# ON $\tilde{g}$ -CONTINUOUS FUNCTIONS

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**Abstract.** In this paper, we introduce a new class of continuous functions as an application of $\tilde{g}$ -closed sets called, $\tilde{g}$ -continuous functions and we study their properties in topological space.

## 1. Introduction

Balachandran [4], Sundaram [23], Levine [13], Mashhour [16], Abd. El. Monsef [1] and Devi [7] have introduced g-continuity, sg-continuity, semi-continuity, α-continuity, β-continuity, gα-continuity and αg-continuity respectively, which are weaker than continuity. Recently Sheik John [22] has introduced and studied ω-continuous functions in topological spaces. Also Rajesh and Ekici [20] introduced and studied $\tilde{g}$ -semi-continuous functions. The aim of this paper is to introduce a weak form of continuous functions as an application of $\tilde{g}$ -closed [11] sets called, $\tilde{g}$ -continuous functions. Moreover, some properties of $\tilde{g}$ -continuous functions are obtained.

Throughout this paper $(X,\tau)$, $(Y,\sigma)$ and $(Z,\eta)$ represent non-empty topological spaces on which no separation axioms are assumed, unless otherwise mentioned.

## 2. Preliminaries

**Definition 2.1.** A subset A of a space $(X, \tau)$ is called:

(i) semi-open [13] if $A\subseteq cl(int(A))$.

(ii) pre open [17] if $A\subseteq int(cl(A))$.

(iii) α-open [18] if $A\subseteq int(cl(int(A)))$.

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(iv) β-open [1] (semi-pre open [2]) if $A\subseteq cl(int(cl(A)))$.

The complement of the above mentioned sets are called semi-closed, pre closed, α-closed and β-closed (= semi-pre closed), respectively.

For an arbitrary topological space $(X, \tau)$, the intersection of all semi-closed sets containing A is called semi-closure [6] of A and is denoted by scl(A). Similarly, we can define pcl(A), αcl(A) and spcl(A).

**Definition 2.1.** A subset A of a topological space (X,τ) is called:

(i) generalized closed (briefly g-closed) [12] if cl(A)⊆U whenever A⊆U and U is open in (X, τ).

(ii) semi generalized closed (briefly sg-closed) [5] if scl(A)⊆U whenever A⊆U and U is semi-open in (X, τ).

(iii) generalized semi-closed (briefly gs-closed) [3] if scl(A)⊆U whenever A⊆U and U is open in (X, τ).

(iv) α-generalized closed (briefly αg-closed) [14] if αcl(A)⊆U whenever A⊆U and U is open in (X, τ).

(v) generalized α-closed (briefly gα-closed) [15] if αcl(A)⊆U whenever A⊆U and U is α-open in (X, τ).

(vi) generalized semi-preclosed (briefly gsp-closed) [10] if spcl(A)⊆U whenever A⊆U and U is open in (X, τ).

(vii) ω-closed [25] if cl(A)⊆U whenever A⊆U and U is semiopen in (X,τ). The complement of an ω-closed set is called ω-open.

(viii) $^*$g-closed [26] if cl(A)⊆U whenever A⊆U and U is ω-open in (X,τ). The complement of a $^*$g-closed set is called $^*$g-open.

(ix) $^\#$g-semi-closed (briefly $^\#$gs-closed) [27] of scl(A)⊆U whenever A⊆U and U is $^*$g-open in (X, τ). The complement of a $^\#$gs-closed set is called $^\#$gs-open.

(x) $\tilde{g}$ -closed [11] if cl(A)⊆U whenever A⊆U and U is $^\#$gs-open in (X,τ). The complement of $\tilde{g}$ -closed set is called $\tilde{g}$ -open. The class of all $\tilde{g}$ -closed subsets of (X, τ) is denoted by $\tilde{G}C(X,\tau)$.

(xi) $\tilde{g}$ -semiclosed (briefly $\tilde{g}$ s-closed) [24] if scl(A)⊆U whenever A⊆U and U is $^\#$gs-open in (X, τ). The complement of a $\tilde{g}$ s-closed set is called $\tilde{g}$ s-open.

