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DYNAMICAL LIMIT SETS OF MOTION ON TOPOLOGICAL STRUCTURE OF SET $F(T,\, X)$

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Abstract. In this paper is proved that, if T is a real line and $(Y,\, \mathcal{V})$ is a Hausdorff topological space, then the dynamical limit sets $\Omega_x\,(A_x)$ of the motion $\psi(x,\, t)$ are close and invariant on topological structure of the set $F(T,\, X)$, related with the relative topology of coordinatewise convergence.

1. Preliminary remarks

Let T be a real line, $(Y,\, \mathcal{V})$-Hausdorff topological space, and $p_t : Y^T \to Y,\ t \in T$ is the natural projection. If we denote:

$$\forall f \in Y^T,\ \ p_t(f) = f(t),\ \forall t \in T$$

$$p_t^{-1}(V) = \{f \in Y^T / f(t) \in V\},\ \forall t \in T,\ \forall V \in \mathcal{V}$$

$$\mathrm{S} = \bigcup_{t\in T} {p_t}^{-1}(V) = \{f \in Y^T / f(t) \in V,\ t \in T,\ V \in \mathcal{V}\}$$

then the family $\mathcal{S}$ is the subbase for $\mathcal{T}$- topologyof coordinatewise convergence on T. The pair $(X,\, \mathcal{T})$ - is a Hausdorff topological space, where $X = Y^T$. Let we denote $\mathcal{R}$ - relative topology of coordinatewise convergence on T, then the pair $(F,\, \mathcal{R})$- is also Hausdorff topological space, where $F = F(T,\, X)$. Let $F(T,\, X)$ be the set of all continuous functions on a topological Hausdorff space T to a topological Hausdorff space $(X,\, \mathcal{T})$ and $i : F(T,\, X) \to (X,\, \mathcal{T})$. The subset $F(T,\, X)$ we can endow with the relative topology of coordinatewise convergence on T. If we denote:

$$\forall f \in F(T,\ X),\ \ i(f) = f(t) = f_t,\ t \in T$$

$$W(V) = i^{-1}(V) = \{f \in F(T,\, X)/f(t) \in V\},\ \forall t \in T,\ \forall V \in \mathcal{V}$$

$$\mathcal{Q} = \bigcup_{t\in T} \{f/f(t) \in V,\ t \in T,\ V \in \mathcal{V}\}$$

then the family $\mathcal{Q}$ is a base for $\mathcal{R} \subset P(F(T,\, X))$ - relative topology of coordinatewise convergence on T.

Definition 1. *Let be $\psi : H \times G \to H$ a general dinamical system on Hausdorff topological space H. Then for any fix point $x \in H$, the mapping $\psi(x,\ ) : G \to H$ is called the motion through the point $x \in H$.*

Definition 2. *The set of all $\omega\,(\alpha)$ - dynamical limit point of the motion $\psi(x,\,)$: $G \to H$ where $x \in H$ is a fix point in H denote with $\Omega_x\,(A_x)$ and it is called $\omega\,(\alpha)$ - dynamical limit sets of the motion $\psi(x,\,t)$.*

2. The result

Let $F(T,\,X)$ be a set of all continuous functions $f : T \to X$. The set $F(T,\,X)$ we can endow with relative topology of coordinatewise convergence. Let be $\psi(x,\,t)$ a motion through the point $x \in F(T,\,X)$. Then the following theorems are satisfied:

Theorem 1. *The dynamical limit sets Ω_x (A_x) of the motion $\psi(x,\,t)$ are close in $F(T,\,X)$, related with the relative topology of coordinatewise convergence.*

Proof. (for Ω_x). Let $y \in F(T,\,X)$ be an arbitrary point of dynamical limit close set $\overline{\Omega_x}$, $y \in \overline{\Omega_x}$. Then there is a net $(y_a,\,a \in D) \subset \Omega_x$, which tends to a point $y \in \Omega_x$. We can prove that the point $y \in F(T,\,X)$ is ω - dynamical limit point of the motion $\psi(x,\,t)$, where $x \in F(T,\,X)$ is a fix point. Let $W_{\varepsilon_0}(V)$ be an arbitrary open neighborhood of the point $y \in F(T,\,X)$. The net $(y_a,\,a \in D) \subset \Omega_x$ tends to a fix point $y \in F(T,\,X)$. In this case, there is a point $y_a \in F(T,\,X)$ such that $y_a \in W_{\varepsilon_0}(V)\,[y]$. Also there is an open neighborhood $W(V_{\varepsilon_1})\,[y_a]$ such that:

$$W(V_{\varepsilon_1})\,[y_a] \subseteq W(V_{\varepsilon_0})\,[y]$$

That is

$$y = \psi(x,\,t) \Rightarrow y_a = \psi(x,\,t_a) \Leftrightarrow (\psi(x,\,t_a), y) \in W_{\varepsilon_0}(V) \Leftrightarrow$$
$$\Leftrightarrow y_a \in W_{\varepsilon_0}(V)[y] \Leftrightarrow (\psi(x,\;t_a),\,y) \in W_{\varepsilon_0}(V)$$

holds for all $t_a > t_0$. On the other hand

$$\psi(x,\;t) \in W_{\varepsilon_0}(V)[y],\;\forall t > t_0.$$

That is to say $\psi(x,\,t) = y \in \Omega_x$. Which is what we set out to prove.

