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ON SOME NONLINEAR COMPLEX DIFFERENCE EQUATIONS

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Abstract

In this paper, the differences in the process of solving linear and nonlinear ψ-difference equations are considered.

1. Introduction

Definition 1.1. *Let us consider the equation of the following type*

$$\overline{z} = g(z) \tag{1.1}$$

where $g(z)$ is an arbitrary analytical function. Equation (1.1) *can define a closed or an open contour, or a set of isolated points. Further, the set of points in a complex plane which is defined by equation* (1.1) *is called* **K-contour**. *In practice, a great number of important contours (line, circle etc) can be shown as type* (1.1).

Definition 1.2. *Let $g(z)$ be an analytical function and $w = w(z, \overline{z})$ be a continuous complex function, which can be developed into a convergent power series by z and $\overline{z}$. Then the compound function $w = w\big(z, g(z)\big)$ is an analytical function for which we will use the symbol $\alpha_{g(z)}w$. The geometrical meaning of this operator is as follows: If $\overline{z} = g(z)$ is an equation of a closed contour, then functions $w = w(z, \overline{z})$ and $\alpha_{g(z)}w$ have the same limit value on the mentioned contour.*

Definition 1.3. *Let* L_0*:* $\overline{z} = g_0(z)$, L_1*:* $\overline{z} = g_1(z), \ldots,$ L_n*:* $\overline{z} = g_n(z)$ *be given closed contours which limit the areas* $G_0, G_1, \ldots, G_n$ *and let* $G_0 \subset G_1 \subset \ldots \subset G_n$*. In* [1] *Čanak M. introduced the so-called areolar* ψ*-differences as:*

$$\begin{aligned}
&\frac{\alpha_{g_1} w - \alpha_{g_0} w}{g_1 - g_0} = \psi(g_0, g_1)\\
&\vdots\\
&\frac{\alpha_{g_n} w - \alpha_{g_{n-1}} w}{g_n - g_{n-1}} = \psi(g_{n-1}, g_n)\\
&\frac{\psi(g_1, g_2) - \psi(g_0, g_1)}{g_2 - g_0} = \psi(g_0, g_1, g_2)\\
&\vdots\\
&\frac{\psi(g_{n-1}, g_n) - \psi(g_{n-2}, g_{n-1})}{g_n - g_{n-2}} = \psi(g_{n-2}, g_{n-1}, g_n)\\
&\vdots\\
&\frac{\psi(g_1, g_2, \ldots, g_n) - \psi(g_0, g_1, \ldots, g_{n-1})}{g_n - g_0} = \psi(g_0, g_1, \ldots, g_{n-1}, g_n).
\end{aligned} \tag{1.2}$$

On the base of the mentioned differences, Čanak M. constructs the sequence of functions:

$$\begin{aligned}
&\frac{\alpha_{g_0} w - w(z, \overline{z})}{g_0 - \overline{z}} = \psi(\overline{z}, g_0)\\
&\frac{\psi(g_0, g_1) - \psi(\overline{z}, g_0)}{g_1 - \overline{z}} = \psi(\overline{z}, g_0, g_1)\\
&\vdots\\
&\frac{\psi(g_0, g_1, \ldots, g_n) - \psi(\overline{z}, g_0, g_1, \ldots, g_{n-1})}{g_n - \overline{z}} = \psi(\overline{z}, g_0, g_1, \ldots, g_n).
\end{aligned} \tag{1.3}$$

The equations which contain unknown complex functions and their differences of the type (1.3) for the system of contours $\overline{z} = g_i(z)$, $(i = 0, 1, \ldots, n)$ are called **complex areolar** ψ**-difference equations.** Some difference equations were solved by Čanak M. in [1]. It was shown that the general solution of the complex difference equation of the first order contains an arbitrary analytical function, and the general solution of the complex difference equation of the n-th order contains n arbitrary analytical functions.

ψ-difference equations can be used in the theory of complex interpolation and for approximative solution of the complex differential equations ([1], [2]).

