Математички Билтен
29 (LV)
2005 (61–70)
Скопје, Македонија

ISSN 0351–336X

SOME SOLUTIONS TO THE RICCATI DIFFERENTIAL EQUATION

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Abstract

In this paper we give some conditions for existence of quasi-periodic solutions with a linear quasi-period and a constant quasi-periodic coefficient to the Riccati differential equation and we find these solutions.

1. Reducibility to the Riccati differential equation with respect to QPS

Let the Riccati differential equation

$$y'(x) + f(x)y(x) + g(x)y^2(x) + h(x) = 0 \quad (g(x) \neq 0) \tag{1}$$

be given. We want to find QPS $y = y(x)$ for (1), i.e. the solution that satisfies the relation

$$y(x+\omega) = \lambda\big(x, \omega(x)\big)y(x) = \lambda(x)y(x), \quad x, x+\omega \in D_y \tag{2}$$

where $\omega = \omega(x)$ is QP and $\lambda = \lambda(x)$ is QPC for the function $y = y(x)$.

The following theorem holds.

Theorem 1.1. *If DE* (1) *has QPS $y = y(x)$ with QP $\omega = \omega(x)$ and QPC $\lambda(x,\omega)$, then it is reduced to the algebraic equation with respect to the*

QPS $y=y(x)$:

$$\begin{aligned}&\Big(-\frac{1}{1+\omega'}\lambda(x,\omega)\cdot g(x)+\lambda^2(x,\omega)\cdot g(x+\omega)\Big)y^2(x)+\\&+\Big(\frac{1}{1+\omega'}\frac{d\lambda}{dx}-\frac{1}{1+\omega'}\lambda(x,\omega)\cdot f(x)+\lambda(x,\omega)\cdot f(x+\omega)\Big)y(x)+\\&+\Big(-\frac{1}{1+\omega'}\lambda(x,\omega)\cdot h(x)+h(x+\omega)\Big)=0\,,\end{aligned}\tag{3}$$

or to the linear nonhomogeneous DFE of first order with respect to $y(x)$

$$\begin{aligned}&\Big(\frac{1}{1+\omega'}\lambda(x,\omega)-\lambda^2(x,\omega)\frac{g(x+\omega)}{g(x)}\Big)y'(x)+\\&+\Big(\frac{1}{1+\omega'}\frac{d\lambda}{dx}+\lambda(x,\omega)f(x+\omega)-\lambda^2(x,\omega)\frac{g(x+\omega)}{g(x)}f(x)\Big)y(x)+\\&+\Big(h(x+\omega)-\lambda^2(x,\omega)\frac{g(x+\omega)}{g(x)}h(x)\Big)=0\end{aligned}\tag{4}$$

Proof. Using the same reducible method as in the papers [1] and [2], under the conditions of the theorem we have the system

$$\left.\begin{aligned}&y'(x)+f(x)y(x)+g(x)y^2(x)+h(x)=0\\&y'(t)+f(t)y(t)+g(t)y^2(t)+h(t)_{/t=x+\omega}=0\\&y(t)=\lambda(x,\omega)y(x)\\&\frac{d}{dx}y(t)=\frac{d\lambda(x,\omega)}{dx}y(x)+\lambda(x,\omega)y'(x)\end{aligned}\right\}\tag{5}$$

from where

$$y'(x)=-f(x)y(x)-g(x)y^2(x)-h(x)\,,\tag{6}$$

$$y^2(x)=-\frac{1}{g(x)}\left(h(x)+f(x)y(x)+y'(x)\right)\tag{7}$$

and

$$y'(t)=\frac{1}{t'}\Big(\frac{d\lambda}{dx}y(x)+\lambda(x,\omega)y'(x)\Big)\,.\tag{8}$$

Substituting (6) or (7) and (8) in the second equation of the system (5), after short transformations, we obtain (3) and (4). □

