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NEW FIXED POINT THEOREMS FOR T_f TYPE CONTRACTIVE CONDITIONS IN 2-BANACH SPACES

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Abstract. S. Gähler ([14]), 1965, defined the term 2-normed space and A. White ([3]),1968, defined 2-Banach spaces. In paper [12] are proven several theorems about them. The term contractive mapping in 2-normed space was defined by Hatikrishnan and K. T. Ravindran in [8] and R. Kannan ([9]) and S. K. Chatterjea ([15]) theorems are proven by M. Kir and H. Kiziltunc in [4] The further generalizations of these results are elaborated in [1], [6], [10] and [16]. In this paper will be proven several theorems about fixed point for determined types of T_f-contractive mapping in 2-Banach space.

1. INTRODUCTION

The theory of fixed point has been rapidly developing the last decades. Certain classical results are generated for 2-normed and 2-Banach spaces, defined by S. Gähler ([14]) and A. White ([3]), 1965 and 1969. P. K. Hatikrishnan and K. T. Ravindran in [8] proved that contractive mapping has a unique fixed point in closed and bounded subset in 2-Banach space. In [4] M. Kir and H. Kiziltunc in 2-normed space made generalizations for the theorems about fixed point of R. Kannan ([9]) and S. K. Chatterjea ([15]). For 2-Banach spaces in [1], [6], [10] and [16], by application of Sequentialy Convergent Mappings, several generalizations for already known theorems about fixed point and sharing fixed point are given. Some of the previously mentioned results are related to the class Θ of monotony increasing continuous functions $f:[0,+\infty)\to\mathbb{R}$ such that $f^{-1}(0)=\{0\}$. If $f\in\Theta$, then $f^{-1}(0)=\{0\}$implies that $f(t)>0$, for all $t>0$.

2. MAINS RESULTS

Definition 1. Let $(L,||\cdot,\cdot||)$ be a 2-normed space. A mapping $T:L\to L$ is sequentially convergent if, for each sequence $\{y_n\}$, hold the following if $\{Ty_n\}$ is convergent then $\{y_n\}$ also is convergent. A mapping T is said

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subsequentially convergent if, holds the following for every sequence $\{y_n\}$, if $\{Ty_n\}$ is convergent then $\{y_n\}$ has a convergent subsequence.

Definition 2. Let $(L,||\cdot,\cdot||)$ be a 2-normed space, $S,T:L\to L$ and $f\in\Theta$. A mapping S is said to be T_f-contraction if there exist $\alpha\in(0,1)$ such that

$$f(||TSx-TSy,z||)\le \alpha f(||Tx-Ty,z||),$$

for all $x,y,z\in L$.

Theorem 1. Let$(L,||\cdot,\cdot||)$ be a 2-Banach space $S:L\to L$, $f\in\Theta$ and the mapping $T:L\to L$ be continuous, injection and subsequentially convergent. If there exists $\alpha,\beta\ge 0$ such that $\alpha+2\beta\in(0,\frac{1}{2})$ and

$$f(||TSx-TSy,z||)\le(\alpha+\beta)f(||Tx-TSx,z||)+\beta f(||Ty-TSy,z||) \quad (1)$$

for all $x,y,z\in L$, then S has a unique fixed point. If T is sequentially convergent, then for each $x_0\in X$ the sequence $\{S^nx_0\}$ converges to the fixed point.

Proof. Let x_0 be any point on L and let the sequence $\{x_n\}$ be defined as $x_{n+1}=Sx_n$, $n=0,1,2,3,....$ For $\lambda=\frac{\alpha+2\beta}{1-(\alpha+2\beta)}$ and since $\alpha+2\beta\in(0,\frac{1}{2})$, $\alpha,\beta\ge0$, we get that $\lambda\in(0,1)$. The inequality (1) implies

$$\begin{aligned} f(||Tx_{n+1}-Tx_n,z||) &= f(||TSx_n-TSx_{n-1},z||)\\ &\le(\alpha+\beta)f(||Tx_{n-1}-TSx_{n-1},z||)+\beta f(||Tx_n-TSx_n,z||)\\ &=(\alpha+\beta)f(||Tx_{n-1}-Tx_n,z||)+\beta f(||Tx_n-Tx_{n+1},z||)\end{aligned}$$