In this paper, the differences between solving linear and nonlinear complex ψ-difference equations are observed.

2. A Linear Homogeneous Complex Difference Equation

Definition 2.1. *Let* L_0: $\overline{z} = g_0(z)$, L_1: $\overline{z} = g_1(z), \ldots,$ Ln: $\overline{z}=g_n(z)$ *be given closed contours which limit the areas* $G_0, G_1, \ldots, G_n$ *and let* $G_0 \subset G_1 \subset \ldots \subset G_n$. *A linear, areolar complex* ψ*-difference equation of n-th order is an equation of the type:*

$$\begin{aligned} &\psi_w(\overline{z}, g_0, \ldots, g_{n-1}) + a_1(z)\psi_w(\overline{z}, g_0, \ldots, g_{n-2}) \\ &+ a_2(z)\psi_w(\overline{z}, g_0, \ldots, g_{n-3}) + \cdots \\ &+ a_{n-2}(z)\psi_w(\overline{z}, g_0, g_1) + a_{n-1}(z)\psi_w(\overline{z}, g_0) + a_n(z)w = 0 \end{aligned} \tag{2.1}$$

where $a_1(z), a_2(z), \ldots, a_n(z)$ *are given analytical functions.*

The process of solving this equation is very similar to the process of solving the corresponding complex differential equations.

The complex equation of the type

$$D^n w + a_1(z)D^{n-1}w + \cdots + a_{n-1}(z)Dw + a_n(z)w = 0 \tag{2.2}$$

is studied by Kečkić J. [1], where $a_k(z)$ $(k = 1, 2, \ldots, n)$ are analytical functions and

$$Dw = (u'_x - v'_y) + i(u'_y + v'_x) = 2w'_{\overline{z}}$$

$$\left(D^n w = D\left(D^{n-1}w\right)\right)$$

is the known differential Kolossov's operator. Kečkić showed that the general solution of the equation (2.2) is:

$$\begin{aligned} w(z, \overline{z}) = \varphi_1(z)\exp\left(\frac{r_1(z)}{2}\overline{z}\right) + \varphi_2(z)\exp\left(\frac{r_2(z)}{2}\overline{z}\right) + \cdots \\ + \varphi_n(z)\exp\left(\frac{r_n(z)}{2}\overline{z}\right) \end{aligned} \tag{2.3}$$

where $\varphi_1(z), \varphi_2(z), \ldots, \varphi_n(z)$ are arbitrary analytical functions and

$$r_1(z) \neq r_2(z) \neq \cdots \neq r_n(z)$$

are the roots of the corresponding auxiliary equation

$$r^n + a_1(z)r^{n-1} + \cdots + a_{n-1}(z)r + a_n(z) = 0. \tag{2.4}$$

Let us consider at first the so-called modified difference equation

$$\psi_w(\overline{z}, g_0) = w(z, \overline{z}) \tag{2.5}$$

where L_0: $\overline{z} = g_0(z)$ is a given contour. The general solution of the equation is of the type:

$$w_0(z, \overline{z}) = \frac{\xi(z)}{1 + g_0(z) - \overline{z}}, \tag{2.6}$$

where $\xi(z)$ is an arbitrary analytical function. Based on relation (2.6), for the difference equation (2.1) we find the solution of the type:

$$w(z, \overline{z}) = \frac{1}{1 + b(z) - \overline{z}} \qquad (\xi(z) = 1), \tag{2.7}$$

where $b(z)$ is an unknown analytical function. After ample calculation, the following sequence of functions is obtained:

$$\begin{aligned}
w(z, \overline{z}) &= \frac{1}{1 + b(z) - \overline{z}} \\
\psi_w(\overline{z}, g_0) &= \frac{1}{[1 + b(z) - g_0][1 + b(z) - \overline{z}]} \\
\psi_w(\overline{z}, g_0, g_1) &= \frac{1}{[1 + b(z) - g_0][1 + b(z) - g_1][1 + b(z) - \overline{z}]} \\
&\vdots \\
\psi_w(\overline{z}, g_0, g_1, \ldots, g_{n-1}) &= \frac{1}{[1 + b(z) - g_0] \cdots [1 + b(z) - g_{n-1}][1 + b(z) - \overline{z}]}.
\end{aligned} \tag{2.8}$$