2. Quasi-periodic solutions with QP $\omega = kx + m$ and QPC $\lambda = \text{const.}$

Theorem 2.1. *If DE (1) has QPS $y = y(x)$ with QP $\omega = kx + m$, $k \neq -1$ and QPC $\lambda = \text{const.} \neq 0$, then it is reduced to the algebraic equation with respect to QPS*

$$\begin{aligned}
&\lambda\Big(-\frac{1}{1+k}g(x) + \lambda g\big((1+k)x+m\big)\Big)y^2(x)+\\
&+\lambda\Big(-\frac{1}{1+k}f(x) + f\big((1+k)x+m\big)\Big)y(x)+\\
&+\Big(-\frac{1}{1+k}\lambda h(x) + h\big((1+k)x+m\big)\Big) = 0
\end{aligned} \tag{9}$$

or to the equation

$$\begin{aligned}
&\Big(\frac{1}{1+k}g(x) - \lambda g\big((1+k)x+m\big)\Big)y'(x)+\\
&+\big(f\big((1+k)x+m\big)g(x) - \lambda f(x)g\big((1+k)x+m\big)\big)y(x)+\\
&+\Big(\frac{1}{\lambda}g(x)h\big((1+k)x+m\big) - \lambda g\big((1+k)x+m\big)h(x)\Big) = 0\,.
\end{aligned} \tag{10}$$

Proof. Substituting in eq. (3) and eq. (4) $\omega = kx + m, \omega' = k$, $t = x + \omega$, $t' = 1 + k$ we obtain eq. (9) and eq. (10).

Theorem 2.2. *If*

1° *DE (1) has QPS $y_1(x)$ with QP $\omega = kx + m$ and QPC $\lambda = \text{const.} \neq 0$.*

2° *the coefficients $f(x)$, $g(x)$, $h(x)$ in DE (1) are QPF with QP $\omega = kx + m$ and QPC $\lambda_1 = \dfrac{1}{1+k}$, $\lambda_2 = \dfrac{1}{\lambda(1+k)}$, $\lambda_3 = \dfrac{\lambda}{1+k}$ respectively,*

3° *the general solution for the equation*

$$z' - \big(f(x) + 2g(x)y_1(x)\big)z - g(x) = 0 \tag{11}$$

is QPF with QP $\omega = kx + m$ and QPC $\dfrac{1}{\lambda}$, then every solution for DE (1) is QPF with QP $\omega = kx + m$ and QPC λ.

Proof. Under the given conditions follows that the general solutions for DE (1) is $y(x) = y_1(x) + \dfrac{1}{z(x)}$, where $z = z(x)$ is QPS for the eq. (11).

Since the coefficients $f(x)$, $g(x)$, $h(x)$ satisfy the relations

$$\begin{aligned} f\big(x+(kx+m)\big) &= \frac{1}{1+k}\, f(x), \\ g\big(x+(kx+m)\big) &= \frac{1}{\lambda(1+k)}\, g(x), \\ h\big(x+(kx+m)\big) &= \frac{\lambda}{1+k}\, h(x) \end{aligned}$$

we get

$$\begin{aligned} &y\big(x+(kx+m)\big) = y_1\big(x+(kx+m)\big)+ \\ &+\frac{1}{z\big(x+(kx+m)\big)} = \lambda\Big(y_1(x)+\frac{1}{z(x)}\Big) = \lambda y(x) \end{aligned}$$

and

$$\begin{aligned} &y'(t)+f(t)y(t)+g(t)y^2(t)+h(t)_{t=x+\omega} = \\ &= \frac{\lambda}{1+k}\, y'(x)+\frac{1}{1+k}\, f(x)\cdot\lambda y(x)+\frac{1}{\lambda(1+k)}\, g(x)\cdot\lambda^2 y^2(x)+\frac{\lambda}{1+k}\, h(x) = \\ &= \frac{\lambda}{1+k}\big(y'(x)+f(x)y(x)+g(x)y^2(x)+h(x)\big) = \frac{\lambda}{1+k}\cdot 0 = 0\,. \qquad \square \end{aligned}$$

Remark 2.1. The general solution to DE (11) is QPF with a linear QP $\omega = kx - kx_0\big(x, x+k(x-x_0)\in D_y\big)$ and QPC $\dfrac{1}{\lambda}$ if $\lambda = 1$ or $\lambda = -1$.

Remark 2.2. If DE (11) has at least one QPS with QP $\omega = kx - kx_0$ and QPC $\dfrac{1}{\lambda}$, then it has many QPS with QP $\omega = kx - kx_0$ and QPC $\dfrac{1}{\lambda}$.