**Definition 2.2.** A function f:(X,τ)→(Y,σ) is called:

(i) semi-continuous [13] if $f^{-1}$(V) is semi-open in (X,τ) for every open set V in (Y,σ).

(ii) pre continuous [17] if $f^{-1}$(V) is pre closed in (X,τ) for every closed set V in (Y,σ).

(iii) α-continuous [16] if $f^{-1}$(V) is α-closed in (X,τ) for every closed set V in (Y,σ).

(iv) β-continuous [1] if $f^{-1}$(V) is β-closed in (X,τ) for every closed set V in (Y,σ).

(v) g-continuous [4] if $f^{-1}(V)$ is g-closed in (X,τ) for every closed set V in (Y,σ).
(vi) sg-continuous [23] if $f^{-1}(V)$ is sg-closed in (X,τ) for every closed set V in (Y,σ).
(vii) gs-continuous [9] if $f^{-1}(V)$ is gs-closed in (X,τ) for every closed set V in (Y,σ).
(viii) gα-continuous [7] if $f^{-1}(V)$ is gα-closed in (X, τ) for every closed set V in (Y,σ).
(ix) gsp-continuous [10] if $f^{-1}(V)$ is gsp-closed in (X, τ) for every closed set V in (Y,σ).
(x) ω-continuous [25] if $f^{-1}(V)$ is ω-closed in (X, τ) for every closed set V in (Y,σ).
(xi) $^{\#}$g-semicontinuous (briefly $^{\#}$gs-continuous) [27] if $f^{-1}(V)$ is $^{\#}$gs-closed in (X,τ) for every closed set V in (Y,σ).
(xii) $\tilde{g}$ -semicontinuous (briefly $\tilde{g}$ s-continuous) [20] if $f^{-1}(V)$ is $\tilde{g}$ s-closed in (X,τ) for every closed set V in (Y,σ).
(xiii) pre-$^{\#}$gs-open (resp. pre-$^{\#}$gs-closed) [27] if the image of every $^{\#}$gs-open (resp. $^{\#}$gs-closed) set in (X,τ) is $^{\#}$gs-open (resp. $^{\#}$gs-closed) in (Y,σ).

**Definition 2.3.** A space (X, τ) is called:
(i) a $T_{1/2}$ space [12] if every g-closed set is closed.
(ii) a $T_b$ space [8] if every gs-closed set is closed.
(iii) an $_{\alpha}T_b$ space [8] if every αg-closed set is closed.
(iv) a $T_{\omega}$ space [20] if every ω-closed set is closed.
(v) a $_{gs}T^{\#}_{1/2}$ space [27] if every $^{\#}$gs-closed set is closed.
(vi) a $T_{\tilde{g}}$ space [19] if every $\tilde{g}$ -closed set is closed.
(vii) an α-space [18] if every α-closed set is closed.

## 3. $\tilde{g}$ -Continuous functions

We introduce some notions that we use in the sequel.

**Definition 3.1.** A function f:(X,τ)→(Y,σ) is called $\tilde{g}$ -continuous if $f^{-1}(V)$ is $\tilde{g}$ -closed in (X,τ) for every closed set V in (Y,σ).

**Example 3.2.** Let X={a,b,c}, τ={∅,{a},{a,b},X}, Y={p,q} and let σ be the discrete topology on Y. We define a function f:(X,τ)→(Y,σ) by f(a)=f(b)=p and f(c)=q. Then f is $\tilde{g}$ -continuous, since $\widetilde{GC}(X,\tau)$ ={∅,{b},{c},{b, c},X}.

**Proposition 3.3.** Every continuous function is $\tilde{g}$ -continuous.

**Proof:** According to the Theorem 3.2 [11], every closed set is $\tilde{g}$ -closed.

**Example 3.4.** The function f in the Example 3.2 is $\tilde{g}$ -continuous but it is not continuous, because for the open set U = {q} in (Y, σ), $f^{-1}(U)$ = {c} is not open in (X, τ).

Thus, the class of all $\tilde{g}$ -continuous functions properly contains the class of all continuous functions. In the following propositions we show that the class of all $\tilde{g}$ -continuous functions is properly contained in the classes of various generalized continuous functions in topological spaces.