By substituting (2.8) in (2.1), the corresponding algebraic auxiliary equation of the n-th order considering $b = b(z)$ is obtained:

$$\begin{aligned}
&1 + (1 + b - g_{n-1})a_1(z) + (1 + b - g_{n-1})(1 + b - g_{n-2})a_2(z) + \cdots \\
&+ (1 + b - g_{n-1})(1 + b - g_{n-2}) \cdots (1 + b - g_0)a_n(z) = 0.
\end{aligned} \tag{2.9}$$

Let us suppose that $b_1(z) \neq b_2(z) \neq \cdots \neq b_n(z)$. Then there exist n linearly independent particular solutions of the difference equation (2.1) of the following type:

$$w_k(z,\bar{z}) = \frac{1}{1+b_k(z)-\bar{z}}, \quad k = 1,2,\ldots,n. \tag{2.10}$$

If $C_1(z)$ and $C_2(z)$ are two arbitrary analytical functions, by using the property:

$$\alpha_g[C_1(z)w_1 + C_2(z)w_2] = C_1(z)\alpha_g w_1 + C_2(z)\alpha_g w_2$$

we set the following:

Theorem 2.1. *If $w_1(z,\bar{z}),\ldots,w_n(z,\bar{z})$ are linearly independent particular solutions of the difference equation (2.1), then the general solution is:*

$$W(z,\bar{z}) = C_1(z)w_1 + C_2(z)w_2 + \cdots + C_n(z)w_n \tag{2.11}$$

where $C_1(z),\ldots,C_n(z)$ are arbitrary analytical functions.

3. Some Nonlinear Complex Difference Equations

3.1. A Remark on Complex Difference Equation of Riccati Type

Let L: $\bar{z} = g(z)$ be a given simple, smooth and closed contour and let $a(z,\bar{z})$, $b(z,\bar{z})$, $c(z,\bar{z})$ be given continuous functions. A complex difference equation of Riccati type is the equation:

$$\psi_w(\bar{z},g) = a(z,\bar{z})w^2 + b(z,\bar{z})w + c(z,\bar{z}). \tag{3.1}$$

It was shown by Čanak M. in [2] that the general solution of the equation (3.1) is:

$$w(z,\bar{z}) = \tag{3.2}$$
$$= \frac{-[1+(g-\bar{z})b]+\sqrt{[1+(g-\bar{z})b]^2-4a(g-\bar{z})[(g-\bar{z})c-\varphi(z)]}}{2(g-\bar{z})a}$$

where $\varphi(z)$ is an arbitrary analytical function. The contour $\bar{z} = g(z)$ is a singular line for the general solution (3.2) and

$$\lim_{\overline{z}\to g(z)} w(z,\overline{z}) = \varphi(z)$$

exists.

There is a connection between the complex differential equation of Riccati type:

$$w'_{\overline{z}} = a(z,\overline{z})w^2 + b(z,\overline{z})w + c(z,\overline{z}) \tag{3.3}$$

and the differential equation of the II order:

$$w''_{\overline{z},\overline{z}} = A(z,\overline{z})w\,. \tag{3.4}$$

Namely, by substitution $w'_{\overline{z}} = wu(z,\overline{z})$, where $u(z,\overline{z})$ is a new unknown function, the equation (3.4) is transformed into:

$$u'_{\overline{z}} + u^2 = A(z,\overline{z})\,.$$

Because of that, it is of an interest to observe the corresponding difference equation.