(For A_x). Let $y \in F(T,\,X)$ be an arbitrary point of dynamical limit close set $\overline{A_x}$, $y \in \overline{A_x}$. Then there is a net $(-y_a,\,a \in D) \subset A_x$, which tends to a point $y \in A_x$. We can prove that the point $y \in F(T,\,X)$ is α - dynamical limit point of the motion $\psi(x,\,t)$, where $x \in F(T,\,X)$ is a fix point. Let $W_{\varepsilon_0}(V)$ be an arbitrary open neighborhood of the point $y \in F(T,\,X)$. The net $(-y_a,\,a \in D) \subset A_x$ tends to a fix point $y \in F(T,\,X)$. In this case, there is a point $y_a \in F(T,\,X)$ such that $y_a \in W_{\varepsilon_0}(V)\,[y]$. Also there is an open neighborhood $W(V_{\varepsilon_1})\,[y_a]$ such that:

$$W(V_{\varepsilon_1})\,[y_a] \subseteq W(V_{\varepsilon_0})\,[y]$$

That is

$$\Leftrightarrow y_a \in W_{\varepsilon_0}(V)[y] \Leftrightarrow (\psi(x,\,t_a),\,y) \in W_{\varepsilon_0}(V)$$

holds for all $t_a < -t_0$. By suppose $y_a = \psi(x, t_a) \to y$ when the net $(-y_a,\,a \in D) \to -\infty$. On the other hand

$$(\psi(x,\;t) \in W_{\varepsilon_0}(V)\,[y],\;\forall t < -t_0,$$

for any arrbitrary moment of time $-t_0 \in T$. That is to say $\psi(x,\;t) = y \in A_x$. Which is what we set out to prove. □

Theorem 2. *The dynamical limit sets Ω_x (A_x) of the motion $\psi(x, t)$ are invariant in $F(T, X)$, related with the relative topology of coordinatewise convergence.*

Proof. (For Ω_x). Let us show that the dynamical limit set Ω_x is an invariant. For this purpose, let $y \in F(T, X)$ be an arbitrary point of Ω_x, $y \in \Omega_x$. Then, there is a net $(t_a, a \in D) \subset T$ which tends to $+\infty$, such that the corresponding net $(\psi(x, t_a), a \in D) \subset F(T, X)$ where $\psi(x, t_a) = y_a$, tends to the point $y \in F(T, X)$. In this case

$$\lim_{a\to+\infty}(t_a + t) = +\infty, \ \forall t \in T$$

$$\psi(x, t_a + t) = \psi(\psi(x, t_a), t) = \psi(y_a, t) = y_a$$

That is to say

$$\lim_{a\to+\infty} \psi(x, t_a + t) = \lim_{a\to+\infty} \psi(\psi(x, t_a), t) = \psi(x, t) = y$$

The last equality shows us that an arbitrary point $y \in \Omega_x$, $\forall t \in T$. From this follows that

$$\psi(x, T) \subseteq \Omega_x.$$

The last condition shows us that the all trajectory $\psi(x, T)$ of motion $\psi(x, t)$ through the fix point $x \in F(T, X)$ is contained in the dynamical limit set Ω_x . That is to say, that the dynamical limit set Ω_x is composed from full trajectories, that is to say that the set Ω_x is an invariant set. Which is what we set out to prove.

(For A_x). We should prove that the dynamical limit set A_x is an invariant. For this purpose, let $z \in F(T, X)$ be an arbitrary point of A_x , $z \in A_x$. Then, there is a net $(-t_a, a \in D) \subset T$ which tends to $-\infty$, such that the corresponding net $(\psi(x, -t_a), a \in D) \subset F(T, X)$ where $\psi(x, -t_a) = z_a$, tends to the point $z \in F(T, X)$. In this case

$$\lim_{a\to+\infty}(-t_a - t) = -\infty, \ \forall t \in T$$

$$\psi(x, -t_a - t) = \psi(\psi(x, -t_a), -t) = \psi(y_a, -t) = z_a$$

That is to say

$$\lim_{a\to+\infty} \psi(x, -t_a - t) = \lim_{a\to+\infty} \psi(\psi(x, -t_a), -t) = \psi(x, -t) = z$$

The last equality shows us that an arbitrary point $z \in A_x$, $\forall t \in T$. From this follows that

$$\psi(x, -T) \subseteq A_x.$$

The last condition shows us that the all trajectory $\psi(x, -T)$ of the motion $\psi(x, t)$ through the fix point $x \in F(T, X)$ is contained in the dynamical limit set A_x . That is to say, the dynamical limit set A_x is composed from full trajectories, so that the set A_x is an invariant set. Which is what we set out to prove. □

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ДИНАМИЧКИ ГРАНИЧНИ МНОЖЕСТВА НА ДВИЖЕЊЕ ВО ТОПОЛОШКАТА СТРУКТУРА НА МНОЖЕСТВА $F(T, X)$

Неки Дервиши

Р е з и м е

Во оваа работа е докажано со две теореми дека: Ако T е реална права и $(Y, \mathcal{V})$ - Хаусдорфов тополошки простор, тогаш динамичките гранични множества $\Omega_x(A_x)$ се затворени и инваријантни на тополошката структура на множества $F(T, X)$, во врска со релативна топологија на конвергенција по координати.

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