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DISCRETE SEMIFLOW ON UNIFORM STRUCTURE OF SET $F(S \times T, X)$

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Abstract. In this paper is proved that, if T is a discrete topological abelian semigroup with the identity, and if X is a Hausdorff uniform space, then the mapping $\delta(f, t) = f_t$, defines a general discret semiflow on uniform structure of set $F(S \times T, X)$ relative to the relative uniformity of pointwise convergence.

1. Preliminary remarks

Let T be a Hausdorff topological abelian group, $(Y, \mathcal{V})$ Hausdorff topological space, and $p_t : Y^T \to Y$, $t \in T$ natural projection. If we denote:

$$\begin{gathered} h_t : (Y^T)^2 \to Y^2,\ \forall t \in T \\ \forall (f,\, g) \in (Y^T)^2,\ h_t(f, g) = (f(t),\, g(t)),\ (f(t),\, g(t)) \in Y^2 \\ h_t^{-1}(V) = \{(f, g) \in (Y^T)^2 / (f(t),\, g(t)) \in V\},\ \forall t \in T,\ \forall V \in \mathcal{V} \\ S = \bigcup_{t \in T} \left\{ h^{-1}(V) \ / \ V \in \mathcal{V} \right\} \subset \mathcal{U} \end{gathered}$$

then the family $\mathcal{S}$ is the subbase for a uniformity $U \subset P((Y^T)^2)$, (in $S \subset X = Y^T$), and the pair $(Y^T, \mathcal{U})$ is Hausdorff uniform space (because, the product of Hausdorff spaces is a Hausdorff space). If we denote

$$\begin{gathered} q_t = n_t \times n_t,\ n_t : X^{S \times T} \to X,\ \forall t \in T \\ q_t^{-1}(B) = \{(f, g) \in X^2 \ / \ (f_t(s),\, g_t(s)) \in B\},\ \forall t, s \in T,\ \forall B \in \mathcal{V} \\ \mathcal{M} = \{q_t^{-1}(B) \ / \ t, s \in T,\ B \in \mathcal{V}\} \quad \text{or} \\ \mathcal{M} = \bigcup_{t \in T} \{q_t^{-1}(B) \ / \ s \in T,\ B \in \mathcal{V}\} \subset \mathcal{N} \end{gathered}$$

then the family $\mathcal{M}$- is a subbase for auniformity $N \subset P((X^{S \times T})^2)$, (in $X^{S \times T}$), and the pair $(X^{S \times T}, \mathcal{N})$, is Hausdorff uniform space (because, the product of Hausdorff uniform space is a Hausdorff space).

Let $F(S \times T,\, X)$ be the set of all continuous functions on a topological Hausdorff space $S \times T$ to a uniform Hausdorff space $(X^{S \times T}, \mathcal{N})$ and $i : F(S \times T,\, X) \to X^{S \times T}$. The subset $F(S \times T,\, X)$ we can endow with the relative uniformity of pointwise convergence on $S\times T$. If we denote:

$$h:(F(S\times T,\,X))^2\rightarrow (X^{S\times T})^2$$
$$\forall (f,g)\in (F(S\times T,\,X))^2,\ h(f,g)=(f_t(s),\,g_t(s)),\ (f(s),\,g(s))\in B$$
$$h^{-1}(B)=B\cap (F(S\times T,X))^2=W(B),\ \forall B\in \mathcal{N}$$
$$W(B)=h^{-1}(B)=\{(f,g)\in (F(S\times T,\,X))^2/\,(f_t(s),\,g_t(s))\in B\},\ \forall t,s\in T,\ \forall B\in \mathcal{V}$$
$$\mathcal{B}=\{W(B)\ /\ B\in \mathcal{N}\}\subset \mathcal{P}$$

then the family $\mathcal{B}$-is a base for a uniformity $\mathcal{P}\subset P(F(S\times T,\,X))^2$, (in $F(S\times T,\,X)$) which is called the relative uniformity of pointwise convergence, and the pair $(F(S\times T,\,X),\,\mathcal{P})$ is a Hausdorff space (because, each subspace of a Hausdorff space, is a Hausdorff space).

Definition. *Let H be a topological space and S is a discrete topological semigroup with the identity. The mapping $\lambda: H\times S\rightarrow H$ is said to be a general discrete semiflow on H, if satisfying the following axioms:*

(a_1) *(Identity property)*

$\lambda(x,\,0)=x,\ \forall x\in H$

(where 0 is the identity of semigroup S).