**Proposition 3.5.** Every $\tilde{g}$ -continuous function is ω-continuous.

**Proof:** According to the Theorem 3.4 [11], every $\tilde{g}$ -closed set in ω-closed.

**Example 3.6.** Let X=Y={a,b,c}, τ={∅,{a},{b,c},X} and let σ={∅,{a},Y}. We define a function f:(X,τ)→(Y,σ) by f(a)=b, f(b)=c and f(c)=a. Then the function f is ω-continuous but it is not $\tilde{g}$ -continuous, since for the open set U={a} in (Y,σ), $f^{-1}(U)$={c} is not $\tilde{g}$ -open in (X,τ).

**Proposition 3.7.** Every $\tilde{g}$ -continuous function is g-continuous.

**Proof:** According to the Theorem 3.4 [11], every $\tilde{g}$ -closed set is g-closed.

**Example 3.8.** Let X={a,b,c}, τ={∅,{a},X} and (Y,σ) be the topological space of the Example 3.2. We define a function f: (X,τ)→ (Y, σ) by f(a) = f(b) = p and f(c) = q. Then f is g-continuous but it is not $\tilde{g}$ -continuous.

**Proposition 3.9.** Every $\tilde{g}$ -continuous function is sg-continuous and hence β-continuous. **Proof:** It follows from the Theorem 3.11 of [11].

**Example 3.10.** Let X=Y={a,b,c}, τ={∅,{a},X} and let σ={∅,{a},{a,b},{a,c},Y}. Let f:(X,τ)→(Y,σ) be the identity function. Then f is both sg-continuous and β-continuous but it is not $\tilde{g}$ -continuous.

**Proposition 3.11.** Every $\tilde{g}$ -continuous function is pre-continuous.

**Proof:** It follows from the Theorem 3.14 of [11].

**Example 3.12.** The function f in the Example 3.10 is pre-continuous but it is not $\tilde{g}$ -continuous.

**Proposition 3.13.** Every $\tilde{g}$ -continuous function is gs-continuous.

**Proof:** It follows from the Theorems 3.11 and 3.6 of [11].

**Example 3.14.** The function f in Example 3.10 is gs-continuous but f is not $\tilde{g}$ -continuous.

**Proposition 3.15.** Every $\tilde{g}$ -continuous function is $\tilde{g}$ s-continuous.

**Proof:** It follows from the Theorem 3.7 of [11].

**Example 3.16.** The function f in the Example 3.8 is $\tilde{g}$ s-continuous but it is not $\tilde{g}$ -continuous.

**Proposition 3.17.** Every $\tilde{g}$ -continuous function is $^{\#}$gs-continuous.

**Proof:** It follows from the Theorem 3.9 of [11].

**Example 3.18.** Let X=Y={a,b,c}, τ = {∅,{a},{b,c},X} and let σ={∅,{a,b},Y}. We define a function f:(X,τ)→(Y,σ) by f(a)=b, f(b)=c and f(c)=a. Then f is $^{\#}$gs-continuous but it is not $\tilde{g}$ -continuous, since for the open set U={a,b} in (Y,σ), $f^{-1}$(U)={a,c} is not $\tilde{g}$ open in (X,τ).

**Remark 3.19.** The following examples show that $\tilde{g}$ -continuity is independent of α-continuity and semi-continuity.

**Example 3.20.** The function f in the Example 3.10 is both α-continuous and semi-continuous but not $\tilde{g}$ -continuous.

**Example 3.21.** Let X=Y={a,b,c} τ={∅,{a,b},X} and let σ={∅,{a},Y}. Let f:(X,τ)→(Y,σ) be the identity function. Then f is $\tilde{g}$ -continuous but it is not not α-continuous and semi-continuous.

## 4. Characterizations of $\tilde{g}$ -continuous functions

Now we give some characterization of $\tilde{g}$ -continuous functions.

**Theorem 4.1.** A function f:(X,τ)→(Y,σ) is $\tilde{g}$ -continuous if and only if $f^{-1}$(U) is $\tilde{g}$ -open in (X,τ) for every open set U in (Y,σ).

