mathcal{V}_2),(\mathcal{V}_0,\mathcal{V}_1,\mathcal{V}_2,\mathcal{V}_3),...,(\mathcal{V}_0,\mathcal{V}_1,\mathcal{V}_2,...,\mathcal{V}_{n-1})}(t_1, ..., t_{n-1}, x), \ t_n = 0 \end{cases}$$

So we obtain, for example the functions: $f_{(\mathcal{V}_0,\mathcal{V}_1),(\mathcal{V}_0,\mathcal{V}_1,\mathcal{V}_2),(\mathcal{V}_0,\mathcal{V}_1,\mathcal{V}_2,\mathcal{V}_3),...,(\mathcal{V}_0,\mathcal{V}_1,\mathcal{V}_2,...,\mathcal{V}_n)}$, $f_{(\mathcal{V}_0),(\mathcal{V}_0,\mathcal{V}_1,\mathcal{V}_2),(\mathcal{V}_0,\mathcal{V}_1,\mathcal{V}_2,\mathcal{V}_3),...,(\mathcal{V}_0,\mathcal{V}_1,\mathcal{V}_2,...,\mathcal{V}_n)}$, and so on.

Because of the same reasons we obtain the functions that in the finite sequence of the index have two members less. For example, in such a way we obtain the functions: $f_{(\mathcal{V}_0,\mathcal{V}_1,\mathcal{V}_2),(\mathcal{V}_0,\mathcal{V}_1,\mathcal{V}_2,\mathcal{V}_3),...,(\mathcal{V}_0,\mathcal{V}_1,\mathcal{V}_2,...,\mathcal{V}_n)}$, $f_{(\mathcal{V}_0,\mathcal{V}_1),(\mathcal{V}_0,\mathcal{V}_1,\mathcal{V}_2,\mathcal{V}_3),...,(\mathcal{V}_0,\mathcal{V}_1,\mathcal{V}_2,...,\mathcal{V}_n)}$, $f_{(\mathcal{V}_0),(\mathcal{V}_0,\mathcal{V}_1,\mathcal{V}_2,\mathcal{V}_3),...,(\mathcal{V}_0,\mathcal{V}_1,\mathcal{V}_2,...,\mathcal{V}_n)}$, and so on.

On the other hand, if it happens that there is a covering that "is missing" (or more coverings that "are missing") in some member of the index, then we define them by completing them by the ones that are missing, starting from left to the right.

For example, in the index of the function $f_{(\mathcal{V}_0),(\mathcal{V}_0,\mathcal{V}_2)}$ we notice that $\mathcal{V}_1$ "is missing" in the second member of the sequence of the index, so we complete the member by it and obtain $f_{(\mathcal{V}_0),(\mathcal{V}_0,\mathcal{V}_1,\mathcal{V}_2)}$. So, by definition,

$$f_{(\mathcal{V}_0),(\mathcal{V}_0,\mathcal{V}_2)} = f_{(\mathcal{V}_0),(\mathcal{V}_0,\mathcal{V}_1,\mathcal{V}_2)}.$$

Another example is the function $f_{(\mathcal{V}_0),(\mathcal{V}_0,\mathcal{V}_3),(\mathcal{V}_0,\mathcal{V}_3,\mathcal{V}_8)}$. We notice that in the second member of the sequence of the index $\mathcal{V}_1$ and $\mathcal{V}_2$ are missing, and in the third member $\mathcal{V}_1$, $\mathcal{V}_2$, $\mathcal{V}_4$, $\mathcal{V}_5$, $\mathcal{V}_6$ and $\mathcal{V}_7$are missing, so we define $f_{(\mathcal{V}_0),(\mathcal{V}_0,\mathcal{V}_3),(\mathcal{V}_0,\mathcal{V}_3,\mathcal{V}_8)}$ in the following way:

$$f_{(\mathcal{V}_0),(\mathcal{V}_0,\mathcal{V}_3),(\mathcal{V}_0,\mathcal{V}_3,\mathcal{V}_8)} = f_{(\mathcal{V}_0,\mathcal{V}_1),(\mathcal{V}_0,\mathcal{V}_1,\mathcal{V}_2,\mathcal{V}_3),(\mathcal{V}_0,\mathcal{V}_1,\mathcal{V}_2,\mathcal{V}_3,\mathcal{V}_4,\mathcal{V}_5,\mathcal{V}_6,\mathcal{V}_7,\mathcal{V}_8)}.$$

We check that for the functions defined in such way, the coherence condition holds. It is enough to check it for the functions that do not have less members in the index, but there are missing coverings in some of the members. We may also remark that if some covering shows in a certain member of the finite sequence of the index, the same will have to show in all of the following members, because of the way the coherent proximate net is defined.