Remark 2.3. If DE (1) has QPS $y_1(x)$ then its general solution is

$$y(x) = y_1(x)+\frac{1}{z(x)} = y_1(x)+\frac{1}{Ca(x)+b(x)},$$

where $a(x) = e^{\int \left(f(x)+2g(x)y_1(x)\right)dx}$, $b(x) = a(x)\int \dfrac{g(x)}{a(x)}\,dx$.

Example 2.1. Let DE (1) be

$$y'(x)-\frac{1}{x-x_0}\, y(x)+\frac{1}{(x-x_0)^{2s}}\, y^2(x)-(2s-1)(x-x_0)^{2s-2} = 0\,.$$

It has a particular solution $y_1 = (x-x_0)^{2s-1}$, which is QPF with QP $\omega = -2(x-x_0)$ and QPC $\lambda = (-1)^{2s-1} = -1$. Since the coefficients $f(x)$,

$g(x)$, $h(x)$ are QPF with the same QP $\omega = -2(x - x_0)$ and QPC $\lambda_1 = -1$, $\lambda_2 = 1$ $\lambda_3 = 1$ respectively, according to the Theorem 2.2. every solution is QPF. Indeed, its general solution is

$$y = (x - x_0)^{2s-1}\Big(1 + \frac{2s}{2Cs(x - x_0)^{2s} - 1}\Big)$$

for $s \neq 0$, and $y = \dfrac{1}{x - x_0}\Big(1 + \dfrac{1}{C + \ln|x - x_0|}\Big)$ for $s = 0$, which are QPF with QP $\omega = -2(x - x_0)$ and QPC $\lambda = -1$.

Theorem 2.3. *If*

1° *DE* (1) *has two QPS* $y_1(x)$ *and* $y_2(x)$,*both with QP* $\omega = kx + m$ *and QPC* $\lambda = \text{const.} \neq 0$,

2° *the coefficients* $f(x)$, $g(x)$, $h(x)$ *in DE* (1) *are QPF with QP* $\omega = \text{const.}$ *and QPC* $\lambda_1 = \dfrac{1}{1+k}$, $\lambda_2 = \dfrac{1}{1+k}$, $\lambda_3 = \dfrac{\lambda}{1+k}$ *respectively,*

3° $z(x) = \dfrac{C\phi(x) - 1}{C\phi(x)} \cdot \dfrac{1}{y_2(x) - y_1(x)}$, *for* $\phi(x) = e^{-\int g(x)\left(y_1(x)-y_2(x)\right)dx}$, *is a general QPS for* (8) *with QP* $\omega = kx + m$ *and QPC* $\frac{1}{\lambda}$, *then every solution for the DE* (1) *is QPF with QP* $\omega = kx + m$ *and QPC* λ.

Proof. It can be proved in a similar manner as the previous theorem. □

Remark 2.4. If DE (1) has two QPS $y_1 = y_1(x)$ and $y_2 = y_2(x)$ with the same QP $\omega = kx + m$ and a constant QPC λ, then its general solution is

$$y(x) = y_1(x) + \frac{1}{z(x)} = \frac{1}{1 - C\phi(x)}\, y_1(x) + \Big(1 - \frac{1}{1 - C\phi(x)}\Big)\, y_2(x) =$$
$$= \mu(x, C) y_1(x) + \big(1 - \mu(x, C)\big) y_2(x)\,.$$

i.e. the solution is in the form $y(x) = y_1(x) + \dfrac{1}{C_1 a(x) + b(x)}$, where

$$a(x) = \frac{-1}{\phi(x)\big(y_1(x) - y_2(x)\big)}, \quad b(x) = \frac{-1}{y_1(x) - y_2(x)}\,.$$

Example 2.2. Let DE (1) be

$$y'(x) - \frac{(x-2\pi)\sin x + \big((x-2\pi)^2 - 1\big)\cos x + \cos^2 x + (x-2\pi)^2 \cos^3 x}{(x - 2\pi)\cos x(\cos x - 1)}\, y(x) +$$
$$+ \frac{\sin x - (x - 2\pi)\cos x + (x - 2\pi)\cos^2 x}{(x - 2\pi)\cos x(cosx - 1)}\, y^2(x) + (x - 2\pi)^2 \cos x = 0\,.$$