**Proof:** Let f:(X,τ)→(Y,σ) be $\tilde{g}$ -continuous and let U be any open in (Y,σ). Then $f^{-1}(U^c)$ is $\tilde{g}$ -closed in (X,τ). But $f^{-1}(U^c)=(f^{-1}(U))^c$ and so $f^{-1}$(U) is $\tilde{g}$ -open in (X,τ).

The proof of the converse statement is similar.

**Theorem 4.2.** A function f:(X,τ)→(Y,σ) is $\tilde{g}$ -continuous if and only if f: (X, $\tau^{\tilde{g}}$ )→(Y,σ) is continuous.

**Proof:** Let f:(X,τ)→(Y,σ) be $\tilde{g}$ -continuous. Then $f^{-1}(U)\in\tau^{\tilde{g}}$ for every open set U in (Y,σ). Therefore, f:(X, $\tau^{\tilde{g}}$ )→(Y,σ) is continuous.

The proof of the converse statement is similar.

**Remark 4.3.** The composition of two $\tilde{g}$ -continuous functions need not be $\tilde{g}$ -continuous.

**Example 4.4.** Let X=Y=Z={a,b,c}, τ={∅,{a},{b},{a,b},{a,c},X}, σ={∅,{a,b},Y} and η={∅,{b},Z}. Let g:(Y,σ)→(Z,η) be the identity function. We define a function f:(X,τ)→(Y,σ) by f(a)=a, f(b)=c and f(c)=b. Then both f and g are $\tilde{g}$ -continuous. Let A={b} be an open set in (Z,η). Then $(g\circ f)^{-1}(A)=f^{-1}(g^{-1}(A))$={c}, which is not $\tilde{g}$ -open in (X, τ). Therefore, g ∘f is not $\tilde{g}$ -continuous.

**Theorem 4.5.** Let (X,τ) and (Z,η) are topological spaces and (Y,σ) be a $T_{\tilde{g}}$ -space. If f:(X,τ)→(Y,σ) and g:(Y,σ)→(Z,η) are $\tilde{g}$ -continuous functions, then their composition g ∘f: (X,τ)→(Z,η) is also $\tilde{g}$ -continuous.

**Proof:** Let G be any closed subset of (Z,η). Since g is $\tilde{g}$-continuous and (Y,σ) is a $T_{\tilde{g}}$-space, $g^{-1}(G)$ is closed in (Y, σ). Since f is $\tilde{g}$-continuous, $(g\circ f)^{-1}(G) = f^{-1}(g^{-1}(G))$ is $\tilde{g}$-closed in (X, τ). Thus, g ∘f is $\tilde{g}$-continuous.

**Theorem 4.6.** Let (X,τ) and (Z,η) be topological spaces and (Y,σ) be a $T_{1/2}$-space (resp. $T_b$-space, ${}_{\alpha}T_b$-space, α-space, ${}_{gs}T^{\#}_{1/2}$-space, $T_{\omega}$-space and $T_{\tilde{g}}$-space). Then the composition g ∘f:(X,τ)→(Z,η) of the $\tilde{g}$-continuous function f:(X,τ)→(Y,σ) and the g-continuous (resp. gs-continuous, αg-continuous, α-continuous, #gs-continuous, ω-continuous and $\tilde{g}$-continuous) function g:(Y,σ)→(Z,η) is $\tilde{g}$-continuous.

**Proof:** It is similar to the proof of Theorem 4.5.

**Proposition 4.7.** If f:(X,τ)→(Y,σ) is $\tilde{g}$-continuous and g:(Y,σ)→(Z,η) is continuous, then the composition g ∘f: (X,τ)→(Z,η) is $\tilde{g}$-continuous.

**Proof:** Let G be any closed set in (Z,η). Then $g^{-1}(G)$ is closed in (Y, σ). Since f is $\tilde{g}$-continuous, $(g\circ f)^{-1}(G)=f^{-1}(g^{-1}(G))$ is $\tilde{g}$-closed in (X, τ) and so g ∘f is $\tilde{g}$-continuous.

We have the following proposition for the restriction of a $\tilde{g}$-continuous function.

**Proposition 4.8.** If f: (X,τ)→(Y,σ) is a $\tilde{g}$-continuous function and A is a $\tilde{g}$-closed subset of (X,τ), then the restriction $f_A$: $(A,\tau_A)$→(Y,σ) is $\tilde{g}$-continuous also.

