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VECTOR VALUED FIELDS

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Abstract. In this paper we introduce (n,m)-fields and fully commutative (n,m)-fields, as a continuation of the vector valued groups and fully commutative vector valued groups. Further we give some examples of the introduced structures.

1. Preliminaries

In this section we will recall the basic definitions and results which will be used in the next two sections.

For a positive integer i, G^i denotes the i-th Cartesian power of G. We will use the notation $x=a_1^i$ instead of $x=(x_1,\dots,a_i)$ for the elements of G^i. Since we will use also a_1^i for the i-th power of a_1, in order not to confuse $(a_1,\dots,a_1)$ with $(a_1,\dots,a_p)$, only a_1^p, with the exponent "p" or constant integer will denote a power. Let n, m and k be positive integers and let n=m+k. The following notions are defined in [1].

Definition 1.1. A map $[\ ]: G^n \to G^m$ is called an (n,m)-operation, and the pair $(G,[\ ])$ is called an (n,m)-groupoid. An (n,m)-groupoid $(G,[\ ])$ is called an (n,m)-semigroup if the operation $[\ ]$ is associative, i.e. for each $1 \le i \le k$ and each $x_1^{n+k}\in G^{n+k}$,

$$[x_1^i[x_{i+1}^{i+n}]x_{i+n+1}^{n+k}] = [x_1^n[x_{n+1}^{n+k}]]. \tag{1.1}$$

An (n,m)-semigroup $(G,[\ ])$ is called an (n,m)-group if for each $a\in G^k$ and $b\in G^m$, the equations

$$[ax] = b = [ya] \tag{1.2}$$

have solutions $x,y\in G^m$.

For example (see [2]), let $(G,*)$ be an arbitrary group. Then the pair $(G^2,[\ ])$ is (4,2)-group, where $[xyzt] = (x*z,y*t)$. We will use this in example 2 in section 3.

There are several definitions for the commutative analogs of the above notions. We will use the following conventions. Let $G^{(m)}$

be the m-th symmetric product of G, i.e. $G^{(m)} = G^m/\approx$ where $\approx$ is the equivalence on G^m defined by

$$x_1^m \approx y_1^m \iff x_1^m \text{ is a permutation of } y_1^m.$$

The following notions are defined in [4] and [5].

Definition 1.2. A map $[\ ]: G^{(n)} \to G^{(m)}$ is called fully commutative (n,m)-operation on G, and the pair (G,[ ]) is called fully commutative (n,m)-groupoid. A fully commutative (n,m)-groupoid is called fully commutative (n,m)-semigroup if the operation [ ] is associative, i.e. for each $1 \le i \le k$, and each $x_1^{n+k} \in G^{(n+k)}$, the identity (1.1) is satisfied. A fully commutative (n,m)-semigroup (G,[ ]) is called fully commutative (n,m)-group, if for each $a \in G^{(k)}$ and $b \in G^{(m)}$, the equation

$$[ax] = b \tag{1.3}$$

has solution $x \in G^{(m)}$.

Further we will use the initials "f.c." instead of "fully commutative". In [10] and [9] many examples of f.c. (n,m)-groups are given. All of them are built on the field of complex numbers $\mathbb{C}$, or a subset of $\mathbb{C}$. Here we give only one example which we will need later.

In [9] it is proven that there is a bijection ν between $\mathbb{C}^{(m)}$ and $\mathbb{C}^m$, defined by

$$\nu(z_1,\dots,z_m) = (a_1,\dots,a_m)$$

where $a_1 = \sum_{1\le i\le m} z_i$, $a_2 = \sum_{1\le i<j\le m} z_i z_j$, ... , $a_m = z_1 z_2 \dots z_m$.

Let us define a f.c. (n,m)-operation [ ] on $\mathbb{C}$, as follows

$$[z_1^n] = w_1^m \iff$$

$$\sum_{1\le i\le n} z_i = \sum_{1\le i\le m} w_i,\quad \sum_{1\le i<j\le n} z_i z_j = \sum_{1\le i<j\le m} w_i w_j,\ \dots,\ \sum z_{i_1}\dots z_{i_m} =$$

$$= w_1 w_2 \dots w_m.$$

Using the fact that $\mathbb{C}$ is algebraically closed field, one can verify that $(\mathbb{C},[\ ])$ is a f.c. (n,m)-group. It is called additive (n,m)-group, and is denoted by $(\mathbb{C},[\ ]_+)$. We notice that in [9] are studied more general classes of such f.c. groups called affine and projective (n,m)-groups, and also topological (n,m)-groups.

