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ABOUT THE STRUCTURE OF CERTAIN THEOREMS OF DIFFERENTIAL CALCULUS

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Abstract. The author presents the structure of certain theorems of differential calculus in the spirit of some eminent mathematicians mentioned in the paper.

1⁰. At the V Congress of the MFA of Yugoslavia, 1970, Skopje, Professor Danilo Blanuša in his paper pointed out: "As a mathematician I can not help but search for the beauty in any aspect of mathematical research. It is not enough to prove something, you need to prove it elegantly. A new mathematical theory is a testimony of invention and creativity of its creator. It is not just a chain of logical deduction. Mathematics is the art" [1].

As a contribution to such a consideration in the differential calculus, the author has also demonstrated the monotony and bounders of sequences $(a_n) = \left(\left(1+\frac{1}{n}\right)^n\right)$, $n \in \mathbf{N}$, $n \geq 2$. Namely, the structure of inequality which comes from [2]:

$$\frac{n+1}{n+1-k} \geq \left(1+\frac{1}{n}\right)^k, \quad 0 \leq k \leq n, \quad n \geq 2, \tag{1}$$

is such that from (1) follows the monotony and bounders of sequences $(a_n) = \left(\left(1+\frac{1}{n}\right)^n\right)$ [3]. Indeed, from (1), multiplying by $\binom{n}{k} = \frac{n!}{k!(n-k)!}$ follows monotony of sequences (a_n) from

$$\binom{n+1}{k} \geq \frac{(n+1)^k}{n^k}\binom{n}{k} \text{ therefore } \binom{n+1}{k}\frac{1}{(n+1)^k} \geq \binom{n}{k}\frac{1}{n^k} \tag{2}$$

ie. the sequence (a_n) is increasing . For $k = \frac{n}{2}$, n–even, from (1) follows:

$$\left(1+\frac{1}{n}\right)^{\frac{n}{2}} \leq \frac{n+1}{n+1-\frac{n}{2}} = 2\frac{n+1}{n+2} < 2, \text{ e.q. } a_n = \left(1+\frac{1}{n}\right)^n < 4, \text{ for all } n\text{–even.} \tag{3}$$

From $a_{2n-1} < a_{2n} < 4$, for all n, there follows the existence of limes of sequences (a_n).

2⁰. Let us consider the case of classical means over segment $[a,b]$ (harmonic, geometric and arithmetic) that are expanding the base of two-dimensional consideration [4]:

$$M_x(a,b) = \frac{a^x + b^x}{a^{x-1} + b^{x-1}}, \quad x \in \mathbf{R}. \tag{4}$$

Namely, the paper [4] gives two analytical properties of the author:

$$M_x(a,b) + M_{2-x}(a,b) = a + b \quad \Leftrightarrow \quad M_{1-t}(a,b) + M_{1+t} = a + b \tag{5}$$

and

$$M_x(a,b) \cdot M_{1-x}(a,b) = a \cdot b \quad \Leftrightarrow \quad M_{\frac{1}{2}-t}(a,b) \cdot M_{\frac{1}{2}+t}(a,b) = a \cdot b \tag{6}$$

obtained by transcription ($x = 1 \pm t$ and $x = \frac{1}{2} \pm t$ respectively) from which we read the graphics are given by the relations to the right of equivalence, based on relations (5) *central symmetrical to the point* $A(1, M_1(a,b))$, while the graphs of functions based on relations (6) *axial symmetric with respect to the straight line* $x = \frac{1}{2}$ $(t = 0)$. This structure proves to be useful with the introduction of appropriate diagrams of means and introduction of the curves $y(x) = M_x(a,b)$ and $y_1(x) = M_{1-x}(a,b)$.

In accordance with relations (5) and (6), we come to the new concepts; namely, *the complementary environments in relation to the number $a+b$, and reciprocal environments in relation to the number of $a \cdot b$.* Thus, through a considered approach, a set of basic means is expanded by the set of complementary and reciprocal ones, given on the diagram means (figure 1)[*)1]. Thus, in the set of basic means we consider the followings two means $M_{3/2}(a,b)$ and $M_2(a,b)$ as a complement of $G(a,b)$ and $H(a,b)$ means respectively.

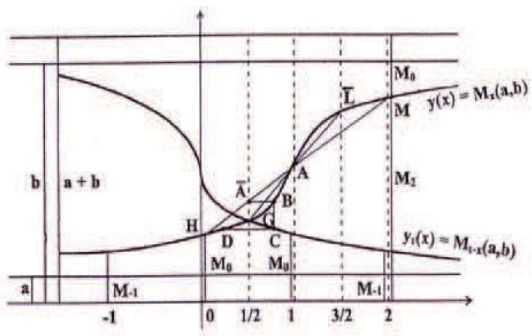

FIGURE 1

On the basis of the diagram we have the relations [5]:

$$M_x(a,b) \le G(a,b) = \sqrt{a \cdot b} \le M_{1-x}(a,b), \quad \text{for } x \le \frac{1}{2} \tag{7}$$

[1*)] Let us quote Weyl: "Beauty is closely connected with symmetry".

and

$$M_x(a,b) \geq G(a,b) = \sqrt{a \cdot b} \geq M_{1-x}(a,b), \quad \text{for } x \geq \frac{1}{2}. \tag{8}$$

Next, on the basis of the previous method in the paper [6], in the process of determining the means from the reciprocal menas, the case of $R\left[\,\overline{M}_x(a,b)\,\right] = \frac{A(a,b)+H(a,b)}{2} = \overline{A}$, it follows by using the introduced reciprocity again, that there is the reflection of the mean $R\left[\,\overline{M}_x(a,b)\,\right]$; namely, there is a point D in the diagram with index $x_0 = 1 - \overline{x}$, where is

$$\overline{x} = \frac{\log(a+3b) - \log(b+3a)}{\log\, b - \log\, a} \in \left(0, \frac{1}{2}\right). \tag{9}$$

Previous consideration justifies the introduction of complementary and reciprocal means with curves $y(x) = M_x(a,b)$ and $y_1(x) = M_{1-x}(a,b)$ on the corresponding diagram.