Analogously,

$$f(||Tx_{n+1}-Tx_n,z||)\le\beta f(||Tx_{n-1}-Tx_n,z||)+(\alpha+\beta)f(||Tx_n-Tx_{n+1},z||).$$

If We summarize last two inequalities , we get the following

$$f(||Tx_{n+1}-Tx_n,z||)\ \le\lambda f(||Tx_n-Tx_{n-1},z||), \quad (2)$$

for each $n=1,2,3,...$ and each $z\in L$. The inequality (2) implies that

$$f(||Tx_{n+1}-Tx_n,z||\ )\le\lambda^n f(||Tx_1-Tx_0,z||) \quad (3)$$

for each $n=1,2,3,...$ and each $z\in L$. Further, the inequalities (1) and (3) imply that for all $m,n\in\mathbb{N}$ $n>m$ and for each $z\in L$

$$\begin{aligned} f(||Tx_n-Tx_m,z||) &= f(||TSx_{n-1}-TSx_{m-1},z||)\\ &\le(\alpha+\beta)f(||TSx_{n-1}-Tx_{n-1},z||)+\beta f(||TSx_{m-1}-Tx_{m-1},z||)]\\ &=(\alpha+\beta)f(||Tx_n-Tx_{n-1},z||)+\beta f(||Tx_m-Tx_{m-1},z||)\\ &\le[(\alpha+\beta)\lambda^{n-1}+\beta\lambda^{m-1}]f(||Tx_1-Tx_0,z||)\end{aligned}$$

holds true. Analogously,

$f(||Tx_n - Tx_m, z||) \leq [(\alpha+\beta)\lambda^{m-1} + \beta\lambda^{n-1}]f(||Tx_1 - Tx_0, z||)]$.

by summarizing the last two inequalities, we get that

$f(||Tx_n - Tx_m, z||) \leq \frac{\alpha+2\beta}{2}(\lambda^{m-1} + \lambda^{n-1})f(||Tx_1 - Tx_0, z||)]$.

The last inequality implies that $\lim\limits_{m,n\to\infty} f(||Tx_n - Tx_m, z||) = 0$, for each $z \in L$, and since $f \in \Theta$ we get that $\lim\limits_{m,n\to\infty} ||Tx_n - Tx_m, z|| = 0$, for each $z \in L$. Therefore, the sequence $\{Tx_n\}$ is Caushy sequence. But, L is 2-Banach space, and therefore the sequence $\{Tx_n\}$ is a convergent sequence. The mapping $T : L \to L$ is subsequentially convergent, therefore the sequence $\{x_n\}$ consists a convergent subsequence $\{x_{n(k)}\}$, i.e. it exists $u \in L$ so that $\lim\limits_{k\to\infty} x_{n(k)} = u$. The continuity of T implies that $\lim\limits_{k\to\infty} Tx_{n(k)} = Tu$. Further, $\{Tx_{n(k)}\}$ is a subsequence of the convergent sequence $\{Tx_n\}$, therefore $\lim\limits_{n\to\infty} Tx_n = \lim\limits_{k\to\infty} Tx_{n(k)} = Tu$.

It will be proven that $u \in L$ is fixed point for the mapping S. For each $z \in L$

$$\begin{aligned}
f(||TSu - Tx_{n+1}, z||) &= f(||TSu - TSx_n, z||) \\
&\leq (\alpha+\beta)f(||TSu - Tu, z||) + \beta f(||TSx_n - Tx_n, z||) \\
&= (\alpha+\beta)f(||TSu - Tu, z||) + \beta f(||Tx_{n+1} - Tx_n, z||)
\end{aligned}$$

holds true. Analogously,

$f(||TSu - Tx_{n+1}, z||) \leq \beta f(||TSu - Tu, z||) + (\alpha+\beta)f(||Tx_{n+1} - Tx_n, z||)$,

therefore

$f(||TSu - Tx_{n+1}, z||) \leq \frac{\alpha+2\beta}{2}[f(||TSu - Tu, z||) + f(||Tx_{n+1} - Tx_n, z||)]$.

For $n \to \infty$, in the inequality above, $\lim\limits_{n\to\infty} Tx_n = Tu$ and the properties of f and the 2-norm imply that

$f(||TSu - Tu, z||) \leq \frac{\alpha+2\beta}{2}[f||TSu - Tu, z||) + f(0)]$,

for each $z \in L$, holds true. But $1 - \frac{\alpha+2\beta}{2} > 0$and $f^{-1}(0) = \{0\}$. Therefore, the above inequality implies that $||TSu - Tu, z|| = 0$, for each $z \in L$, i.e. $TSu = Tu$. Finally, T is injection, and therefore $Su = u$, that is the mapping S has a fixed point.