Let (G,[ ]) be a given f.c. (n,m)-semigroup, $n-m = k \geq 1$, and let q be the least non-negative integer, such that $m+q \equiv 0 \pmod{k}$, and let s be the least non-negative integer such that $k(s-1)<m \leq ks$ and $m+q = ks$ [5]. The following definition is given in [5], [9].

Definition 1.3. Define a binary operation $*$ on $G^{(m)}$ by:

$$a*b = [acb] \tag{1.4}$$

where $c \in G^{(q)}$ for $q \geq 1$; and c is empty symbol for $q = 0$, i.e. $a*b = [ab]$ for $q = 0$.

It is proved in [5], [9] that $(G^{(m)},*)$ is a semigroup. We say that $(G^{(m)},*)$ is a derived semigroup for (G,[ ]). In case $q \geq 1$, the operation $*$ depends on c, but any two derived groups of a f.c. (n,m)-group, are isomorphic [9]. Analogous definition of derived semigroup and results hold in the ordinary case [2], [3], [8], [6], i.e. not fully commutative case. Further in both cases, the induced binary operation "$*$" in the derived groups will be denoted by "+".

Let (G,[ ]) be a f.c. (n,m)-semigroup and let $G^{(+)} = \bigcup_{r\geq 1} G^{(r)}$. If $x \in G^{(r)}$, then we say that the dimension of x is r, i.e. dim(x)=r. We notice that the mapping $[\ ]: G^{(n)} \to G^{(m)}$ induces a mapping $[\ ]': G^{(+)} \to G^{(+)}$, and we define a relation $\doteq$ on $G^{(+)}$ as follows [5], [7]:

$u \doteq v$ iff there is $a \in G^{(+)}$ such that $[au]' = [av]'$.

Then $u \doteq v$ implies $\dim(u) \equiv \dim(v) \pmod{k}$, "$\doteq$" is a congruence on $G^{(+)}$ and the factor structure $G^{(+)}/\doteq$ is a commutative semigroup. The commutative semigroup $G^{(+)}/\doteq$ is called universal commutative semigroup for (G,[ ]) and it is denoted by $G^{(v)}$ ([5]). If (G,[ ]) is a f.c. (n,m)-group, then $G^{(v)}$ is commutative group, and $G^{(v)}$ is called universal commutative group for (G,[ ]) ([5], [7]).

In this paper we will introduce two classes of vector valued fields. The first is f.c. (n,m)-fields and the second is (n,m)-fields. The both definitions introduce new structures over a given field, like vector spaces. Further we give an example of f.c. (m+k,m)-field on $\mathbb{C}$, an example of (2m,m)-field and an example of $(2^{p+1},2^p)$-field on $\mathbb{C}$. Here the following two unsolved problems naturally arise:

1. Whether each f.c. (m+k,m)-field for $m \geq 2$, must be constructed over an algebraically closed field?

2. Whether there exist an (n,m)-field such that $m \nmid n$?

2. Introducing (n,m)-fields

Before we introduce (n,m)-fields, first we will prove the following proposition.

Proposition 2.1. The ordered triple $(\mathbb{C}^{(m)},+,\cdot)$, where the operations "+" and "·" are defined as follows

$$(a_1^m) + (b_1^m) = [a_1^m\ b_1^m]_+ \tag{2.1}$$

$$(a_1^m) \cdot (b_1^m) = [c_{11}\ c_{12}\ \dots\ c_{1m}\ \dots\ c_{mm}]_+, \quad (c_{ij}=a_i\cdot b_j), \tag{2.2}$$

satisfies the following conditions:

(i) $(\mathbb{C}^{(m)},+)$ is an Abelian group,

(ii) $(\mathbb{C}_*^{(m)},\cdot)$ is an Abelian group, where

$$\mathbb{C}_*^{(m)} = \{(z_1,\dots,z_m) \mid z_1^p + \dots + z_m^p \neq 0 \text{ for each } p\in\{1,\dots,m\}\},$$

(iii) The operation "·" is distributive with respect to the operation "+".