3^0. The famous Russian mathematician Andrei Kolmogorov emphasized structure of certain statements in mathematics in his papers: Contemporary Mathematics (1936) and Mathematics (1954, 1974) which deals with "the transition of the basic concepts of mathematics from a lower level of generality and abstraction to a higher level of generality and abstraction and that this transition makes a new stage in the development of mathematics, which is very important for scientific and teaching activities in this field" [7].

In accordance with previously stated, in the context "$\eta - \xi$" statements for the theorem 6 [8], we give a geometrical interpretation of result in sense of two multiple specifications. Based on the statement 6 from [8] it is true that:

$$\left\{\begin{array}{lll} (77_1) & \exists \eta \in (a,b) : f(\eta) = \varphi(\eta) & (\eta - \text{relation}) \\ (78_1) & \exists \xi_b \in (\eta, b) : T^f_{\xi_b}(a) = T^{\varphi}_{\xi_b}(a) & \\ (78'_1) & \exists \xi_a \in (a, \eta) : T^f_{\xi_a}(b) = T^{\varphi}_{\xi_a}(b) & (\xi - \text{relations}) \end{array}\right. \tag{10}$$

where φ is a secant of function f in relation to function φ ($\varphi(a) = f(a)$ and $\varphi(b) = f(b)$). Its corollary, for example (78_1), is that for tangent in existential points (result of the first specification):

$$M_2(\xi_b, f(\xi_b)) \quad \text{and} \quad N_2(\xi_b, \varphi(\xi_b)), \tag{11}$$

has intersection in a point P on line $x = a$ (figure 2). Analogous geometric interpretation we have for the relationship $(78'_1)$.

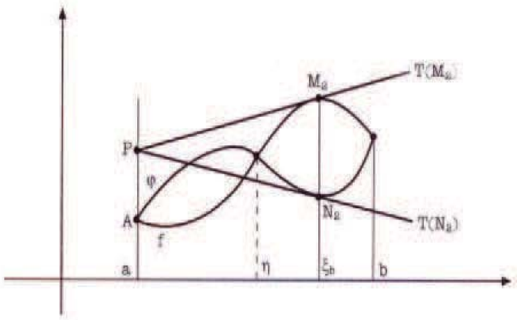

FIGURE 2

We are pointing at the second specification based on the first specification. Namely, for the case when the curved secant φ is line:

$$\varphi(x) = f(a) + \frac{f(b) - f(a)}{b - a}(x - a), \tag{12}$$

the revious relationships (78_1) and $(78'_1)$ change into relations (with relation (77_1)):

$$\begin{cases} \exists \xi_b \in (\eta, b) : T^f_{\xi_b}(a) = f(a) \\ \exists \xi_a \in (a, \eta) : T^f_{\xi_a}(b) = f(b) \end{cases} \tag{13}$$

respectively, which is then reduced to the other form, the well known Flett relations:

$$\begin{cases} T^f_{\xi_b}(a) = f(a) \ : \ f(\xi_b) + f'(\xi_b)(a - \xi_b) = f(a) \Rightarrow \dfrac{f(\xi_b) - f(a)}{\xi_b - a} = f'(\xi_b), \\ T^f_{\xi_a}(b) = f(b) \ : \ f(\xi_a) + f'(\xi_a)(b - \xi_a) = f(b) \Rightarrow \dfrac{f(\xi_a) - f(b)}{\xi_a - b} = f'(\xi_a). \end{cases} \tag{14}$$

Thus, Flett's relation $T^f_{\xi_b}(a) = f(a)$, under appropriate conditions of the existence of the point η, with relation

$$\left(f'(a) - \frac{f(b) - f(a)}{b - a}\right) \cdot \left(f'(b) - \frac{f(b) - f(a)}{b - a}\right) > 0. \tag{15}$$

gives the consequent figure 3.

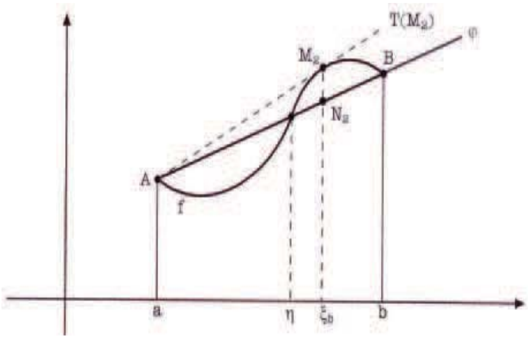

FIGURE 3

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ЗА СТРУКТУРАТА НА ОПРЕДЕЛЕНИ ТЕОРЕМИ ОД ДИФЕРЕНЦИЈАЛНО СМЕТАЊЕ

Јован В. Малешевиќ

Р е з и м е

Авторот ја презентира структурата на определени теореми од диференцијалното сметање во духот на некои еминентни математичари спомнати во трудот

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