**Proof:** Let G be any closed set in (Y,σ). Since f is $\tilde{g}$-continuous, $f^{-1}(G)$ is $\tilde{g}$-closed in (X, τ). Let $f^{-1}(G)\cap A = A_1$. Then $A_1$ is $\tilde{g}$-closed in (X, τ) by Corollary 3.23 [11]. We have $(f_A)^{-1}(G) = f^{-1}(G)\cap H = H_1$. Let U be any #gs-open set of $(A,\tau_A)$ such that $A_1\subseteq U$. Since U is #gs-open in $(A,\tau_A)$, U=F∩H for some #gs-open set F in (X,τ). Now $A_1\subseteq F\cap A$ and $A_1\subseteq F$. Since $A_1$ is $\tilde{g}$-closed in (X, τ), $cl(A_1)\subseteq F$. We have $cl_A(A_1)=cl(A_1)\cap A\subseteq F\cap A=U$ and therefore $A_1$ is $\tilde{g}$-closed in $(A,\tau_A)$. Hencce $f_A$ is $\tilde{g}$-continuous.

**Theorem 4.9.** Let f:(X,τ)→(Y,σ) be a function and $\{A_\alpha: \alpha\in\Lambda\}$ a $\tilde{g}$-open cover of X. If the restriction $f_{A\alpha}:(A_\alpha,\tau_{A\alpha})\to(Y,\sigma)$ is $\tilde{g}$-continuous for each α∈Λ, then f is $\tilde{g}$-continuous.

**Proof:** Let U be any open set in (Y,σ). Then $(f_{A\alpha})^{-1}(U)=f^{-1}(U)\cap A$ is $\tilde{g}$-open in $A_\alpha$, since $f_{A\alpha}$ is $\tilde{g}$-continuous. By Theorem 4.6 [11], $f^{-1}(U)\cap A_\alpha\in\tau^{\tilde{g}}$ for each α∈Λ. Since an arbitrary union of $\tilde{g}$-open sets is $\tilde{g}$-open, $\bigcup_{\alpha\in\Lambda}(f^{-1}(U)\cap A_\alpha)=f^{-1}(U)\in\tau^{\tilde{g}}$. Therefore, f is $\tilde{g}$-continuous.

**Definition 4.10**:[27] A function $f:(X,\tau)\rightarrow(Y,\sigma)$ is called $^{\#}$gs -irresolute if $f^{-1}(V)$ is a $^{\#}$gs-closed in $(X,\tau)$ for every $^{\#}$gs-closed set V of $(Y,\sigma)$.

**Proposition 4.11.** If A is any $\tilde{g}$ -closed in $(X,\tau)$ and $f:(X,\tau)\rightarrow(Y,\sigma)$ is $^{\#}$gs -irresolute and closed, then f(A) is $\tilde{g}$ -closed in $(Y,\sigma)$.

**Proof:** Let F be any $^{\#}$gs-open in $(Y,\sigma)$ such that $f(A)\subseteq F$. Then $A\subseteq f^{-1}(F)$ and $cl(A)\subseteq f^{-1}(F)$. Thus $f(cl(A))\subseteq F$ and f(cl(A)) is a closed set. Now, $cl(f(A))\subseteq cl(f(cl(A))\subseteq f(cl(A))\subseteq U$, so f(A) is $\tilde{g}$ -closed in $(Y,\sigma)$.

**Theorem 4.12.** Let $f:(X,\tau)\rightarrow(Y,\sigma)$ is $\tilde{g}$ -continuous pre-$^{\#}$gs-open function. If A is $\tilde{g}$ -open (or $\tilde{g}$ -closed) subset of $(Y,\sigma)$, then $f^{-1}(A)$ is $\tilde{g}$ -open (or $\tilde{g}$ -closed ) in $(X,\tau)$.