It has particular solutions $y_1 = (x - 2\pi)$ and $y_2(x) = (x - 2\pi)$ which are QPF with QP $\omega = -2x + 4\pi$ and QPC $\lambda = -1$. Since the coefficients $f(x)$, $g(x)$, $h(x)$ are QPF with the same QP $\omega = -2x + 4\pi$ and QPC $-1, 1, 1$, respectively, according to the Theorem 2.3. every solution is QPF. Indeed, the general solution is

$$y = (x - 2\pi)\Big(1 + \frac{C\cos x\, e^{\frac{(x-2\pi)^2}{2} - (x-2)\sin x + \cos x}}{C\cos x\, e^{\frac{x-2\pi)^2}{2} - (x-2\pi)\sin x + \cos x} - 1}(\cos x - 1)\Big),$$

that is QPF with QP $\omega = -2x + 4\pi$ and QPC $\lambda = -1$.

Theorem 2.4. *If*

1° *DE* (1) *has QPS* $y_1(x)$, $y_2(x)$ *and* $y_3(x)$ *with QP* $\omega = kx + m$ *and QPC* $\lambda = \text{const.} \neq 0$,

2° *the coefficients* $f(x)$, $g(x)$, $h(x)$ *in DE* (1) *are QPF with QP* $\omega = kx + m$ *and QPC* $\lambda_1 = \dfrac{1}{1+k}$, $\lambda_2 = \dfrac{1}{\lambda(1+k)}$, $\lambda_3 = \dfrac{\lambda}{1+k}$ *respectively, and*

3° $z(x) = \dfrac{k\psi(x) - 1}{k\psi(x)}\dfrac{1}{y_2(x) - y_1(x)}$, *where* $\psi(x) = \dfrac{y_3(x) - y_1(x)}{y_3(x) - y_2(x)}$, *is a general QPS for* (8) *with QP* $\omega = kx + m$ *and QPC* $\dfrac{1}{\lambda}$, *then every solution for DE* (1) *is QPF with QP* $\omega = kx + m$ *and QPC* λ.

Proof. It can be proved in a similar manner as the Theorem 2.2.

Remark 2.5. The general solution for DE (1) is

$$y(x) = y_1(x) + \frac{1}{z(x)} = \frac{1}{1 - K\psi(x)}y_1(x) + \Big(1 - \frac{1}{1 - K\psi(x)}\Big)y_2(x) =$$
$$= \nu(x, K)y_1(x) + \big(1 - \nu(x, K)\big)y_2(x)\,.$$

i.e. $y(x) = y_1(x) + \dfrac{1}{C_2 a(x) + b(x)}$, where $a(x) = \dfrac{-1}{\psi(x)\big(y_1(x) - y_2(x)\big)}$, $b(x) = \dfrac{-1}{y_1(x) - y_2(x)}$.

Example 2.3. Let DE (1) be

$$y' - \frac{(x-2\pi)^3\sin x + \cos x + (x-2\pi)^2\cos^2 x - \sin^2 x - \big(1 + (x-2\pi)^2\big)\cos^3 x - (x-2\pi)\sin^3 x}{(x - 2\pi)\big((x - 2\pi) - \sin x\big)(1 - \cos x)\big((x - 2\pi)\cos x - \sin x\big)}y +$$
$$+ \frac{(x - 2\pi) + \big(1 - (x - 2\pi)\big)\sin x - (x - 2\pi)\cos x - \sin x\cos x}{(x - 2\pi)\big((x - 2\pi) - \sin x\big)(1 - \cos x)\big((x - 2\pi)\cos x - \sin x\big)}y^2 +$$
$$+ \frac{1 + (x-2\pi)\sin x - (x-2\pi)\sin x\cos x - \cos^3 x - 2(x-2\pi)\sin^3 x}{(x - 2\pi - \sin x)(1 - \cos x)\big((x - 2\pi)\cos x - \sin x\big)} = 0\,.$$

It has particular solutions $y_1 = (x - 2\pi)$, $y_2(x) = (x - 2\pi)\cos x$ and $y_3(x) = \sin x$. They are QPF with QP $\omega = -2x + 4\pi$ and QPC $\lambda = -1$. Since the coefficients $f(x)$, $g(x)$, $h(x)$ are QPF with the same QP $\omega = -2x + 4\pi$ and QPC $-1, 1, 1$, respectively, according to the Theorem 2.4. every solution is QPF. Indeed, the general solution is

$$y = (x - 2\pi)\Big(1 + \frac{C\big(\sin x - (x - 2\pi)\big)(\cos x - 1)}{C\big(\sin x - (x - 2\pi)\big) - \sin x + (x - 2\pi)\cos x}\Big),$$

which is QPF with QP $\omega = -2x + 4\pi$ and QPC $\lambda = -1$.