For example, let the covering $\mathcal{V}_l$ be missing in the j-th member of the index. Then, by definition we have:

$$\begin{aligned}
&f_{(\mathcal{V}_0),(\mathcal{V}_0,\mathcal{V}_1),...,(\mathcal{V}_0,\mathcal{V}_1,...,\hat{\mathcal{V}}_l,...,\mathcal{V}_j),...,(\mathcal{V}_0,\mathcal{V}_1,\mathcal{V}_2,...,\mathcal{V}_n)}(t_1,t_2,...,t_n,x)\\
&= f_{(\mathcal{V}_0),(\mathcal{V}_0,\mathcal{V}_1),...,(\mathcal{V}_0,\mathcal{V}_1,...,\mathcal{V}_l,...,\mathcal{V}_j),...,(\mathcal{V}_0,\mathcal{V}_1,\mathcal{V}_2,...,\mathcal{V}_n)}(t_1,t_2,...,t_n,x)
\end{aligned}$$

$$= \begin{cases} f_{(\mathcal{V}_0,\mathcal{V}_1),(\mathcal{V}_0,\mathcal{V}_1,\mathcal{V}_2),...,(\mathcal{V}_0,\mathcal{V}_1,...,\mathcal{V}_l,...,\mathcal{V}_j),...,(\mathcal{V}_0,\mathcal{V}_1,\mathcal{V}_2,...,\mathcal{V}_n)}(t_2,...,t_n,x), \ t_1 = 1 \\ f_{(\mathcal{V}_0),(\mathcal{V}_0,\mathcal{V}_1),(\mathcal{V}_0,\mathcal{V}_1,\mathcal{V}_2),...,<(\mathcal{V}_0,\mathcal{V}_1,\mathcal{V}_2,...,\mathcal{V}_i)>,...,(\mathcal{V}_0,\mathcal{V}_1,\mathcal{V}_2,...,\mathcal{V}_n)}(t_1,...,\hat{t}_i,...,t_n,x), t_i = t_{i+1} \\ f_{(\mathcal{V}_0),(\mathcal{V}_0,\mathcal{V}_1),...,(\mathcal{V}_0,\mathcal{V}_1,...,\mathcal{V}_l,...,\mathcal{V}_j),...,(\mathcal{V}_0,\mathcal{V}_1,\mathcal{V}_2,...,\mathcal{V}_{n-1})}(t_1,...,t_{n-1},x), \ t_n = 0 \end{cases}$$

$$= \begin{cases} f_{(\mathcal{V}_0,\mathcal{V}_1),...,(\mathcal{V}_0,\mathcal{V}_1,...,\hat{\mathcal{V}}_l,...,\mathcal{V}_j),...,(\mathcal{V}_0,\mathcal{V}_1,\mathcal{V}_2,...,\mathcal{V}_n)}(t_2,...,t_n,x), \ t_1 = 1 \\ f_i(t_1,...,\hat{t}_i,...,t_n,x), \quad t_i = t_{i+1} \\ f_{(\mathcal{V}_0),(\mathcal{V}_0,\mathcal{V}_1),...,(\mathcal{V}_0,\mathcal{V}_1,...,\hat{\mathcal{V}}_l,...,\mathcal{V}_j),...,(\mathcal{V}_0,\mathcal{V}_1,\mathcal{V}_2,...,\mathcal{V}_{n-1})}(t_1,...,t_{n-1},x), \ t_n = 0 \end{cases}$$

where $f_i(t_1,...,\hat{t}_i,...,t_n,x)$

$$= \begin{cases} f_{(\mathcal{V}_0),...,<(\mathcal{V}_0,\mathcal{V}_1,\mathcal{V}_2,...,\mathcal{V}_i)>,...,(\mathcal{V}_0,\mathcal{V}_1,...,\hat{\mathcal{V}}_l,...,\mathcal{V}_j),...,(\mathcal{V}_0,\mathcal{V}_1,\mathcal{V}_2,...,\mathcal{V}_n)}(t_1,...,\hat{t}_i,...,t_n,x), \ i < j \\ f_{(\mathcal{V}_0),...,<(\mathcal{V}_0,\mathcal{V}_1,...,\hat{\mathcal{V}}_l,...,\mathcal{V}_i)>,...,(\mathcal{V}_0,\mathcal{V}_1,\mathcal{V}_2,...,\mathcal{V}_n)}(t_1,...,\hat{t}_i,...,t_n,x), \ i = j \\ f_{(\mathcal{V}_0),...,(\mathcal{V}_0,\mathcal{V}_1,...,\hat{\mathcal{V}}_l,...,\mathcal{V}_j),...,<(\mathcal{V}_0,\mathcal{V}_1,\mathcal{V}_2,...,\mathcal{V}_i)>,...,(\mathcal{V}_0,\mathcal{V}_1,\mathcal{V}_2,...,\mathcal{V}_n)}(t_1,...,\hat{t}_i,...,t_n,x), \ i > j \end{cases}$$

So, the coherence condition holds.