Proof. Since $(\mathbb{C},[\ ]_+)$ is a f.c. (2m,m)-group, the condition (i) is satisfied.

In order to prove (ii) and (iii), we will use the semigroup $\mathbb{C}^{(+)}$. Since the f.c. (n,m)-group $(\mathbb{C}^{(m)},[\ ]_+)$ is induced by the f.c. (m+1,m)-group $(\mathbb{C}^{(m)},[\ ]_+)$ (see [9]), we can temporary suppose that k=1. In this case (k=1) we notice that in $\mathbb{C}^{(+)}$ it holds

$$z_1^r \doteq w_1^s \iff \Sigma z_{i_1} z_{i_2} \dots z_{i_p} = \Sigma w_{i_1} w_{i_2} \dots w_{i_p} \text{ for } 1\leq p\leq m.$$

From $z_1 + \dots + z_r = w_1 + \dots + w_s$ and $\sum_{1\leq i<j\leq r} z_i z_j = \sum_{1\leq i<j\leq m} w_i w_j$, it

follows $\sum_{i=1}^{r} z_i^2 = \sum_{i=1}^{s} w_i^2$, and then from $\Sigma z_i z_j z_\ell = \Sigma w_i w_j w_\ell$ it follows $\sum_{i=1}^{r} z_i^3 = \sum_{i=1}^{s} w_i^3$ and so on. Moreover it holds

$$z_1^r \cong w_1^s \iff \sum_{i=1}^{r} z_i^p = \sum_{i=1}^{s} w_i^p \text{ for } 1 \le p \le m. \qquad (2.3)$$

Now we define mappings $+, \cdot : \mathbb{C}^{(+)} \times \mathbb{C}^{(+)} \to \mathbb{C}^{(m)}$ as follows

$$(a_1^r) + (b_1^s) = (c_1^m) \iff \sum_{i=1}^{r} a_i^p + \sum_{i=1}^{s} b_i^p = \sum_{i=1}^{m} c_i^p \text{ for } 1 \le p \le m \qquad (2.4)$$

and

$$(a_1^r) \cdot (b_1^s) = (c_1^m) \iff \sum_{i=1}^{r} \sum_{j=1} (a_i b_j)^p = \sum_{i=1}^{m} c_i^p \text{ for } 1 \le p \le m \qquad (2.5)$$

i.e.

$$(a_1^r) \cdot (b_1^s) = (c_1^m) \iff (\sum_{i=1}^{r} a_i^p)(\sum_{i=1}^{s} b_i^p) = \sum_{i=1}^{m} c_i^p \text{ for } 1 \le p \le m. \qquad (2.6)$$

Using (2.3), (2.4) and (2.6) it is easy to verify that if

$a_1^r \cong b_1^u$ and $c_1^s \cong d_1^v$, then

$$(a_1^r) + (c_1^s) \cong (b_1^u) + (d_1^v) \text{ and } (a_1^r) \cdot (c_1^s) \cong (b_1^u) \cdot (d_1^v).$$

Hence "+" and "$\cdot$" are mappings $+, \cdot : \mathbb{C}^{(v)} \times \mathbb{C}^{(v)} \to \mathbb{C}^{(m)}$, where $\mathbb{C}^{(v)}$ was defined by $\mathbb{C}^{(+)}/\cong$. It is easy to verify that their restrictions on $\mathbb{C}^{(m)} \times \mathbb{C}^{(m)}$ are given by (2.1) and (2.2).

Now let us return to the proofs of (ii) and (iii). Namely, using the identities (2.4) and (2.6), it is easy to verify that (iii) is satisfied and that the operation "$\cdot$" is commutative and associative. Besides, if a_1^m and b_1^m are such that $\sum_{i=1}^{m} a_i^p \neq 0$ and $\sum_{i=1}^{m} b_i^p \neq 0$, then $(a_1^m) \cdot (b_1^m) = (c_1^m)$ where $\sum_{i=1}^{m} c_i^p \neq 0$ for $1 \le p \le m$, and hence $(\mathbb{C}_*^{(m)}, \cdot)$ is Abelian semigroup. The element $e = (1, 0, \dots, 0) \in \mathbb{C}^{(m)}$ is neutral element and the equation

$$(a_1^m) \cdot (x_1^m) = e, \text{ i.e. } (\sum_{i=1}^{m} a_i^p)(\sum_{i=1}^{m} x_i^p) = 1 \text{ for } 1 \le p \le m$$

uniquely determines the sums $\sum_{i=1}^{m} x_i^p$ for $p \in \{1, \dots, m\}$, hence $(x_1^m) \in \mathbb{C}^{(m)}$ is uniquely determined, and (ii) is proved. ◊