Let $u, v \in X$ be two fixed points for S, i.e. $Su = u$ and $Sv = v$. So, (1) implies that

$$\begin{aligned}
f(||Tu - Tv, z||) &= f(||TSu - TSv, z||) \\
&\leq (\alpha+\beta)[f(||Tu - TSu, z||) + \beta f(||Tv - TSv, z||)] = 0,
\end{aligned}$$

for each $z \in L$, holds true, that is $||Tu - Tv, z|| = 0$, for each $z \in L$. Therefore, $Tu = Tv$. But, T is injection, and therefore $u = v$, that is T has a unique fixed point. Finally, if T is sequentially convergent, by switching the sequence $\{n(k)\}$ with the sequence$\{n\}$ the arbitrarily of $x_0 \in L$ and the above stated, imply that for each $x_0 \in L$ the sequence $\{S^n x_0\}$ converges to the unique fixed point for S. □

Corollary 1. Let $(L, ||\cdot,\cdot||)$ be a 2-Banach space, $S : L \to L$ and $f \in \Theta$. If there exist $\alpha, \beta \geq 0$ such that $\alpha + 2\beta \in (0, \frac{1}{2})$ and

$$f(||Sx - Sy, z||) \leq (\alpha + \beta) f(||x - Sx, z||) + \beta f(||y - Sy, z||) \tag{4}$$

for all $x, y, z \in L$, then S has a unique fixed point and for each $x_0 \in X$ the sequence $\{S^n x_0\}$ converges to the fixed point.

Proof. The mapping $Tx = x$, for each $x \in L$ is continuous, injection and sequentially convergent. Therefore, the corollary is directly implied by Theorem 1 for $Tx = x$. □

Corollary 2. Let $(L, ||\cdot,\cdot||)$ be a 2-Banach space, $S : L \to L$ and the mapping $T : L \to L$ is continuous, injection and subsequentially convergent. If there exist $\alpha, \beta \geq 0$ such that $\alpha + 2\beta \in (0, \frac{1}{2})$ and

$$||TSx - TSy, z|| \leq (\alpha + \beta)||Tx - TSx, z|| + \beta||Ty - TSy, z||$$

for all $x, y, z \in L$, then S has a unique fixed point. If T is sequentially convergent, then for each $x_0 \in L$ the sequence $\{S^n x_0\}$ converges to the fixed point.

Proof. The function $f(t) = t$, $t \geq 0$ is monotony increasing and $f^{-1}(0) = \{0\}$. Therefore, the corollary is direct implication of Theorem 1 for $f(t) = t$. □

Theorem 2. Let $(L, ||\cdot,\cdot||)$ be a 2-Banach space, $S : L \to L$, $f \in \Theta$ and the mapping $T : L \to L$ is continuous, injection and subsequentially convergent. If there exist $\alpha, \beta \geq 0$ such that $\alpha + 2\beta \in (0, \frac{1}{2})$ and

$$f(||TSx - TSy, z||^2) \leq (\alpha+\beta) f(||Tx - TSx, z||^2) + \beta f(||Ty - TSy, z||^2) \tag{5}$$

for all $x, y, z \in L$, then S has a unique fixed point. If T is sequentially convergent, then for each $x_0 \in X$ the sequence $\{S^n x_0\}$ converges to the fixed point.

Proof. Let x_0 be any point in L and let the sequence $\{x_n\}$ be defined as $x_{n+1} = Sx_n$, $n = 0, 1, 2, 3, ...$ The inequality (5), analogously as the proof of Theorem 1, implies that

$$f(||Tx_{n+1} - Tx_n, z||^2) \ \leq \lambda f(||Tx_n - Tx_{n-1}, z||^2) \tag{6}$$

for each $n = 0, 1, 2, 3, ...$ and each $z \in L$, for $\lambda = \frac{\alpha+2\beta}{1-(\alpha+2\beta)}$, holds true.