**Proof:** Let A be any $\tilde{g}$ -open set in $(Y,\sigma)$ and let F be $^{\#}$gs-open set in $(X,\tau)$ such that $F\subseteq f^{-1}(A)$. Then $f(F)\subseteq A$. According to the assumption f(F) is $^{\#}$gs-open and A is $\tilde{g}$ -open in $(Y,\sigma)$. From the Theorem 4.4 [11] it follows that $f(F)\subseteq int(A)$, so $F\subseteq f^{-1}(int(A))$. Since f is $\tilde{g}$ -continuous and int(A) is open in $(Y, \sigma)$, $f^{-1}(int(A))$ is $\tilde{g}$ -open in $(X,\tau)$. Thus $F\subseteq int(f^{-1}(int(A)))\subseteq int(f^{-1}(A))$. From the Theorem 4.4 [18] it follows that $f^{-1}(A)$ is $\tilde{g}$ -open in $(X,\tau)$. By taking complements, we can show that if A is $\tilde{g}$ -closed in $(Y,\sigma)$, $f^{-1}(A)$ is $\tilde{g}$ -closed in $(X, \tau)$.

**Definition 4.13.** Let x be a point of $(X,\tau)$ and V be a subset of X. Then V is called a $\tilde{g}$ -neighbourhood of x in $(X, \tau)$ if there exists a $\tilde{g}$ -open set U of $(X, \tau)$ such that $x\in U\subseteq V$.

The intersection of all $\tilde{g}$ -closed sets containing A is called $\tilde{g}$ -closure of A [21] and is denoted by $\tilde{g}$ -cl(A). Since $\tilde{g}$ -closed set coincide with their $\tilde{g}$ -closure we can conclude that A is $\tilde{g}$ -closed if and only if $\tilde{g}$ -cl(A)=A.

**Theorem 4.14.** Let A be a subset of $(X,\tau)$. Then $x\in \tilde{g}$ -cl(A) if and only if for any $\tilde{g}$ -neighbourhood $W_x$ of x in $(X, \tau)$, $A\cap W_x \neq \emptyset$.

**Proof:** Let $x\in \tilde{g}$ -cl(A). We suppose that there exists a $\tilde{g}$ -neighbourhood W of the point x in $(X, \tau)$ such that $W\cap A=\emptyset$. Since W is a $\tilde{g}$ -neighbourhood of x in $(X, \tau)$, by Definition 4.13, there exists a $\tilde{g}$ -open set $U_x$ such that $x \in U_x \subseteq W$. Therefore, $U_x\cap A=\emptyset$, so $A\subseteq(U_x)^c$. Since $(U_x)^c$ is a $\tilde{g}$ -closed set containing A, we have $\tilde{g}$ -cl(A)$\subseteq(U_x)^c$ and therefore $x\notin \tilde{g}$ -cl(A), which is a contradiction.

We suppose that for each $\tilde{g}$ -neighbourhood $W_x$ of x in $(X,\tau)$, $A\cap W_x \neq\emptyset$. Let $x\notin \tilde{g}$ -cl(A). Then there exists a $\tilde{g}$ -closed set F of $(X,\tau)$ such that $A\subseteq F$ and $x\notin F$. Thus, $x\in F^c$ and $F^c$ is $\tilde{g}$ -open and hence $F^c$ is a $\tilde{g}$ -neighbourhood of x in $(X,\tau)$. But $A\cap F^c=\emptyset$, which is a contradiction.

**Theorem 4.15.** Let $f:(X,\tau)\rightarrow(Y,\sigma)$ be a function. Then the following statements are equivalent:

a) f is $\tilde{g}$ -continuous.

b) the inverse image of every open set in (Y, σ) is $\tilde{g}$ -open in (X, τ).

c) for each point x in (X,τ) and each open set V in (Y,σ) with f(x)∈V, there is a $\tilde{g}$ -open set U in (X,τ) such that x∈U, f(U)⊆V.

d) the inverse image of every closed set in (Y,σ) is $\tilde{g}$ -closed in (X,τ).

e) for each x in (X,τ), the inverse image of every neighbourhood of f(x) is a $\tilde{g}$ -neighborhood of x.

f) for each x in (X,τ) and each neighbourhood N for f(x), there is a $\tilde{g}$ -neighbourhood W of x such that f(W)⊆N.

g) for each subset A of (X,τ), f( $\tilde{g}$ -cl(A))⊆cl(f(A)).

h) for each subset B of (Y, σ), $\tilde{g}$ -cl(f(B))⊆$f^{-1}$(cl(B)).