Similarly, we show that the coherence condition holds also in the case when there are more than one coverings missing in the members of the sequence.

We proved that Φ is surjection.

Next step is to show that $\Phi : SSh^{\infty}(Cpt) \to SSh^2(Cpt)$ is an injection.

Let $\Phi(f^{\infty}) = (f_n, f_{n,n+1})$ and $\Phi(f'^{\infty}) = (f'_n, f'_{n,n+1})$. We will show that if $(f_n, f_{n,n+1}) \approx (f'_n, f'_{n,n+1})$ by a homotopy $(F_n, F_{n,n+1})$ in $SSh^2(Cpt)$, then there exists a homotopy F^{∞} in $SSh^{\infty}(Cpt)$ which connects f^{∞} and f'^{∞}.

We have th homotopy $F_n$that connects f_n and f'_n (picture 3).

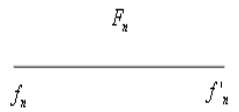

FIGURE 3

$$F_n = \Phi(F_{(\mathcal{V}_0, \mathcal{V}_1, \mathcal{V}_2, \ldots, \mathcal{V}_n)}).$$

$F_{n,n+1}$ is a homotopy that connects $f_{n,n+1}$ and $f'_{n,n+1}$ (picture 4).

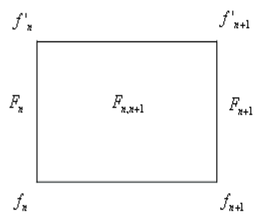

FIGURE 4

$$F_{n,n+1} = \Phi(F_{(\mathcal{V}_0, \mathcal{V}_1, \mathcal{V}_2, \ldots, \mathcal{V}_n), (\mathcal{V}_0, \mathcal{V}_1, \mathcal{V}_2, \ldots, \mathcal{V}_{n+1})}).$$

At picture 5 (below) there is a given illustration of the homotopies that we have.

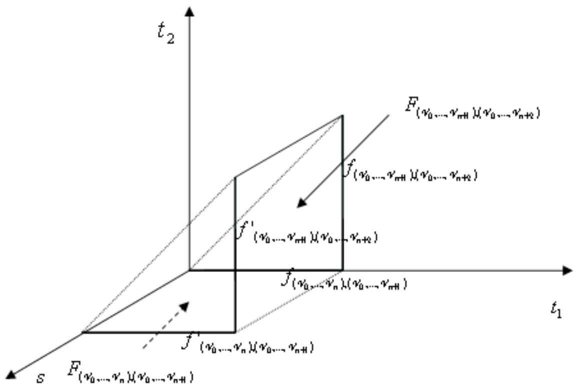

FIGURE 5

Let K^2 be the part of Δ^2 where $t_2 = 0$ or $t_1 = 1$. We define a function $F'_{(\mathcal{V}_0,\mathcal{V}_1,\mathcal{V}_2,\ldots,\mathcal{V}_n),(\mathcal{V}_0,\mathcal{V}_1,\mathcal{V}_2,\ldots,\mathcal{V}_n,\mathcal{V}_{n+1}),(\mathcal{V}_0,\mathcal{V}_1,\mathcal{V}_2,\ldots,\mathcal{V}_{n+1},\mathcal{V}_{n+2})} : K^2 \times I \times X \to Y$ in the following way:

$$F'_{(\mathcal{V}_0,\ldots,\mathcal{V}_n),(\mathcal{V}_0,\ldots,\mathcal{V}_{n+1}),(\mathcal{V}_0,\ldots,\mathcal{V}_{n+2})}(t_1,t_2,s,x) = \begin{cases} F_{(\mathcal{V}_0,\ldots,\mathcal{V}_n),(\mathcal{V}_0,\ldots,\mathcal{V}_{n+1})}(t_1,t_2,s,x), \ t_2 = 0 \\ F_{(\mathcal{V}_0,\ldots,\mathcal{V}_{n+1}),(\mathcal{V}_0,\ldots,\mathcal{V}_{n+2})}(t_1,t_2,s,x), \ t_1 = 1 \end{cases}$$

There exists a continuous retraction $R^2 : \Delta^2 \times I \to K^2 \times I$, so that $R^2|_{\Delta^2} = r^2$, the previously defined retraction. We now define the function:

$F_{(\mathcal{V}_0,\mathcal{V}_1,\mathcal{V}_2,\ldots,\mathcal{V}_n),(\mathcal{V}_0,\mathcal{V}_1,\mathcal{V}_2,\ldots,\mathcal{V}_n,\mathcal{V}_{n+1}),(\mathcal{V}_0,\mathcal{V}_1,\mathcal{V}_2,\ldots,\mathcal{V}_{n+1},\mathcal{V}_{n+2})} : \Delta^2 \times I \times X \to Y$ in the following way:

$$\begin{aligned} &F_{(\mathcal{V}_0,\ldots,\mathcal{V}_n),(\mathcal{V}_0,\ldots,\mathcal{V}_{n+1}),(\mathcal{V}_0,\ldots,\mathcal{V}_{n+2})}(t_1,t_2,s,x) \\ &\qquad = F'_{(\mathcal{V}_0,\ldots,\mathcal{V}_n),(\mathcal{V}_0,\ldots,\mathcal{V}_{n+1}),(\mathcal{V}_0,\ldots,\mathcal{V}_{n+2})}(R^2(t_1,t_2),s,x) \end{aligned}$$