From the proof of the proposition we notice that for each $p \in \{1, \dots, m\}$ the mapping $h_p : \mathbb{C}^{(+)} \to \mathbb{C}$ defined by

$$h_p(z_1^r) = \sum_{i=1}^{r} z_i^p \qquad (2.7)$$

induces homomorphisms h_p: $(\mathbb{C}^{(m)},\dotplus) \to (\mathbb{C},+)$ and h_p: $(\mathbb{C}_*^{(m)},\cdot) \to (\mathbb{C}\setminus\{0\},\cdot)$ of groups and also

$$z_1^r \doteq w_1^s \Rightarrow h_p(z_1^r) = h_p(w_1^s).$$

The invertible elements of the operation "$\cdot$" are all z_1^m such that $h_p(z_1^r) \neq 0$ for $p\in\{1,\dots,m\}$. We also notice that $(\mathbb{C}^{(m)},\dotplus)$ is the derived group for the f.c. (2m,m)-group $(\mathbb{C},[\ ]_+)$.

The previous proposition together with this discussion, leads us to the following definition.

Definition 2.1. Let $(F,+,\cdot)$ be a field, and $[\ ]$: $F^{(n)} \to F^{(m)}$ and $*$: $F^{(m)}\times F^{(m)} \to F^{(m)}$ be given maps. The ordered triple $(F,[\ ],*)$ is called f.c. (n,m)-field, if the following axioms are satisfied:

(i) $(F,[\ ])$ is a f.c. (n,m)-group;

(ii) The ordered triple $(F^{(m)},\dotplus,*)$ is a commutative ring with unit (note that $(F^{(m)},\dotplus)$ is the derived group of the f.c. (n,m)-group $(F,[\ ])$);

(iii) There exist m mappings h_i: $F^{(m)} \to F$ $(i\in\{1,\dots,m\})$ such that

$$h_i(a\dotplus b) = h_i(a) + h_i(b) \text{ and } h_i(a*b) = h_i(a)\cdot h_i(b),$$

i.e. h_i: $(F^{(m)},\dotplus,*) \to (F,+,\cdot)$ is a homomorphism of rings;

(iv) If $h_p(a) \neq 0$ for each $p\in\{1,\dots,m\}$, then $a\in F^{(m)}$ is an invertible element.

It is easy to verify from (ii), (iii) and (iv) that the set

$$F_*^{(m)} = \{a\in F^{(m)} \mid h_p(a) \neq 0 \text{ for } p\in\{1,\dots,m\}\}$$

is an Abelian group with respect to the operation $*$. We will call the mappings h_i projections. It is easy to verify that the f.c. (m+k,m)-field induces a f.c. (m+kt,m)-field. Finally we notice that each field $(F,+,\cdot)$ is a f.c. (2,1)-field. It is sufficient to put $[ab] = a+b$, $a*b = a\cdot b$ and $h_1(a) = a$.

Analogously to the definition 1, we will define now (ordinary) (n,m)-field.

Definition 2.2. Let $(F,+,\cdot)$ be a field, $[\ ]: F^n \to F^m$ and $*: F^m \times F^m \to F^m$. The ordered triple $(F,[\ ],*)$ is called (n,m)-field, if the following axioms are satisfied:

(i) $(F,[\ ])$ is an (n,m)-group;

(ii) The ordered triple $(F^m,+,*)$ is a commutative ring with unit (note that $(F^m,+)$ is the derived group of the (n,m)-group $(F,[\ ])$);

(iii) There exist m mappings $h_i: F^m \to F$ $(i\in\{1,\dots,m\})$ such that

$$h_i(a+b) = h_i(a) + h_i(b) \text{ and } h_i(a*b) = h_i(a)\cdot h_i(b),$$

i.e. $h_i: (F^m,+,*) \to (F,+,\cdot)$ is a homomorphism of rings;

(iv) If $h_i(a) \neq 0$ for each $p\in\{1,\dots,m\}$, then $a\in F^m$ is an invertible element.