**Proof:** (a)⇔(b) It follows from the Theorem 4.1.

(a)⇔(c) Let V be any open set in (Y,σ) and let x∈$f^{-1}$(V). Then, f(x)∈V, so there exists a $\tilde{g}$ -open set $U_x$ such that x∈$U_x$ and f($U_x$) ⊆ V. Now, x∈$U_x$⊆$f^{-1}$(V) and $f^{-1}(V)=\bigcup_{x\in f^{-1}(V)} U_x$. Since $T_{\tilde{g}}$ forms a topology, $f^{-1}$(V) is $\tilde{g}$ -open in (X,τ) and therefore, f is $\tilde{g}$ -continuous. Conversely, let f(x)∈V. Then x ∈$f^{-1}$(V) $\in \tau^{\tilde{g}}$, since f is $\tilde{g}$ -continuous. Let U=$f^{-1}$(V). Then x∈U and f(U)⊆V.

(b)⇔(d) It follows from the fact that if A is a subset of (Y,σ), then $f^{-1}(A^c)=(f^{-1}(A))^c$.

(b)⇒(e) Let x∈X and let N be any neighbourhood of f(x). Then there exists an open set U in (Y,σ) such that f(x)∈U⊆N. Consequently, $f^{-1}$(U) is a $\tilde{g}$ - open set and x∈$f^{-1}$⊆ $f^{-1}$(N). Thus, $f^{-1}$(N) is a $\tilde{g}$ - neighbourhood of x.

(e)⇒(f) Let x∈X and let N be any neighbourhood of f(x). According to the assumption W=$f^{-1}$(N) is a $\tilde{g}$ -neighbourhood of x and f(W)=f($f^{-1}$(N))⊆N.

(f)⇒(c) Let x∈X and let V be an open set containing f(x). Then V is a neighbourhood of f(x). According to the assumption, there exists a $\tilde{g}$ -neighbourhood W of x such that f(W)⊆V. Hence there exists a $\tilde{g}$ -open set U in (X,τ) such that x∈U⊆W and f(U)⊆f(W)⊆V.

(g)⇔(d) Let A be any subset of (X,τ). Since A⊆$f^{-1}$(f(A)), we have A ⊆ $f^{-1}$(cl(f(A)). Since cl(f(A)) is a closed set in (Y, σ), by assumption $f^{-1}$(cl(f(A))) is a $\tilde{g}$ -closed set containing A, so $\tilde{g}$ -cl(A)⊆$f^{-1}$(cl(f(A))). Thus f( $\tilde{g}$ -cl(A))⊆f($f^{-1}$(cl(f(A))))⊆cl(f(A)). Conversely, let F be any closed subset of (X,τ). Then f( $\tilde{g}$ -cl($f^{-1}$(F)))⊆cl(f($f^{-1}$(F)))⊆cl(F)=F, so $\tilde{g}$ -cl($f^{-1}$(F))⊆$f^{-1}$(F). Hence $f^{-1}$(F) is $\tilde{g}$ -closed.

(g)⇔(h) Let B be any subset of (Y,σ). For A=$f^{-1}$(B) we get

$f(\tilde{g}\text{-cl}(f^{-1}(B)))\subseteq cl(f(f^{-1}(B)))\subseteq cl(B)$. Then $\tilde{g}\text{-cl}(f^{-1}(B))\subseteq f^{-1}(cl(B))$. Conversely, let $B=f(A)$, where A is a subset of $(X,\tau)$. Then $\tilde{g}\text{-cl}(A)\subseteq \tilde{g}\text{-cl}(f^{-1}(B))\subseteq f^{-1}(cl(f(A)))$ and so $f(\tilde{g}\text{-cl}(A)) \subseteq cl(f(A))$.

**Definition 4.16.** A topological space $(X,\tau)$ is $\tilde{g}$-compact if every $\tilde{g}$-open cover of X has a finite subcover.

**Theorem 4.17.** Let f: $(X,\tau)\to(Y,\sigma)$ be a surjective $\tilde{g}$-continuous function. If X is $\tilde{g}$-compact, then Y is compact.