For $s = 0$,

$$\begin{aligned} &F_{(\mathcal{V}_0,\ldots,\mathcal{V}_n),(\mathcal{V}_0,\ldots,\mathcal{V}_{n+1}),(\mathcal{V}_0,\ldots,\mathcal{V}_{n+2})}(t_1,t_2,0,x) \\ &= F'_{(\mathcal{V}_0,\ldots,\mathcal{V}_n),(\mathcal{V}_0,\ldots,\mathcal{V}_{n+1}),(\mathcal{V}_0,\ldots,\mathcal{V}_{n+2})}(R^2(t_1,t_2),0,x) \\ &= \begin{cases} F_{(\mathcal{V}_0,\ldots,\mathcal{V}_n),(\mathcal{V}_0,\ldots,\mathcal{V}_{n+1})}(R^2(t_1,t_2),0,x), \ t_2 = 0 \\ F_{(\mathcal{V}_0,\ldots,\mathcal{V}_{n+1}),(\mathcal{V}_0,\ldots,\mathcal{V}_{n+2})}(R^2(t_1,t_2),0,x), \ t_1 = 1 \end{cases} \\ &= \begin{cases} f_{(\mathcal{V}_0,\ldots,\mathcal{V}_n),(\mathcal{V}_0,\ldots,\mathcal{V}_{n+1})}(r^2(t_1,t_2),x), \ t_2 = 0 \\ f_{(\mathcal{V}_0,\ldots,\mathcal{V}_{n+1}),(\mathcal{V}_0,\ldots,\mathcal{V}_{n+2})}(r^2(t_1,t_2),x), \ t_1 = 1 \end{cases} \\ &= f_{(\mathcal{V}_0,\ldots,\mathcal{V}_n),(\mathcal{V}_0,\ldots,\mathcal{V}_{n+1}),(\mathcal{V}_0,\ldots,\mathcal{V}_{n+2})}(t_1,t_2,x). \end{aligned}$$

- For $s=1$,

$$\begin{aligned}
&F_{(\mathcal{V}_0,\ldots,\mathcal{V}_n),(\mathcal{V}_0,\ldots,\mathcal{V}_{n+1}),(\mathcal{V}_0,\ldots,\mathcal{V}_{n+2})}(t_1,t_2,1,x)\\
&=F'_{(\mathcal{V}_0,\ldots,\mathcal{V}_n),(\mathcal{V}_0,\ldots,\mathcal{V}_{n+1}),(\mathcal{V}_0,\ldots,\mathcal{V}_{n+2})}(R^2(t_1,t_2),1,x)\\
&=\begin{cases} F_{(\mathcal{V}_0,\ldots,\mathcal{V}_n),(\mathcal{V}_0,\ldots,\mathcal{V}_{n+1})}(R^2(t_1,t_2),1,x), & t_2=0\\ F_{(\mathcal{V}_0,\ldots,\mathcal{V}_{n+1}),(\mathcal{V}_0,\ldots,\mathcal{V}_{n+2})}(R^2(t_1,t_2),1,x), & t_1=1\end{cases}\\
&=\begin{cases} f'_{(\mathcal{V}_0,\ldots,\mathcal{V}_n),(\mathcal{V}_0,\ldots,\mathcal{V}_{n+1})}(r^2(t_1,t_2),x), & t_2=0\\ f'_{(\mathcal{V}_0,\ldots,\mathcal{V}_{n+1}),(\mathcal{V}_0,\ldots,\mathcal{V}_{n+2})}(r^2(t_1,t_2),x), & t_1=1\end{cases}\\
&=f'_{(\mathcal{V}_0,\ldots,\mathcal{V}_n),(\mathcal{V}_0,\ldots,\mathcal{V}_{n+1}),(\mathcal{V}_0,\ldots,\mathcal{V}_{n+2})}(t_1,t_2,x).
\end{aligned}$$

So we have defined the functions having three members in the index.

Let us assume that the functions having up to $n-1$ members in the finite sequence of the index are defined.

