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THE NULL SPACES CODIMENSION AND THE EXISTENCE OF THE INTERPOLATING SPLINE-FUNCTION IN BANACH SPACE

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Abstract

Using the results in papers [2] and [3] in this paper we prove the existence of the interpolating spline-function by the null space codimension of operators A and T.

Introduction

Let X, Y, Z be Bannach spaces. Suppose A is a bounded linear operator of X into Z and T is bounded linear operator of X into Y. The null space and the rang of operator A will be denoted by $N(A)$ and $R(A)$.

Let $R(A) = Z$. For fixed element $z \in Z$ we write by

$$I_z = \{x \in X : A_x = z\} = A^{-1}(z).$$

Definition 1. *The element $s \in I_z$ for wich*

$$\|T_s\| = \inf\{\|T_x\| : x \in I_z\}$$

is called the interpolating spline-function for z in connection with the operators A and T and is denoted by $s = s(z, A, T)$.

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The following theorem have been proved in the case that X, Y and Z are Hilbert spaces (see [1]).

Theorem 1. *Suppose:*
(i) $N(A)\dot{+}N(T)$ *is a closed set in* X
(ii) $N(A)\cap N(T) = \{0\}$. *Then for each* $z \in Z$ *there exists a unique interpolating spline-function* $s = s(z, A, T)$.

If X, Z be Banach spaces and Y is a reflexiv Banach space is proved the following theorem (see [2]).

Theorem 2. *Suppose:*
(i) $TA^{-1}(z)$ *is a closed and bounded set in* Y
(ii) $N(A)\cap N(T) = \{0\}$, *then there exists a unique interpolating spline-function* $s = s(z, A, T)$.

The main result

If X is a reflexive Banech space, it is proved the following

Theorem 3. *Suppose:*
(i) $N(T)$ *is a finite codimensional subspace in* X
(ii) $N(A)\cap N(T) = \{0\}$, *then for each* $z \in Z$ *exists a unique interpolating spline-function* $s = s(z, A, T)$.

Proof. According to Theorem 2 it is enough to show that the set $TA^{-1}(z)$ $(z \in Z)$ is closed in Y. Since $TA^{-1}(z)$ is a translation of the set $TN(A)$ it is enough to show that the set $TN(A)$ is closed in Y. Let $y_0 \in \mathrm{cl}\, TN(A)$, then there exists a sequence $(y_n)\subseteq TN(A)$, $\|y_n - y_0\| \to 0$ $(n \to \infty)$. Hence there exists $(x_n) \subseteq N(A)$ such that $Tx_n = y_n$ $(n \in \mathrm{N})$. Since $N(T)$ is a finite codimensional subspace in X then exists finite dimensional subspace F in X such that $X = N(T)\dot{+}F$. The subspace F is Banach space because it is finite dimensional subspace in Banach spaces X. We denote by T_1 the restriction of operator T in F. Operator T_1 is biection. According to Theorem of continuity of inverse, the inverse T_1^{-1} exists and is bounded. Since $x_n = t_n + f_n (t_n \in N(T), f_n \in F)$ $(n \in N)$, $Tx_n = T_1 f_n \Rightarrow f_n = T_1^{-1}Tx_n$ $(n \to \infty)$. Consequently the sequence (f_n) is bounded in F. Further, we denote by A_1 the restriction of the operator A in $N(T)$. Operator A is a biection. Let us prove that A_1 is $1-1$. Let $x, y \in N(T) \Rightarrow x - y \in N(T)$, then $Ax = Ay$ implicies that $x - y \in N(A)$. Since $N(A)\cap N(T) = \{0\}$, then $x = y$. According to Theorem of continuity of inverse, the invers A_1^{-1} exists and is bounded. Since $x_n = t_n + f_n (t_n \in N(T),\ f_n \in F)$ $(n \in \mathrm{N})$, $A_1 t_n = -Af_n \Rightarrow t_n = -A_1^{-1}f_n$. Consequently the sequence (t_n) is bounded in $N(T)$. Hence $(x_n) \subseteq N(T)$ is a bounded sequence in reflexive Banach space X. Therefore, the sequence (x_n) contains a subsequence $(x_{1,n})$ which converges weakly to $x_0 \in X$.

Since $f \circ T \in X^*(f \in Y^*)$, $(Tx_{1,n})$ converges weakly to $Tx_0 \in Y$ and $Tx_0 = y_0$. Also, the sequence $(Ax_{1,n})$ is weakly convergent to Ax_0. Since $Ax_{1,n} = 0$ $(n \in \mathbf{N})$, $Ax_0 = 0 \Rightarrow x_0 \in N(A) \Rightarrow y_0 = Tx_0 \in TN(A)$.

□

This completes the proof.

Corollary 1. *Suppose:*

(i) *$N(T)$ is a finite dimensional subspace in X*

(ii) *$N(A) \cap N(T) = \{0\}$, then for each $z \in Z$ exists a unique interpolating spline-function $s = s(z, A, T)$.*

Proof. Since $N(T)$ is a finite dimensional subspace in X, then exist a closed subspace F in X such that $X = N(T) + F$.

The subspace F is Banach space, because it is closed subspace in Banach space X.

□

Analogues results stay for the case of null-space $N(A)$.

References

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ПРОСТОРИ СО НУЛА КОДИМЕНЗИЈА И ПОСТОЕЊЕ НА ИНТЕРПОЛАЦИОНА СПЛАЈН ФУНКЦИЈА ВО БАНАХОВ ПРОСТОР

Рамадан Шх. Зејнуллаху

Р е з и м е

Тргнувајќи од резултатите во трудовите [2] и [3], во оваа работа докажана е егзистенција на интерполиорачка сплајн функција со помош на нула просторната кодимензија на операторите A и T.

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