Let $A = A(z,\overline{z})$ be a given continuous complex function and let L_0: $\overline{z} = g_0(z)$, L_1: $\overline{z} = g_1(z)$ be given contours. We will observe the following difference equation:

$$\psi_w(\overline{z}, g_0, g_1) = A(z,\overline{z})w\,. \tag{3.5}$$

From (3.5), it directly follows that:

$$\psi_w(\overline{z}, g_0, g_1) = \frac{\psi(g_0,g_1) - \psi(\overline{z},g_0)}{g_1 - \overline{z}} = A(z,\overline{z})w \Rightarrow$$

$$\psi(g_0,g_1) - \psi_w(\overline{z},g_0) = (g_1 - \overline{z})A(z,\overline{z})w \Rightarrow$$

$$\psi_w(\overline{z},g_0) = \psi(g_0,g_1) - (g_1 - \overline{z})A(z,\overline{z})w \Rightarrow \tag{3.6}$$

$$\frac{\alpha_{g_0}w - w(z,\overline{z})}{g_0 - \overline{z}} = \psi(g_0,g_1) - (g_1 - \overline{z})A(z,\overline{z})w \Rightarrow$$

$$\alpha_{g_0}w - w(z,\overline{z}) = (g_0 - \overline{z})\psi(g_0,g_1) - (g_0 - \overline{z})(g_1 - \overline{z})A(z,\overline{z})w \Rightarrow$$

$$w(z,\overline{z}) = \frac{\alpha_{g_0}w - (g_0 - \overline{z})\psi(g_0,g_1)}{1 - (g_0 - \overline{z})(g_1 - \overline{z})A(z,\overline{z})}\,.$$

As the general solution must contain two arbitrary analytical functions, and considering relation (3.6) we suppose that it is in the following form:

$$w(z,\overline{z}) = \frac{\varphi_1(z) - \varphi_2(z)(g_0 - \overline{z})}{1 - (g_0 - \overline{z})(g_1 - \overline{z})A(z,\overline{z})} \tag{3.7}$$

where the unknown analytical functions $\alpha_{g_0} w$ and $\psi(g_0, g_1)$ are included in the arbitrary analytical functions $\varphi_1(z)$ and $\varphi_2(z)$. Since equation (3.7) satisfies equation (3.5), it follows that it is its general solution.

3.2. Complex Difference Equation of the Bicadze Type

In monograph [4], Bicadze considers the following complex differential equation:

$$w'_{\overline{z}} = \frac{w + \overline{w}}{2(z + \overline{z})} \tag{3.8}$$

which is very important in the theory of axial symmetry stationary gravitation field. After a large calculation, he has proved that the general solution of equation (3.8) is:

$$w = \int_0^{\eta} \frac{\partial \rho(\xi,\tau)}{\partial \xi}\, d\tau + \xi \int_0^{\xi} \frac{\partial \rho(t,y)}{\partial y}\Big|_{y=0} \frac{dt}{t} + c\xi + i\rho(\xi,\eta) \tag{3.9}$$

with

$$\rho(\xi,\eta) = \xi^2 \int_0^1 f(\eta + i\xi - 2i\xi t)\sqrt{t(1-t)}\, dt, \quad \xi + i\eta = z$$

where c is an arbitrary real constant and $f(\tau)$ is an arbitrary analytical function.

In this paper, the corresponding difference equation is considered:

$$\psi(\overline{z}, g_0) = \frac{w + \overline{w}}{2(z + \overline{z})} \tag{3.10}$$

where L: $\overline{z} = g_0(z)$ is the equation of a given closed contour.