Theorem 2.5. *Let DE* (1) *have one QPS with QP* $\omega = kx + m$ *and QPC* $\lambda = \frac{\lambda_3}{\lambda_1}$, *and let the coefficients* $f(x) \neq 0$, $g(x)$, $h(x)$ *be QPF with the same QP* $\omega = kx+m$ *and QPC* $\lambda_1 \neq \frac{1}{1+k}$, $\lambda_2 = \frac{1}{\lambda(1+k)}$, λ_3 *respectively. Then, QPS for DE* (1) *is*

$$y = -\frac{h(x)}{f(x)} \tag{12}$$

if the relation

$$\Big(\frac{h(x)}{f(x)}\Big)' - g(x)\Big(\frac{h(x)}{f(x)}\Big)^2 = 0 \tag{13}$$

is satisfied.

Proof. From the Theorem 2.1., under the conditions of the theorem, we have that QPS to DE (1) is also QPS to the equation

$$\Big(\lambda_1 - \frac{1}{1+k}\Big)f(x)y = -\frac{1}{\lambda}\Big(\lambda_3 - \lambda\frac{1}{1+k}\Big)h(x)$$

from where we get

$$y = \mu_1 \cdot \frac{h(x)}{f(x)} \tag{14}$$

where $\mu_1 = -\frac{\lambda_3(1+k) - \lambda}{\lambda\big(\lambda_1(1+k) - 1\big)}$ for $\lambda_1 \neq \lambda\lambda_2$ i.e. $\lambda_1 - \lambda_3\lambda_2 \neq 0$. Solution (14) is QPF with QP $\omega = kx + m$ and QPC $\lambda = \frac{\lambda_3}{\lambda_1}$, for which $\mu_1 = -1$. Thus, from (14) we obtain (12). Since the solution (12) is also the solution to DE (1), we obtain that the coefficients f, g, h have to satisfy the relation (13). □

Corollary 2.1. *Under the conditions of the Theorem* 2.5. *QPS for DE* (1) *is given by*

$$y = \frac{1}{\int_{\overline{x}_0}^{x} g(x)dx + C(\overline{x}_0)}, \quad C(\overline{x}_0) = -G(\overline{x}_0), \quad G'(x) = g(x)\,. \tag{15}$$

Proof. From the relation (13) we have

$$\frac{h(x)}{f(x)} = -\frac{1}{\int\limits_{\overline{x}_0}^{x} g(x)dx + C(\overline{x}_0)}. \quad (16)$$

Since, under the conditions of the Theorem 2.5., QPS for DE (1) is $y = -\frac{h(x)}{f(x)}$, we obtain (15). □

Example 2.4. The Riccati equation

$$y' - (x-x_0)^r y + s(x-x_0)^{-s-1} \cdot y^2 + (x-x_0)^{r+s} = 0$$

has coefficients $f(x) = -(x-x_0)^r$, $g(x) = s(x-x_0)^{-s-1}$, $h(x) = (x-x_0)^{r+s}$, which are QPF with the same QP $\omega = kx - kx_0$ and QPC $\lambda_1 = (1+k)^r$, $\lambda_2 = (1+k)^{-s-1}$, $\lambda_3 = (1+k)^{r+s}$ respectively, and they satisfy the condition (13). Thus, according to the Theorem 2.5., QPS for the given DE is $y = -\frac{h(x)}{f(x)} = (x-x_0)^s$ ($\omega = kx - kx_0$, $\lambda = (1+k)^s = \frac{\lambda_3}{\lambda_1}$), or using (15):

$$y = \frac{1}{\int\limits_{\overline{x}_0}^{x} g(x)dx + C_0} = \frac{1}{\int\limits_{\overline{x}_0}^{x} s(x-x_0)^{-s-1}\,dx + C(\overline{x}_0)} = (x-x_0)^s.$$

Theorem 2.6. *Let the coefficients $f(x)$, $g(x)$, $h(x)$ in DE (1) be QPF with QP $\omega = kx+m$ and QPC $\lambda_1 \neq \frac{1}{1+k}$, $\lambda_2 \neq \frac{1}{\lambda(1+k)}$, λ_3 respectively. If DE (1) has QPS $y = y(x)$ with QP $\omega = kx+m$ and QPC λ, then $y = 0$ or $y(x) = C$.*