Generally, for the function $F_{(\mathcal{V}_0,\ldots,\mathcal{V}_i),(\mathcal{V}_0,\ldots,\mathcal{V}_{i+1}),\ldots,(\mathcal{V}_0,\ldots,\mathcal{V}_{i+n})}$ having n members in the finite sequence of the index, the functions $F_{(\mathcal{V}_0,\ldots,\mathcal{V}_i),(\mathcal{V}_0,\ldots,\mathcal{V}_{i+1}),\ldots,(\mathcal{V}_0,\ldots,\mathcal{V}_{i+n-1})}$ and $F_{(\mathcal{V}_0,\ldots,\mathcal{V}_{i+1}),\ldots,(\mathcal{V}_0,\ldots,\mathcal{V}_{i+n})}$ are defined.

We define the set:

$$K^n=\{\underline{t}\in\partial\Delta^n|t_1=1\text{ or }t_n=0\}.$$

Then we define a function $F'_{(\mathcal{V}_0,\ldots,\mathcal{V}_i),(\mathcal{V}_0,\ldots,\mathcal{V}_{i+1}),\ldots,(\mathcal{V}_0,\ldots,\mathcal{V}_{i+n})}:K^n\times I\times X\to Y$ in the following way:

$$\begin{aligned}
&F'_{(\mathcal{V}_0,\ldots,\mathcal{V}_i),(\mathcal{V}_0,\ldots,\mathcal{V}_{i+1}),\ldots,(\mathcal{V}_0,\ldots,\mathcal{V}_{i+n})}(\underline{t},s,x)\\
&=\begin{cases} F_{(\mathcal{V}_0,\ldots,\mathcal{V}_i),(\mathcal{V}_0,\ldots,\mathcal{V}_{i+1}),\ldots,(\mathcal{V}_0,\ldots,\mathcal{V}_{i+n-1})}(\underline{t},s,x), & t_n=0\\ F_{(\mathcal{V}_0,\ldots,\mathcal{V}_{i+1}),\ldots,(\mathcal{V}_0,\ldots,\mathcal{V}_{i+n})}(\underline{t},s,x), & t_1=1\end{cases}
\end{aligned}$$

There exists a continuous retraction $R^n:\Delta^n\times I\to K^n\times I$, so that $R^n|_{\Delta^n}=r^n$.

Finally, we define the function $F_{(\mathcal{V}_0,\ldots,\mathcal{V}_i),(\mathcal{V}_0,\ldots,\mathcal{V}_{i+1}),\ldots,(\mathcal{V}_0,\ldots,\mathcal{V}_{i+n})}$ by:

$$\begin{aligned}
&F_{(\mathcal{V}_0,\ldots,\mathcal{V}_i),(\mathcal{V}_0,\ldots,\mathcal{V}_{i+1}),\ldots,(\mathcal{V}_0,\ldots,\mathcal{V}_{i+n})}(\underline{t},s,x)\\
&=F'_{(\mathcal{V}_0,\ldots,\mathcal{V}_i),(\mathcal{V}_0,\ldots,\mathcal{V}_{i+1}),\ldots,(\mathcal{V}_0,\ldots,\mathcal{V}_{i+n})}(R^n(\underline{t}),s,x)
\end{aligned}$$

- For $s=0$,

$$\begin{aligned}
&F_{(\mathcal{V}_0,\ldots,\mathcal{V}_i),(\mathcal{V}_0,\ldots,\mathcal{V}_{i+1}),\ldots,(\mathcal{V}_0,\ldots,\mathcal{V}_{i+n})}(\underline{t},0,x)\\
&=F'_{(\mathcal{V}_0,\ldots,\mathcal{V}_i),(\mathcal{V}_0,\ldots,\mathcal{V}_{i+1}),\ldots,(\mathcal{V}_0,\ldots,\mathcal{V}_{i+n})}(R^n(\underline{t}),0,x)\\
&=\begin{cases} F_{(\mathcal{V}_0,\ldots,\mathcal{V}_i),(\mathcal{V}_0,\ldots,\mathcal{V}_{i+1}),\ldots,(\mathcal{V}_0,\ldots,\mathcal{V}_{i+n-1})}(R^n(\underline{t}),0,x), & t_n=0\\ F_{(\mathcal{V}_0,\ldots,\mathcal{V}_{i+1}),\ldots,(\mathcal{V}_0,\ldots,\mathcal{V}_{i+n})}(R^n(\underline{t}),0,x), & t_1=1\end{cases}\\
&=\begin{cases} f_{(\mathcal{V}_0,\ldots,\mathcal{V}_i),(\mathcal{V}_0,\ldots,\mathcal{V}_{i+1}),\ldots,(\mathcal{V}_0,\ldots,\mathcal{V}_{i+n-1})}(r^n(\underline{t}),x), & t_n=0\\ f_{(\mathcal{V}_0,\ldots,\mathcal{V}_{i+1}),\ldots,(\mathcal{V}_0,\ldots,\mathcal{V}_{i+n})}(r^n(\underline{t}),x), & t_1=1\end{cases}\\
&=f_{(\mathcal{V}_0,\ldots,\mathcal{V}_i),(\mathcal{V}_0,\ldots,\mathcal{V}_{i+1}),\ldots,(\mathcal{V}_0,\ldots,\mathcal{V}_{i+n})}(\underline{t},x).
\end{aligned}$$