Further, by using the inequality (6), analogously as the proof of Theorem 1, can be proven that the sequence $\{Tx_n\}$ is a convergent sequence. Therefore, $\{x_n\}$ consists a convergent subsequence, i.e. there exist $u \in L$ and a subsequence $\{x_{n(k)}\}$ of the sequence $\{x_n\}$ such that $\lim\limits_{k\to\infty} x_{n(k)} = u$. The continuity of T implies that $\lim\limits_{k\to\infty} Tx_{n(k)} = Tu$, that is $\lim\limits_{n\to\infty} Tx_n = Tu$. Further, the inequality (5), analogously as the proof of Theorem 1, implies that

$$f(||TSu - Tx_{n+1}, z||^2) \leq \frac{\alpha+2\beta}{2}[f(||TSu - Tu, z||^2) + f(||Tx_{n+1} - Tx_n, z||^2)].$$

hold true. For $n \to \infty$, the inequality above is transformed as the following $f(||TSu - Tu, z||^2) \leq \frac{\alpha+2\beta}{2}[f||TSu - Tu, z||^2) + f(0)]$, for each $z \in L$. Analogously as in the theorem 1, from the inequality above we conclude that $Su = u$, that is the mapping S has a fixed point. Let $u, v \in X$ be two fixed point for S, i.e. $Su = u$ and $Sv = v$. Then (5) implies that

$$f(||Tu - Tv, z||^2) = f(||TSu - TSv, z||^2)$$
$$\leq (\alpha + \beta)f(||Tu - TSu, z||^2) + \beta f(||Tv - TSv, z||^2) = 0,$$

for each $z \in L$, holds true. Therefore $u = v$, i.e. S has a unique fixed point. Finally, if T ? sequentially convergent, then when substitute the sequence $\{n(k)\}$ by the sequence $\{n\}$ the arbitrarily of $x_0 \in L$ and the above stated imply that for each $x_0 \in L$ the sequence $\{S^n x_0\}$ converges to the unique fixed point for S. □

Corollary 3. Let $(L, ||\cdot, \cdot||)$ be a 2-Banach space, $S : L \to L$ and $f \in \Theta$. If it exists $\alpha, \beta \geq 0$ such that $\alpha + 2\beta \in (0, \frac{1}{2})$ and

$$f(||Sx - Sy, z||^2) \leq (\alpha + \beta)f(||x - Sx, z||^2) + \beta f(||y - Sy, z||^2),$$

for all $x, y, z \in L$, then S has a unique fixed point and for each $x_0 \in L$ the sequence $\{S^n x_0\}$ converges to this fixed point.

Proof. For $Tx = x$ in the Theorem 2, we get the corollary 3.

Corollary 4. Let $(L, ||\cdot, \cdot||)$ be a 2-Banach space, $S : L \to L$ and the mapping $T : L \to L$ be continuous, injection and subsequentially convergent. If there exist $\alpha, \beta \geq 0$ such that $\alpha + 2\beta \in (0, \frac{1}{2})$ and

$$||TSx - TSy, z||^2 \leq (\alpha + \beta)||Tx - TSx, z||^2 + \beta||Ty - TSy, z||^2),$$

for all $x, y, z \in L$, then S has a unique fixed point. If T is sequentially convergent, then for each $x_0 \in L$ the sequence $\{S^n x_0\}$ converges to the fixed point.

Proof. For $f(t) = t$in theorem 2, we obtain the corollary 4. □

Theorem 3. Let $(L, ||\cdot, \cdot||)$ be a 2-Banach space, $S : L \to L$, the mapping $T : L \to L$ be continuous, injection and subsequentially convergent and $f \in \Theta$ is such that $f(a+b) \leq f(a)+f(b)$, for all $a, b \geq 0$. If there exist $\alpha, \beta \geq 0$ such that $\alpha + 2\beta \in (0, \frac{1}{2})$ and

$$f(||TSx - TSy, z||) \leq$$
$$\leq (\alpha + \beta)f(||Tx - TSy, z||) + \beta f(||Ty - TSx, z||) \tag{7}$$

for all $x, y, z \in L$, then S has a unique fixed point. If T is sequentially convergent, then for each $x_0 \in L$ the sequence $\{S^n x_0\}$ converges to the fixed point.

