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METHOD OF VARIATION OF PARAMETERS IS NOT EXHAUSTED

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Abstract. A possibility to use a generalisation of the method of variation of parameters for nonlinear differential equations is shown.

First of all we consider the ordinary differential equation

$$\ddot{x} = f(t,x,\dot{x}), \tag{1}$$

where $f(t,x,\dot{x})$ satisfies a theorem of uniquenes of a local value problem.

Let us try to get the solutions of this equation on the form

$$x(t) = x_1(t)y_1(t) + x_2(t)y_2(t), \tag{2}$$

where $y_1(t)$ and $y_2(t)$ are given differentiable functions. Suppose that the functions $x_1(t)$ and $x_2(t)$ are determined by the system

$$\dot{x}_i = f_i(t,x_1,x_2,y_1,y_2),\ i=1,2, \tag{3}$$

where f_1 and f_2 are to be determined.

Differentiating (2) with respect to t and using (3) we obtain

$$\dot{x} = (\dot{y}_1(t)x_1(t) + \dot{y}_2(t)x_2(t)) + (y_1(t)f_1(t,x_1,x_2,y_1,y_2) + y_2(t)f_2(t,x_1,x_2,y_1,y_2)). \tag{4}$$

Choosing the second term in (4) to be zero, we have the first requirement for f_1 and f_2:

$$y_1(t)f_1(t,x_1,x_2,y_1,y_2) + y_2(t)f_2(t,x_1,x_2,y_1,y_2) = 0 \tag{5}$$

Taking into consideration (4) and (5) we have the following second derivative of x with respect to t

$$\ddot{x} = x_1 f(t,y_1,\dot{y}_1) + x_2 f(t,y_2,\dot{y}_2) + \dot{y}_1 f_1(t,x_1,x_2,y_1,y_2) + \dot{y}_2 f_2(t,x_1,x_2,y_1,y_2).$$

Hence, the relation (2) gives the solution of the equation (1) :

$$\dot{y}_1 f_1(t,x_1,x_2,y_1,y_2) + \dot{y}_2 f_2(t,x_1,x_2,y_1,y_2) = f(t,x_1y_1+x_2y_2,x_1\dot{y}_1+x_2\dot{y}_2) - x_1 f(t,y_1,\dot{y}_1) - x_2 f(t,y_2,\dot{y}_2).$$

Knowing $y_1(t)$ and $y_2(t)$, we can get the functions f_1 and f_2. But the problem is to solve the system (3).

In particular case when the equation (1) is linear, i.e. it has the form

$$\ddot{x} + p(t)\dot{x} + q(t)x = f(t),$$

$y_1(t)$ and $y_2(t)$ are linear independent solutions of the appropriate homogeneous equations and we get

$$f_1 = \frac{-y_2(t)f(t)}{W}, \quad f_2 = \frac{y_1(t)f(t)}{W}$$

where $W=y_1(t)\dot{y}_2(t)-\dot{y}_1(t)y_2(t)$ is the usual Wronskian. Having solved system (3) we get the well-known Lagranges formula

$$x = C_1y_1(t) + C_2y_2(t) - y_1(t)\int_{t_0}^{t}\frac{y_2(\xi)f(\xi)d\xi}{W} + y_2(t)\int_{t_0}^{t}\frac{y_1(\xi)f(\xi)d\xi}{W}$$

The result above seems not to be serious and useful except at the linear case. The following easely checked proposal shows that this is not always true. Let us consider the equation

$$\frac{d^2y}{dt^2} + \frac{2}{t}\left(\frac{dy}{dt}\right)^2 + f(t)y = 0 \qquad (6)$$

<u>Theorem</u>. The general solution of the equation (6) is determined by the formula (2), where $(x_1(t),x_2(t))$ is the general solution of the Riccati system

$$\frac{dx_1}{dt} = \frac{4x_1x_2y_1y_2^2}{tW}, \quad \frac{dx_2}{dt} = -\frac{4x_1x_2y_1^2y_2}{tW}, \qquad (7)$$

where $y_1=y_1(t)$ and $y_2=y_2(t)$ are particular solutions of the equation (6).

We do not declare the system (6) to be solvable. But the properties of its solutions are well-known (see [1], [2]).

R E F E R E N C E S

[1] Reid, W.T.: Riccati Differential Equations, Academic Press, New York, 1972

[2] Ince, E.L.: Ordinary Differential Equations, Dover Publications, New York, 1964

МЕТОДОТ НА ВАРИЈАЦИЈА НА КОНСТАНТИТЕ НЕ Е ИСЦРПЕН

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Р е з и м е

Во трудот се покажува дека е можна генерализација на методите на Лагранж на варијација на константи врз нелинеарни равенки од II ред.

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