- For $s=1$,

$$
\begin{aligned}
&F_{(\mathcal{V}_0,...,\mathcal{V}_i),(\mathcal{V}_0,...,\mathcal{V}_{i+1}),...,(\mathcal{V}_0,...,\mathcal{V}_{i+n})}(\underline{t},1,x)\\
&=F'_{(\mathcal{V}_0,...,\mathcal{V}_i),(\mathcal{V}_0,...,\mathcal{V}_{i+1}),...,(\mathcal{V}_0,...,\mathcal{V}_{i+n})}(R^n(\underline{t}),1,x)\\
&=\begin{cases} F_{(\mathcal{V}_0,...,\mathcal{V}_i),(\mathcal{V}_0,...,\mathcal{V}_{i+1}),...,(\mathcal{V}_0,...,\mathcal{V}_{i+n-1})}(R^n(\underline{t}),1,x),\ t_n=0\\ F_{(\mathcal{V}_0,...,\mathcal{V}_{i+1}),...,(\mathcal{V}_0,...,\mathcal{V}_{i+n})}(R^n(\underline{t}),1,x),\ t_1=1\end{cases}\\
&=\begin{cases} f'_{(\mathcal{V}_0,...,\mathcal{V}_i),(\mathcal{V}_0,...,\mathcal{V}_{i+1}),...,(\mathcal{V}_0,...,\mathcal{V}_{i+n-1})}(r^n(\underline{t}),x),\ t_n=0\\ f'_{(\mathcal{V}_0,...,\mathcal{V}_{i+1}),...,(\mathcal{V}_0,...,\mathcal{V}_{i+n})}(r^n(\underline{t}),x),\ t_1=1\end{cases}\\
&=f'_{(\mathcal{V}_0,...,\mathcal{V}_i),(\mathcal{V}_0,...,\mathcal{V}_{i+1}),...,(\mathcal{V}_0,...,\mathcal{V}_{i+n})}(\underline{t},x).
\end{aligned}
$$

In such a way we obtain all functions of type $F_{(\mathcal{V}_0,...,\mathcal{V}_i),(\mathcal{V}_0,...,\mathcal{V}_{i+1}),...,(\mathcal{V}_0,...,\mathcal{V}_{i+n})}$, which connect $f_{(\mathcal{V}_0,...,\mathcal{V}_i),(\mathcal{V}_0,...,\mathcal{V}_{i+1}),...,(\mathcal{V}_0,...,\mathcal{V}_{i+n})}$ and $f'_{(\mathcal{V}_0,...,\mathcal{V}_i),(\mathcal{V}_0,...,\mathcal{V}_{i+1}),...,(\mathcal{V}_0,...,\mathcal{V}_{i+n})}$.

Each function whose finite sequence of the index has one member less is already defined, because the coherence condition holds:

$$
\begin{aligned}
&F_{(\mathcal{V}_0),(\mathcal{V}_0,\mathcal{V}_1),(\mathcal{V}_0,\mathcal{V}_1,\mathcal{V}_2),(\mathcal{V}_0,\mathcal{V}_1,\mathcal{V}_2,\mathcal{V}_3),...,(\mathcal{V}_0,\mathcal{V}_1,\mathcal{V}_2,...,\mathcal{V}_n)}(t_1,t_2,...,t_n,s,x)\\
&=\begin{cases} F_{(\mathcal{V}_0,\mathcal{V}_1),(\mathcal{V}_0,\mathcal{V}_1,\mathcal{V}_2),(\mathcal{V}_0,\mathcal{V}_1,\mathcal{V}_2,\mathcal{V}_3),...,(\mathcal{V}_0,\mathcal{V}_1,\mathcal{V}_2,...,\mathcal{V}_n)}(t_2,...,t_n,s,x),\ t_1=1\\ F_{(\mathcal{V}_0),(\mathcal{V}_0,\mathcal{V}_1),(\mathcal{V}_0,\mathcal{V}_1,\mathcal{V}_2),...,(\mathcal{V}_0,\mathcal{V}_1,\hat{\mathcal{V}}_2,...,\mathcal{V}_i),...,(\mathcal{V}_0,\mathcal{V}_1,\mathcal{V}_2,...,\mathcal{V}_n)}(t_1,...,\hat{t}_i,...,t_n,s,x),\\ \qquad t_i=t_{i+1}\\ F_{(\mathcal{V}_0),(\mathcal{V}_0,\mathcal{V}_1),(\mathcal{V}_0,\mathcal{V}_1,\mathcal{V}_2),(\mathcal{V}_0,\mathcal{V}_1,\mathcal{V}_2,\mathcal{V}_3),...,(\mathcal{V}_0,\mathcal{V}_1,\mathcal{V}_2,...,\mathcal{V}_{n-1})}(t_1,...,t_{n-1},s,x),\ t_n=0\end{cases}
\end{aligned}
$$