Proof. Let x_0 be any point on L and the sequence $\{x_n\}$ be defined as the following $x_{n+1} = Sx_n$, $n = 0, 1, 2, 3, ...$ The inequality (7) and the property of f imply the followings

$$f(||Tx_{n+1}-Tx_n,z||)\leq \beta f(||Tx_{n-1}-Tx_n,z||)+\beta f(||Tx_n-Tx_{n+1},z||)$$

and

$$f(||Tx_{n+1}-Tx_n,z||)\leq (\alpha+\beta)f(||Tx_{n-1}-Tx_n,z||)+ \\ +(\alpha+\beta)f(||Tx_n-Tx_{n+1},z||).$$

By summarizing the last two inequalities we obtain the following

$$f(||Tx_{n+1}-Tx_n,z||)\ \leq \lambda f(||Tx_n-Tx_{n-1},z||), \tag{8}$$

for each $n=1,2,3,...$ and each $z\in L$, for $\lambda=\frac{\alpha+2\beta}{2-(\alpha+2\beta)}<1$. Further, by applying the inequality (8), analogously as the proof in theorem 1, we get that the sequence $\{Tx_n\}$ is convergent. Therefore, the sequence $\{x_n\}$ consists of convergent subsequence, i.e. it exists $u\in L$ and a subsequence $\{x_{n(k)}\}$ of the sequence $\{x_n\}$ such that $\lim\limits_{k\to\infty} x_{n(k)}=u$. The continuity of T implies that $\lim\limits_{k\to\infty} Tx_{n(k)}=Tu$, that is $\lim\limits_{n\to\infty} Tx_n=Tu$. Further, the inequality (7), analogously as the proof in theorem 1, implies

$$f(||TSu-Tx_{n+1},z||)\leq \tfrac{\alpha+2\beta}{2}[f(||Tu-Tx_{n+1},z||)+f(||Tx_n-TSu,z||)]$$

For $n\to\infty$in the above inequality, the continuity of f and T and the properties of the 2-norm, imply

$$f(||TSu-Tu,z||)\leq \tfrac{\alpha+2\beta}{2}[f(||TSu-Tu,z||)+f(0)],$$

for each $z\in L$. Therefore, analogously as the proof in theorem 1, we conclude that $Su=u$, that is the mapping S has a fixed point. Let $u,v\in X$ be fixed point on S, i.e. $Su=u$ and $Sv=v$. Then, (5) implies

$$\begin{aligned} f(||Tu-Tv,z||) &= f(||TSu-TSv,z||) \\ &\leq (\alpha+\beta)f(||Tu-TSv,z||)+\beta f(||Tv-TSu,z||) \\ &= (\alpha+2\beta)f(||Tu-Tv,z||), \end{aligned}$$

for each $z\in L$. Therefore, $u=v$, i.e. S has a unique fixed point. Finally, if T is sequentially convergent, then by substituting the sequence $\{n(k)\}$ with the sequence $\{n\}$ and arbitrarily of $x_0\in L$ and the above stated, imply that for each $x_0\in L$the sequence $\{S^n x_0\}$ converges to the unique fixed point on S. □

Corollary 5. Let $(L,||\cdot,\cdot||)$ be a 2-Banach space, $S:L\to L$ and $f\in\Theta$ be such that $f(a+b)\leq f(a)+f(b)$, for all $a,b\geq 0$. If it exist $\alpha,\beta\geq 0$ such that $\alpha+2\beta\in(0,\frac{1}{2})$ and

$$f(||Sx-Sy,z||)\leq(\alpha+\beta)f(||x-Sy,z||+\beta f(||y-Sx,z||),$$

for all $x,y,z\in L$, then S has a unique fixed point and for each $x_0\in L$ the sequence $\{S^n x_0\}$ converges to that point.

Proof. For $Tx = x$, in theorem 3 we get the above corollary. □

Corollary 6. Let $(L, ||\cdot,\cdot||)$ be a 2-Banach space, $S : L \to L$ and the mapping $T : L \to L$ be continuous, injection and subsequentially convergent. If it exists $\alpha, \beta \geq 0$ such that $\alpha + 2\beta \in (0, \frac{1}{2})$ and

$$||TSx - TSy, z|| \leq (\alpha + \beta)||Tx - TSy, z|| + \beta||Ty - TSx, z||$$

for all $x, y, z \in L$, then$S$ has a unique fixed point. If T is sequentially convergent, then for each $x_0 \in L$the sequence $\{S^n x_0\}$ converges to the fixed point.

Proof. For $f(t) = t$, in theorem 2 we get the above corollary. □

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