From (3.10) directly follows:

$$\alpha_{g_0} w - w = \frac{(g_0 - \overline{z})(w + \overline{w})}{2(z + \overline{z})} \Rightarrow$$
$$(g_0 + 2z + \overline{z})w + (g_0 - \overline{z})\overline{w} = 2(z + \overline{z})\alpha_{g_0} w\,. \tag{3.11}$$

We observe the following conjugated equation too:

$$(\overline{g} - z)w + (\overline{g}_0 + 2\overline{z} + z)\overline{w} = 2(z + \overline{z})\overline{\alpha_{g_0} w}\,. \tag{3.12}$$

By solving systems (3.11) and (3.12), the following is obtained:

$$w(z, \overline{z}) = \frac{\alpha_{g_0} w(z + 2\overline{z} + \overline{g}_0) - \overline{\alpha_{g_0} w}\,(g_0 - \overline{z})}{z + \overline{z} + g_0 + \overline{g}_0}\,. \tag{3.13}$$

As the general solution of the equation (3.10) must contain an arbitrary analytical function $\varphi(z)$, by using (3.13) we suppose that it is in the following form:

$$w(z, \overline{z}) = \frac{\varphi(z)(z + 2\overline{z} + \overline{g}_0) + \overline{\varphi(z)}(\overline{z} - g_0)}{z + \overline{z} + g_0 + \overline{g}_0} \tag{3.14}$$

which means that the unknown arbitrary function $\alpha_{g_0} w$ is included in the arbitrary analytical function $\varphi(z)$. As relation (3.14) satisfies equation (3.8), it is its general solution.

4. Conclusion

The problem of studying complex difference equations is particularly similar to the problem of real difference equations. The following analogies can be mentioned:

a) Instead of a sequence of points $x_0, x_1, \ldots, x_n$ on the x-axis, there are the contours L_0: $\overline{z} = g_0(z)$, L_1: $\overline{z} = g_1(z), \ldots$, L_n: $\overline{z} = g_n(z)$;

b) Instead of variable x, there is variable $\overline{z}$;

c) The rule of arbitrary constant is substituted by the arbitrary analytical function $\varphi(z)$;

d) Instead of a continuous real function $y = f(x)$, there is a continuous complex function $w(z, \overline{z})$;

e) The general solution of a complex difference equation of the n-th order contents n arbitrary analytical functions.

There is also a similarity between linear complex difference and differential equations. The general solution of the polydifference equation of $(n+1)$-order is a polynomial of n-th order on $\overline{z}$ with analytical coefficients (see [1]). The process of solving a linear difference equation of the n-th order is reduced to an algebraic equation of n-th order. The general solution contains n linearly independent particular solutions.

In the process of solving nonlinear complex difference equations, there are some interesting particular remarks. If the general solution of Riccati's differential equations is not possible to be solved in finite, closed form, then the corresponding difference equations are possible to be solved. Similar conclusion can be drawn for the differential Bicadze equation. In that case, operator $\alpha_g w(z,\overline{z})$ has an important role. It makes it possible to include the unknown analytical function $\alpha_g w(z,\overline{z})$ into an arbitrary analytical function $\varphi(z)$, which makes the process of solving simpler.

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ЗА НЕКОИ НЕЛИНЕАРНИ КОМПЛЕКНИ ДИФЕРЕНЦНИ РАВЕНКИ

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Р е з и м е

Во овој труд се користат диференции за решавање на линеарни и нелинеарни ψ-диференцни равенки.

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ABOUT FINAL GROUPS OF TRANSFORMATIONS OF THE NONLINEAR DIFFERENTIAL EQUATION OF THE SECOND ORDER

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Abstract

In the given work, the final groups of transformations of the equations of a type

$$y'' = \alpha x^n y^m y'^l \tag{1}$$

are considered allowing to divide a class of the equations (1) on not crossing final subclasses, inside which the solution of one representative of the given subclass guarantees to us the solutions of all other representatives of the same subclass with the help of known transformations.