**Proof:** Let $\{A_i: i\in I\}$ be an open cover of Y. Then $\{f^{-1}(A_i): i\in I\}$ is a $\tilde{g}$-open cover of X. Since X is $\tilde{g}$-compact, it has a finite subcover, say $\{f^{-1}(A_1), f^{-1}(A_2),\dots, f^{-1}(A_n)\}$. Since f is surjective $\{A_1, A_2,\dots, A_n\}$ is an open cover of Y. Hence Y is compact.

**Definition 4.18.** A topological space $(X, \tau)$ is $\tilde{g}$-connected if X cannot be written as the disjoint union of two non-empty $\tilde{g}$-open sets.

**Theorem 4.19.** For a topological space $(X,\tau)$, the following statements are equivalent:

(i) X is $\tilde{g}$-connected.

(ii) the only subsets of X which are both $\tilde{g}$-open and $\tilde{g}$-closed are the empty set $\varnothing$ and X.

(iii) each $\tilde{g}$-continuous function of X into a discrete space Y with at least two points is a constant function.

**Proof:** (i)$\Rightarrow$(ii) Let $S\subset X$ be any proper subset, which is both $\tilde{g}$-open and $\tilde{g}$-closed. Then its complement $X\setminus S$ is also $\tilde{g}$-open and $\tilde{g}$-closed. Then $X=S\cup(X \setminus S)$ is a disjoint union of two non-empty $\tilde{g}$-open sets which contradicts the fact that X is $\tilde{g}$-connected. Hence, $S=\varnothing$ or $S=X$.

(ii)$\Rightarrow$(i) Let $X=A\cup B$ where $A\cap B=\varnothing$, $A\neq\varnothing$, $B\neq\varnothing$ and A and B are $\tilde{g}$-open. Since $A=X\setminus B$, A is $\tilde{g}$-closed. It is a contradiction.

(ii) $\Rightarrow$ (iii) Let $f:(X,\tau)\to(Y,\sigma)$ be a $\tilde{g}$-continuous function where Y is a discrete space with at least two points. Then $f^{-1}(\{y\})$ is both $\tilde{g}$-closed and $\tilde{g}$-open for each $y\in Y$ and $X=\cup\{f^{-1}(\{y\}):y\in Y\}$. According to the assumption, $f^{-1}(\{y\})=\varnothing$ or $f^{-1}(\{y\})=X$. If $f^{-1}(\{y\})=\varnothing$ for all $y\in Y$, f will not be a function. Also can not exists more than one $y\in Y$ such that $f^{-1}(\{y\})=X$. Hence, there exists only one $y\in Y$ such that $f^{-1}(\{y\})=X$ and $f^{-1}(\{y_1\})=\varnothing$, where $y \neq y_1 \in Y$. Thus, f is a constant function.

(iii)$\Rightarrow$(ii) Let $S\neq\varnothing$ be both $\tilde{g}$-open and $\tilde{g}$-closed in X. Let $f:(X,\tau)\to(Y,\sigma)$ be a $\tilde{g}$-continuous function defined by $f(S)=\{a\}$ and $f(X\setminus S)=\{b\}$ where $a\neq b$ and $a, b\in Y$. Since f is constant, $S=X$.

**Theorem 4.20.** Let f:(X,τ)→(Y,σ) is $\tilde{g}$-continuous and onto and X is $\tilde{g}$-connected, then Y is connected.

**Proof:** Suppose Y is not connected. Then Y=A∪B where A∩B=∅, A≠∅, B≠∅ and A,B open in Y. Since f is $\tilde{g}$-continuous and onto, $f^{-1}(A)$ and $f^{-1}(B)$ are disjoint non-empty $\tilde{g}$-open subsets of (X,τ) and their union is X. It is a contradiction. Hence X is $\tilde{g}$-connected.

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## ЗА $\tilde{g}$ -НЕПРЕКИНАТИТЕ ФУНКЦИИ

**Неламегарјан Рајеш, Ердал Екичи**

**Апстракт.** Во овој труд воведуваме нова класа непрекинати функции, наречени $\tilde{g}$ -непрекинати функции, како апликација од $\tilde{g}$ -затворените множества и ги испитуваме нивните особини во тополошки простор.