Proof. Under the conditions of the theorem, QPS for DE (1), i.e. for eq. (9) and eq.(10), is also QPS to the equation

$$\lambda\Big(-\frac{1}{1+k} + \lambda\lambda_2\Big)g(x)y^2(x) + \\ +\lambda\Big(-\frac{1}{1+k} + \lambda_1\Big)f(x)y(x) + \Big(-\frac{1}{1+k}\lambda + \lambda_3\Big)h(x) = 0. \quad (17)$$

or to the equation

$$\lambda\Big(\frac{1}{1+k} - \lambda\lambda_2\Big)y'(x) + \lambda(\lambda_1 - \lambda\lambda_2)f(x)y(x) + (\lambda_3 - \lambda^2\lambda_2)h(x) = 0. \quad (18)$$

Since the coefficients to the eq.(17) and eq.(18) are QPF, theirs QPS are QPS for the equations

$$\lambda^2\Big(-\frac{1}{1+k}+\lambda_1\Big)(-\lambda\lambda_2+\lambda_1)f(x)y(x)+\Big(-\frac{1}{1+k}\lambda+\lambda_3\Big)(\lambda_3-\lambda^2\lambda_2)h(x) = 0, \quad (19)$$

$$\lambda^2(\lambda_1 - \lambda\lambda_2)(\lambda_1 - 1)f(x)y(x) + (\lambda_3 - \lambda^2\lambda_2)(\lambda_3 - \lambda)h(x) = 0\,, \qquad (20)$$

respectively. Thus, from (19), we have:

1) If $\lambda \neq \frac{\lambda_1}{\lambda_2}$, $\lambda_1^2 - \lambda_2\lambda_3 \neq 0$ and $f(x) \neq 0$ then

$$y = \frac{(\lambda^2\lambda_2 - \lambda_3)\left(\lambda_3 - \frac{1}{1+k}\lambda\right)}{\lambda^2(\lambda_1 - \lambda\lambda_2)\left(\lambda_1 - \frac{1}{1+k}\right)} \cdot \frac{h(x)}{f(x)} = \mu_2 \frac{h(x)}{f(x)}\,,$$

from where follows $\lambda = \frac{\lambda_3}{\lambda_1}$, $\mu_2 = -1$ and $y = -\frac{h(x)}{f(x)}$. Since the obtained solution is also a solution for the equations (17) and (1), we get $h(x) = 0$ and $y = 0$.

2) If $\lambda \neq \frac{\lambda_1}{\lambda_2}$, $\lambda_1^2 - \lambda_2\lambda_3 \neq 0$ and $f(x) = 0$, then from (19) follows $h(x) = 0$, but then from (17) and (1) we obtain $y = 0$.

3) If $\lambda = \frac{\lambda_1}{\lambda_2}$, and $h(x) = 0$, then from (17) and (1) follows $y = 0$, or $y = C$ when $Cg(x) + f(x) = 0$.

4) If $\lambda = \frac{\lambda_1}{\lambda_2}$ and $\lambda = \lambda_3$ i.e. $\lambda_1 = \lambda_2\lambda_3$, as well as in the previous case, follows that QPS is $y = 0$, or $y = C$ if $Cg(x) + f(x) = 0$.

5) If $\lambda = \frac{\lambda_1}{\lambda_2}$ and $\lambda^2 = \frac{\lambda_3}{\lambda_2}$, i.e. $\lambda_1^2 = \lambda_2\lambda_3$, then, in a similar manner, we have that QPS is $y = 0$, or $y = C$ if $C^2g(x) + Cf(x) + h(x) = 0$.

In a similar manner, it can be proved that QPS for eq.(20) is $y = 0$, or $y = C$.

Remark 2.6. Some examples in [3] are special cases of this paper's assertions.

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НЕКОИ РЕШЕНИЈА НА РИКАТИЕВА ДИФЕРЕНЦИЈАЛНА РАВЕНКА

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Р е з и м е

Во овој труд се дадени некои услови при кои постојат квазипериодични решенија со линеарен квазипериод и константен квазипериодичен коефициент за Рикатиевата диференцијална равенка и се наоѓаат овие решенија.

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