Because of the same reasons we obtain the functions that in the finite sequence of the index have two or more members less. For example, in such a way we obtain: $F_{(\mathcal{V}_0,\mathcal{V}_1,\mathcal{V}_2),(\mathcal{V}_0,\mathcal{V}_1,\mathcal{V}_2,\mathcal{V}_3),...,(\mathcal{V}_0,\mathcal{V}_1,\mathcal{V}_2,...,\mathcal{V}_n)}$, $F_{(\mathcal{V}_0,\mathcal{V}_1),(\mathcal{V}_0,\mathcal{V}_1,\mathcal{V}_2,\mathcal{V}_3),...,(\mathcal{V}_0,\mathcal{V}_1,\mathcal{V}_2,...,\mathcal{V}_n)}$, $F_{(\mathcal{V}_0),(\mathcal{V}_0,\mathcal{V}_1,\mathcal{V}_2,\mathcal{V}_3),...,(\mathcal{V}_0,\mathcal{V}_1,\mathcal{V}_2,...,\mathcal{V}_n)}$, and so on.

On the other hand, if it happens that there is a covering that "is missing" (or more coverings that "are missing") in some member of the index, then we define them by completing them by the ones that are missing, starting from left to the right. For example, in the morphism $F_{(\mathcal{V}_0),(\mathcal{V}_0,\mathcal{V}_3),(\mathcal{V}_0,\mathcal{V}_3,\mathcal{V}_8)}$ we notice that $\mathcal{V}_1$ and $\mathcal{V}_2$ are missing in the second member of the sequence of the index, and in the third member $\mathcal{V}_1$, $\mathcal{V}_2$, $\mathcal{V}_4$, $\mathcal{V}_5$, $\mathcal{V}_6$ and $\mathcal{V}_7$ "are missing", so we define $F_{(\mathcal{V}_0),(\mathcal{V}_0,\mathcal{V}_3),(\mathcal{V}_0,\mathcal{V}_3,\mathcal{V}_8)}$ in the following way:

$$F_{(\mathcal{V}_0),(\mathcal{V}_0,\mathcal{V}_3),(\mathcal{V}_0,\mathcal{V}_3,\mathcal{V}_8)}=F_{(\mathcal{V}_0,\mathcal{V}_1),(\mathcal{V}_0,\mathcal{V}_1,\mathcal{V}_2,\mathcal{V}_3),(\mathcal{V}_0,\mathcal{V}_1,\mathcal{V}_2,\mathcal{V}_3,\mathcal{V}_4,\mathcal{V}_5,\mathcal{V}_6,\mathcal{V}_7,\mathcal{V}_8)}.$$

We check that for such defined functions the coherence condition holds. It is enough to check it for the functions that do not have less members in the index, but there are missing coverings in some of the members. For example, let the covering $\mathcal{V}_l$ be missing in the j-th member of the index. Then, by definition we have:

$$
\begin{aligned}
&F_{(\mathcal{V}_0),(\mathcal{V}_0,\mathcal{V}_1),...,(\mathcal{V}_0,\mathcal{V}_1,...,\hat{\mathcal{V}}_l,...,\mathcal{V}_j),...,(\mathcal{V}_0,\mathcal{V}_1,\mathcal{V}_2,...,\mathcal{V}_n)}(t_1,t_2,...,t_n,s,x)\\
&=F_{(\mathcal{V}_0),(\mathcal{V}_0,\mathcal{V}_1),...,(\mathcal{V}_0,\mathcal{V}_1,...,\mathcal{V}_l,...,\mathcal{V}_j),...,(\mathcal{V}_0,\mathcal{V}_1,\mathcal{V}_2,...,\mathcal{V}_n)}(t_1,t_2,...,t_n,s,x)
\end{aligned}
$$

$$= \begin{cases} F_{(\mathcal{V}_0,\mathcal{V}_1),(\mathcal{V}_0,\mathcal{V}_1,\mathcal{V}_2),\dots,(\mathcal{V}_0,\mathcal{V}_1,\dots,\mathcal{V}_l,\dots,\mathcal{V}_j),\dots,(\mathcal{V}_0,\mathcal{V}_1,\mathcal{V}_2,\dots,\mathcal{V}_n)}(t_2,\dots,t_n,s,x), \ t_1 = 1 \\ F_{(\mathcal{V}_0),(\mathcal{V}_0,\mathcal{V}_1),(\mathcal{V}_0,\mathcal{V}_1,\mathcal{V}_2),\dots,(\mathcal{V}_0,\mathcal{V}_1,\hat{\mathcal{V}}_2,\dots,\mathcal{V}_i),\dots,(\mathcal{V}_0,\mathcal{V}_1,\mathcal{V}_2,\dots,\mathcal{V}_n)}(t_1,\dots,\hat{t}_i,\dots,t_n,s,x), \\ \hfill t_i = t_{i+1} \\ F_{(\mathcal{V}_0),(\mathcal{V}_0,\mathcal{V}_1),\dots,(\mathcal{V}_0,\mathcal{V}_1,\dots,\mathcal{V}_l,\dots,\mathcal{V}_j),\dots,(\mathcal{V}_0,\mathcal{V}_1,\mathcal{V}_2,\dots,\mathcal{V}_{n-1})}(t_1,\dots,t_{n-1},s,x), \ t_n = 0 \end{cases}$$