The equation (1) is generalized homogeneous, therefore its order can easily be decreased by one by transformation:

$$x = e^t, \quad y = u\, e^{\frac{n-l+2}{m+l-1}t}; \quad u' = p \quad (u'' = pp').$$

After replacement

$$\xi = \frac{2n+m-l+3}{m+l-1}\, u$$

we receive the equation

$$\begin{aligned} pp' - p = &- \frac{(n-l+2)(m+n+1)}{(2n+m-l+3)^2}\,\xi \\ &+ A\xi^m \left(p - \frac{n-l+2}{2n+m-l+3}\,\xi\right)^l. \end{aligned} \tag{2}$$

In a case $2n+m-l+3 \neq 0$, $m+l-1 \neq 0$ is generalization of the equation Abel of 2nd order

$$pp' - p = R(\xi).$$

Let's consider formal transformations of the initial equation (1), taking in to account, that in each concrete case the solution should be checked up. Symbolically, the initial equation will be written as (m,n,l).

1. Let y be independent variable, than we have

$$x'' = -ay^m x^n x'^{3-l}$$

thus we receive transition $(m,n,l) \to (n,m,3-l)$. Let's designate this transformation by the letter r.

2. We shall transform the equation of a type (1) with parameters (μ,ν,λ) as follows. We multiply both parts of the equation on $y^{-\mu}y'^{-\lambda}$, raise to the power both parts of the equation in a degree $\frac{1}{\nu}$ and than differentiate:

$$yy'y^m - \lambda yy''^2 - \mu y'^2 y'' = \nu a^{\frac{1}{\nu}} y^{\frac{\mu+\nu}{\nu}} y'^{\frac{\lambda+\nu}{\nu}} y''^{\frac{\nu-1}{\nu}}.$$

After transformation

$$y' = z, \quad y = e^t, \quad z = w\, e^{\frac{\mu+\nu+1}{\nu-\lambda+2}t}, \quad w' = q, q = pw^{\lambda-1}$$

replacing

$$\xi = \frac{\mu\lambda + 2\nu\lambda + \mu\nu - 2\mu + \lambda - 3\nu - 2}{(\lambda-2)(\lambda-\nu-2)} w^{2-\lambda}$$

we receive

$$pp' - p = -\frac{\nu(\mu+\lambda-1)(\mu+\nu+1)(\lambda-2)}{(\mu\lambda+2\nu\lambda+\mu\nu-2\mu-3\nu-2)^2}\xi \tag{3}$$

$$+ A\xi^{\frac{1}{\lambda-2}}\left[p - \frac{(\lambda-2)(\mu+\nu+1)}{\mu\lambda+2\nu\lambda+\mu\nu-2\mu+\lambda-3\nu-2}\xi\right]^{\frac{\nu-1}{\nu}}.$$

We again have received the equation of a type (2). Let's require, that the factors and parameters of the equations (2) and (3) coincided, then we shall receive transition $(m,n,l) \to (\mu,\nu,\lambda)$. The solution of system of the algebraic equations gives

$$\mu = -\frac{n}{n+1}, \quad \nu = \frac{1}{1-l}, \quad \lambda = \frac{2m+1}{m}$$

i.e.

$$(m, nl) \to \left(-\frac{n}{n+1}, \frac{1}{1-l}, \frac{2m+1}{m}\right).$$

Designating this transformation by the letter f, and applying it repeatedly, we shall receive $\left(\frac{1}{l-2}, -\frac{m}{m+1}, \frac{n-1}{n}\right)$, we receive groups C_3.

3. Let in the initial equation $l = 0$:

$$y'' = ax^n y^m \qquad (m, n, 0). \tag{4}$$

In this special case (equation Emden-Fowler) transformation

$$x = x_1^{-1}, \quad y = \frac{y_1}{x_1}$$

gives

$$y_1'' = ax_1^{-m-n-3} y_1^m$$

i.e. $(m, n, 0) \leftrightarrow (m, -m-n-3, 0)$. Let's designate this transformation by the letter s. As $fs = sf$, the group represents $C_3 \times C_2$ i.e. isomorphic to C_5. We receive groups of transformations of the equation Emden-Fowler:

$$(m, n, 0) \to (m, -m-n-3, 0)$$

$$\to \left(-\frac{n}{n+1}, 1, \frac{2m+1}{m}\right) \to \left(-\frac{1}{2}, -\frac{m}{m+1}, \frac{m+n+4}{m+n+3}\right)$$

$$\to \left(-\frac{m+n+3}{m+n+2}, 1, \frac{2m+1}{m}\right) \to \left(-\frac{1}{2}, \frac{m}{m+1}, \frac{n-1}{n}\right).$$