For (n,m)-field the discussion which followed after the definition 2.1 also holds.

3. Examples

Example 1. Let $F = \mathbb{C}$, and let $(\mathbb{C},[\ ]_+)$ be the additive f.c. $(m+k,m)$-group. Since this f.c. group is induced by the f.c. $(m+1,m)$-group, we obtain that the derived group is $\mathbb{C}^{(m)}$ with the operation "+" defined by (2.1). We define an operation $*$ in $\mathbb{C}^{(m)}$ by (2.2), and projections $h_p: \mathbb{C}^{(m)} \to \mathbb{C}$ by

$$h_p(a_1^m) = \sum_{i=1}^{m} a_i^p \quad (1 \le p \le m). \tag{3.1}$$

Then $(\mathbb{C},[\ ]_+,*)$ is a f.c. $(m+k,m)$-field. The proof is analogous to the proof of proposition 2.1. Indeed that proposition asserts that $(\mathbb{C},[\ ]_+,*)$ is a f.c. $(2m,m)$-field for $k = m$.

Example 2. Now we give an example of $(2m,m)$-field. Let $(F,+,\cdot)$ be an arbitrary field, and let $(F,[\ ])$ be the $(2m,m)$-group defined by

$$[a_1^m b_1^m] = (a_1+b_1,\dots,a_m+b_m). \tag{3.2}$$

Then the derived group is F^m with the following operation

$$(a_1,\dots,a_m) + (b_1,\dots,b_m) = (a_1+b_1,\dots,a_m+b_m). \tag{3.3}$$

In F^m we define operation $*$ by

$$(a_1,\dots,a_m)*(b_1,\dots,b_m) = (a_1\cdot b_1,\dots,a_m\cdot b_m), \qquad (3.4)$$

and projections

$$h_p(a_1,\dots,a_m) = a_p \text{ for } p\in\{1,\dots,m\}.$$

Now it is easy to verify that $(F,[\ ],*)$ is $(2m,m)$-field.

Now we will find a connection between the f.c. $(2m,m)$-field from the example 1 (or the proposition 2.1.) and the $(2m,m)$-field from the example 2 for $F = \mathbb{C}$. Namely we will show that there is an isomorphism ϕ between their derived groups, their multiplicative groups with the operation $*$, and it preserves the projections.

Let us define mapping ϕ: $\mathbb{C}^{(m)} \to \mathbb{C}^m$ by

$$\phi(z_1^m) = (a_1,\dots,a_m),$$

where $a_p = \sum_{i=1}^{m} z_i^p$ for $1 \le p \le m$. Then ϕ is a bijection. Further let $\phi(w_1^m) = (b_1,\dots,b_m)$ such that $b_p = \sum_{i=1}^{m} w_i^p$ for $1 \le p \le m$. The m-tuples $(a_1,\dots,a_m)$ and $(b_1,\dots,b_m)$ in $\mathbb{C}^m$ are added and multiplied according to (3.3) and (3.4). Let $u_1^m, v_1^m \in \mathbb{C}^{(m)}$ are such that

$$u_1^p + \dots + u_m^p = a_p + b_p \qquad p\in\{1,\dots,m\} \text{ and}$$
$$v_1^p + \dots + v_m^p = a_p\cdot b_p \qquad p\in\{1,\dots,m\}.$$

Then according to (2.4) and (2.6) we obtain

$$z_1^m + w_1^m = u_1^m \text{ and } z_1^m * w_1^m = v_1^m,$$

and hence

$$\phi(z_1^m + w_1^m) = \phi(u_1^m) = (a_1+b_1,\dots,a_m+b_m) =$$
$$= (a_1,\dots,a_m) + (b_1,\dots,b_m) = \phi(z_1^m) + \phi(w_1^m),$$

and

$$\phi(z_1^m * w_1^m) = \phi(v_1^m) = (a_1\cdot b_1,\dots,a_m\cdot b_m) =$$
$$= (a_1,\dots,a_m)\cdot(b_1,\dots,b_m) = \phi(z_1^m)\cdot\phi(w_1^m).$$