$$= \begin{cases} F_{(\mathcal{V}_0,\mathcal{V}_1),\dots,(\mathcal{V}_0,\mathcal{V}_1,\dots,\hat{\mathcal{V}}_l,\dots,\mathcal{V}_j),\dots,(\mathcal{V}_0,\mathcal{V}_1,\mathcal{V}_2,\dots,\mathcal{V}_n)}(t_2,\dots,t_n,s,x), \ t_1 = 1 \\ F_i(t_1,\dots,\hat{t}_i,\dots,t_n,s,x), \ t_i = t_{i+1} \\ F_{(\mathcal{V}_0),(\mathcal{V}_0,\mathcal{V}_1),\dots,(\mathcal{V}_0,\mathcal{V}_1,\dots,\hat{\mathcal{V}}_l,\dots,\mathcal{V}_j),\dots,(\mathcal{V}_0,\mathcal{V}_1,\mathcal{V}_2,\dots,\mathcal{V}_{n-1})}(t_1,\dots,t_{n-1},s,x), \ t_n = 0 \end{cases}$$

where

$F_i(t_1,\dots,\hat{t}_i,\dots,t_n,x)$

$$= \begin{cases} F_{(\mathcal{V}_0),\dots,(\mathcal{V}_0,\mathcal{V}_1,\hat{\mathcal{V}}_2,\dots,\mathcal{V}_i),\dots,(\mathcal{V}_0,\mathcal{V}_1,\dots,\hat{\mathcal{V}}_l,\dots,\mathcal{V}_j),\dots,(\mathcal{V}_0,\mathcal{V}_1,\mathcal{V}_2,\dots,\mathcal{V}_n)}(t_1,\dots,\hat{t}_i,\dots,t_n,s,x), \ i < j \\ F_{(\mathcal{V}_0),\dots,(\mathcal{V}_0,\mathcal{V}_1,\dots,\hat{\mathcal{V}}_l,\dots,\mathcal{V}_i),\dots,(\mathcal{V}_0,\mathcal{V}_1,\mathcal{V}_2,\dots,\mathcal{V}_n)}(t_1,\dots,\hat{t}_i,\dots,t_n,s,x), \ i = j \\ F_{(\mathcal{V}_0),\dots,(\mathcal{V}_0,\mathcal{V}_1,\dots,\hat{\mathcal{V}}_l,\dots,\mathcal{V}_j),\dots,(\mathcal{V}_0,\mathcal{V}_1,\hat{\mathcal{V}}_2,\dots,\mathcal{V}_i),\dots,(\mathcal{V}_0,\mathcal{V}_1,\mathcal{V}_2,\dots,\mathcal{V}_n)}(t_1,\dots,\hat{t}_i,\dots,t_n,s,x), \ i > j \end{cases}$$

by which we prove the coherence condition.

Similarly, we show that the coherence condition holds also in the case when there are more than one coverings missing in the members of the sequence.

We proved that there exists a homotopy F^∞ in $SSh^\infty(Cpt)$ that connects f^∞ and f'^∞, and it follows that Φ is an injection.

Now we define a functor $\Phi : SSh^\infty(Cpt) \to SSh^2(Cpt)$ on morphisms of $SSh^\infty(Cpt)$ by

$$\Phi([f^\infty]) = [(f_n, f_{n,n+1})]$$

This functor is well defined and by the previous it is proved that it is an isomorphism between the categories $SSh^\infty(Cpt)$ and $SSh^2(Cpt)$.

□

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ПОТКАТЕГОРИЈАТА НА КОМПАКТНИ МЕТРИЧКИ ПРОСТОРИ ВО КАТЕГОРИЈАТА НА ЈАК ОБЛИК

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Р е з и м е

Во предходниот труд на авторите за прв пат е презентирана внатрешна дефиниција на категоријата на јак облик. Морфизмите во оваа категорија се класите на хомотопија на кохерентни проксимативни мрежи. Во овој труд е даден доказ дека поткатегоријата чии објекти се компактните метрички простори, е изоморфна со категоријата на јак облик на метрички компакти предходно конструирана од Шекутковски, чии морфизми се класите на хомотопија на јаки проксимативни низи.

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