(a_2) *(Group property)*

$\lambda(\lambda(x,\,t),\,s)=\lambda(x,\,t\oplus s),\ \forall x\in H\,\&\,\forall t,\,s\in S,$

(where $\oplus$ is the semigroup operation of S).

(a_3) *(Continuity property)*

The mapping $\lambda: H\times S\rightarrow H$ is continuous in H. In other words, for each neighborhood N of point $\lambda(x,\,t)$, there exist a neighborhood E of $x\in H$ such that $\lambda(E,\,t)\subset N$.

2. The result

Let $F(S\times T,\,X)$ be a set of all continuous functions $f:S\times T\rightarrow X$. The subset $F(S\times T,\,X)$ we can endow with the relative uniformity of pointwise convergence on $S\times T$. If the mapping $\delta: F\times T\rightarrow F$ defined by:

$$\forall (f,\,t)\in F\times T,\ \delta\,(f,\,t)=f_t$$

where $f_t(s)=f(t\oplus s),\ \forall t,\,s\in T$, then satisfied theorems:

Theorem. *Let T be a discrete topological abelian semigroup with the identity, and $F(S\times T,\,X)$ Hausdorff topological space. The mapping $\delta\,(f,\,t)=f_t$, defines a general discret semiflow on uniform structure of set $F(S\times T,\,X)$ relative to the relative uniformity of pointwise convergence.*

Proof. We shall prove the axioms of discret flow:

(a_1) (Identity property). By definition

$$\delta(f(s),\,t)=f_t(s)=f(t\oplus s),\ \forall t,s\in T$$
$$\delta(f(s),\,0)=f_0(s)=f(0\oplus s)=f(s),\ \forall s\in T$$
$$\delta(f,\,0)=f,\ \forall f\in F(T,\,X)$$

(a_2) (Group property). For each $t, s \in T$ and for each $f \in F(T,\, X)$. By the definition

$$\begin{gathered}\delta(f(s),\, t) = f_t(s) = f(t \oplus s),\, \forall t, s \in T,\\ \delta(\delta(f(s),\, t),\, m) = \delta(f_t(s),\, m) = \delta(f(t \oplus s),\, m) = f_m(t \oplus s)\\ f_m(t \oplus s) = f(t \oplus s \oplus m) = f_{t \oplus s}(m)\\ \delta(\delta(f(s),\, t),\, m) = f_{t \oplus s}(m),\, \forall m \in T\end{gathered}$$

or

$$\left.\begin{array}{l}\delta(\delta(f,\, t),\, s) = f_{t \oplus s}\\ f_{t \oplus s} = \delta(f,\, t \oplus s)\end{array}\right\} \Rightarrow \delta(\delta(f,\, t),\, s) = \delta(f,\, t \oplus s)$$

(a_3)(Continuity property). Let us now show that the mapping $\delta : F \times T \to F$ is continuous in uniform structure of set $F(S \times T,\, X)$. Assume that the set $F(S \times T,\, X)$ has the $\mathcal{P}$ relative uniformity of pointwise convergence on T. That is

$$A \in \mathcal{P} \Leftrightarrow A = \{f \in M \cap F(S \times T,\, X)/f_t(s) \in V\},\, s, t \in T,\, V \in \mathcal{V},\, M \in \mathcal{M}.$$

Let $f \in F(S \times T,\, X)$ be an arbitrary point and let $\mathcal{N}_f$ be the family of all open neighborhood of point f relative to the relative uniformity of pointwise convergence. The family $\mathcal{N}_f$ can be directed with the binary relation $(\leq) \subseteq \mathcal{N}_f \times \mathcal{N}_f$ as follows:

$$\forall A_1,\, A_2 \in \mathcal{N}_f,\, A_1 \leq A_2 \Leftrightarrow A_1 \supseteq A_2.$$