4. We shall make in the initial equation (1) following transformations multiplying both parts of the equation on $x^{-n}y^{-m}y'^{l}$ and differentiate. Further, we put

$$x = e^t, \quad y = e^{\int u\,dt}, \quad u' = q, \quad q = pu^l$$

and receive

$$pp' - (m+2l-3)u^{1-l}p - (n-2l+3)u^{-1}p =$$

$$= -(1-l-m)u^{3-2l} - (m+2l-n-3)u^{2-2l} - (2-l+n)u^{1-2l} .$$

Accordingly, from (μ, ν, λ) we shall receive

$$pp' - (\mu+2\lambda-3)u^{1-\lambda}p - (\nu-2\lambda+3)u^{-\lambda}p =$$

$$= -(1-\lambda-\mu)u^{3-2\lambda} - (\mu+2\lambda-\nu-3)u^{2-2\lambda} - (2-\lambda+\nu)u^{1-2\lambda} .$$

In the first equation let's put

$$\xi = \frac{m+2l-3}{2-l}\,u^{2-l},$$

in second

$$\xi = \frac{\nu-2\lambda+3}{1-\lambda}\,u^{1-\lambda},$$

therefore the equation transform in

$$pp' - p = \xi^{\frac{1}{l-2}}\left[p - \frac{(n-2l-m+3)(l-2)}{(n-2l+3)(m+2l-3)}\,\xi\right] - \frac{(l+m-1)(l-2)}{(m+2l-3)^2}\,\xi - \frac{(l-n-2)(l-2)}{(n-2l+3)^2}\,\xi^{\frac{l}{l-2}} \tag{5}$$

$$pp' - p = \xi^{\frac{1}{1-\lambda}}\left[p - \frac{(\nu-2\lambda-\mu+3)(\lambda-1)}{(\nu-2\lambda+3)(\mu+2\lambda-3)}\,\xi\right] - \frac{(\lambda+\mu-1)(\lambda-1)}{(\mu+2\lambda-3)^2}\,\xi^{\frac{3-\lambda}{1-\lambda}} - \frac{(\lambda-\nu-2)(\lambda-1)}{(\nu-2\lambda+3)^2}\,\xi . \tag{6}$$

Equating of the appropriate factors and parameters of the equations (5) and (6) gives not trivial solution only at

$$l = \frac{m+2n+3}{m+n+2}$$

$$\left(m,\, n,\, \frac{m+2n+3}{m+n+2}\right) \leftrightarrow \left(-\frac{m}{m+n+1},\, -\frac{n}{m+n+1},\, \frac{2m+n+3}{m+n+2}\right).$$

The description of transformations can be applied for research of the equation Emden-Fowler and some other nonlinear equations of physics, which is type (1) or reduced to it, and also allowing to find the large number of the equations of a type (1) and the equations Abel of 2nd order, integrated in quadratures and through known special functions.

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ЗА КОНЕЧНИТЕ ГРУПИ ТРАНСФОРМАЦИИ НА НЕЛИНЕАРНИ ДИФЕРЕНЦИЈАЛНИ РАВЕНКИ ОД ВТОР РЕД

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Резиме

Во дадената работа, се разгледуваат конечни групи на трансформации на равенките од обликот

$$y'' = \alpha x^n y^m y'^l \tag{1}$$

коишто дозволуваат да класите равенки (1) се разбијат на поткласи, кои не се сечат, и во кои внатре решението претставено со една дадена поткласа гарантира решение на сите останати претставници на истата поткласа дадени со помош на познатите трансформации.

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