Hence ϕ is an isomorphism between their derived groups and their multiplicative groups with the operation $*$. Moreover, ϕ preserves the projections, because

$$h_p(\phi(z_1^m)) = h_p((a_1,\dots,a_m)) = a_p = \sum_{i=1}^{m} z_i^p = h_p(z_1^m), \text{ i.e. } h_p\circ\phi = h_p.$$

<u>Example 3</u>. We define (4,2)-group on $\mathbb{C}$ by

$$[z_1 z_2 z_3 z_4] = (z_1+z_3, z_2+z_4) \qquad z_1, z_2, z_3, z_4 \in \mathbb{C}.$$

Then the derived group is $\mathbb{C}^2$ with the operation

$$(z_1, z_2) + (z_3, z_4) = (z_1+z_3, z_2+z_4),$$

and we define a multiplicative operation "$*$" in $\mathbb{C}^2$ by

$$(z_1, z_2)*(z_3, z_4) = (z_1 z_3 - z_2 z_4, z_1 z_4 + z_2 z_3).$$

Obviously the multiplicatively operation is associative with unit (1,0) and that the distributive law for multiplication with respect to the addition is satisfied. Further we define mappings $h_1, h_2: \mathbb{C}^2 \to \mathbb{C}$ by $h_1(z_1, z_2) = z_1 + iz_2$ and $h_2(z_1, z_2) = z_1 - iz_2$. Now one can verify that the axiom (iii) from the definition 2.2 is satisfied. Further

$$F_*^2 = \{(z_1, z_2) \mid h_1(z_1, z_2) \neq 0 \text{ or } h_2(z_1, z_2) \neq 0\} =$$

$$= \{(a+ib, c+id) \mid a=-d,\ b=c\} \cup \{(a+ib, c+id) \mid a=d,\ b=-c\}.$$

One can verify that $(z_1, z_2) \in \mathbb{C}^2$ is an invertible element iff $(z_1, z_2) \notin F_*^2$. Indeed,

$$(z_1, z_2)*(w_1, w_2) = (1,0)$$

$$(a+ib, c+id)\cdot(a'+ib', c'+id') = (1,0)$$

yields to a system of linear equations of a',b',c' and d' whose main determinant is

$$\begin{vmatrix} a & -b & -c & d \\ b & a & -d & -c \\ c & -d & a & -b \\ d & c & b & a \end{vmatrix} = [(a+d)^2+(b-c)^2][(a-d)^2+(b+c)^2].$$

Thus the element $(z_1, z_2) \in \mathbb{C}^2$ is an invertible iff $(z_1, z_2) \notin F_*^2$ and hence $(\mathbb{C}, [\ ], *)$ is (4,2)-field.

The pair $(a+ib, c+id) \in \mathbb{C}^2$ can be written as $(a+ib)+j(c+id)$. Then the multiplication in $\mathbb{C}^2$ satisfies the identities $i^2=j^2=-1$ and $i\cdot j=j\cdot i$. The homomorphisms h_1 and h_2 indeed change j by i and -i respectively.

The above example can be generalized for $(2^{p+1}, 2^p)$-field on $\mathbb{C}$. For example for p=2 the operation [ ] is defined analogously to (3.2) for m=4. The derived group consists of

$$(z_1+jz_2) + k(z_3+jz_4), \quad z_1, z_2, z_3, z_4 \in \mathbb{C}.$$

The multiplication is induced by the following identities

$$i^2 = j^2 = k^2 = -1 \text{ and } ij = ji, \ ik = ki, \ jk = kj.$$

The homomorphisms h_1, h_2, h_3 and h_4 are uniquely determined by the following identities:

$$h_1(j) = i, \quad h_1(k) = i,$$
$$h_2(j) = i, \quad h_2(k) = -i,$$
$$h_3(j) = -i, \quad h_3(k) = i,$$
$$h_4(j) = -i, \quad h_4(k) = -i$$

and $h_i(z) = z$ if $z \in \mathbb{C}$.

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ВЕКТОРСКО ВРЕДНОСНИ ПОЛИЊА

Костадин Тренчевски

Р е з и м е

Во овој труд се воведени поимите за (n,m)-поле и потполно комутативно (n,m)-поле, како продолжение на теоријата на векторско вредносни групи и потполно комутативни векторско вредносни групи. Потоа се дадени неколку примери од воведените поими.

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