Then, the ordering pair $(\mathcal{N}_f,\, \leq)$ becomes a directed family. Indeed, for two each open neighborhood $A_1,\, A_2 \in \mathcal{N}_f$ of the point $f \in F(T,\, X)$, their intersection $A_3 = A_1 \cap A_2 \in \mathcal{N}_f$ is also an open neighborhood of the point f such that: $A_1 \leq A_3, ,\, A_1 \leq A_3$. The mapping $g : \mathcal{N}_f \to F(S \times T,\, X)$ which is defined by:

$$\forall A \in \mathcal{N}_f,\, g(A) = g_A$$

defines the net $(g_A,\, A \in \mathcal{N}_f) \subset F(S \times T,\, X)$ which converges at the unique point $f \in F(S \times T,\, X)$. In other words, there exist an open neighborhood $A_0 \in \mathcal{N}_f$ of the point $f \in F(S \times T,\, X)$ such that, for each open neighborhood $A \in \mathcal{N}_f$ of the point $f \in F(S \times T,\, X)$ is fulfilled:

$$A \geq A_0 \Rightarrow g_A \in A \subseteq A_0$$

relative to the $\mathcal{P} \subset P(F(S \times T,\, X))$ relative topology of uniform pointwise convergence. The point f is unique, because the ordering pair $(F(S \times T,\, X),\, \mathcal{P})$, is a Hausdorff space, relative to the relative topology of uniform pointwise convergence. For continuity of mapping $\delta : F \times T \to F$ in $F(S \times T,\, X)$, it will suffice to show that the corresponding net $(\delta(g_A,\, t_0),\, A \in \mathcal{N}_f)$, (where $t_0 \in T$ is a fixed point), converges to a unique point $\delta(f,\, t_0) \in F(S \times T,\, X)$ relative to the relative topology of uniform pointwise convergence.

Suppose contrary that the corresponding net $(\delta(g_A,\, t_0),\, A \in \mathcal{N}_f) \subset F(S \times T,\, X)$ converges to a unique point $\delta(f,\, t_0) \in F(S \times T,\, X)$, but the mapping $\delta : F \times T \to F$ is discontinuous at the point $f \in F(S \times T,\, X)$. In other words, there exist an open neighborhood $A_0 \in \mathcal{N}_{\delta(f,\, t_0)}$ of the point $\delta(f,\, t_0)$ in $F(S \times T,\, X)$ such that, for each open neighborhood $A \in \mathcal{N}_f$ in $F(S \times T,\, X)$ satisfies the condition:

$$\left.\begin{array}{r}(\exists A_0 \in \mathcal{N}_{\delta(f,\, t_0)})(\forall A \in \mathcal{N}_f)\\ \delta(A,\, t_0) \not\subset A_0\end{array}\right\}$$

On the other hand, the net $(\delta(g_A,\, t_0),\, A \in \mathcal{N}_f) \subset F(S \times T,\, X)$ converges to a unique point $\delta(f,\, t_0) \in F(S{\times}T,\, X)$ relative to the $\mathcal{P} \subset P(F(S{\times}T,\, X))$ topology of uniform pointwise convergence. This means that, there exist an open neighborhood $A_0 \in \mathcal{N}_{\delta(f,\, t_0)}$ of the point $\delta(f,\, t_0) \in F(S{\times}T,\, X)$ such that $\delta(g_A,\, t_0) \in A_0$. Hence, we have:

$$\delta(g_A,\, t_0) \in \delta(A,\, t_0) \not\subset A_0.$$

Consequently,

$$\left.\begin{array}{l}(\exists A_0 \in \mathcal{N}_{\delta(f,\, t_0)})(\forall W \in \mathcal{N}_{\delta(f,\, t_0)}) \\ W \geq A_0 \Rightarrow \delta(A_0,\, t_0) \notin W \subseteq A_0\end{array}\right\}$$

The last condition, show that the net $(\delta(g_A,\, t_0),\, A \in \mathcal{N}_f) \subset F(S \times T,\, X)$ does not convergence to a unique point $\delta(f,\, t_0) \in F(S{\times}T,\, X)$, which is impossible. This contradiction show that the mapping $\delta : F \times T \to F$ is continuous in $F(S \times T,\, X)$, relative to the relative uniformity of pointwise convergence. □

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ДИСКРЕТЕН СЕМИТЕК ВО РАМНОМЕРНАТА СТРУКТУРА НА МНОЖЕСТВА $F(S \times T, X)$

Неки Дервиши

Р е з и м е

Во оваа работа е докажано дека, ако е T дискретна абелова тополошка семигрупа, и ако е рамнпомерно Хаусдорфов простор, тогаш мапингот $\delta(f, t) = f_t$ одредува еден генерален дицкретен семитек во рамномерната структура на множества $F(S \times T, X)$, во врска со релативна униформна конвергенцијата по тачкама.

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