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FREE GROUPOIDS WITH $(xy)^2 = x^2y^2$

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A b s t r a c t: We investigate free objects in the variety $\mathcal{V}$ of groupoids which satisfy the law $(xy)^2 = x^2y^2$ [1)]. The main results and necessary preliminary definitions are stated in Introduction. Corresponding generalizations, concerning the law $(xy)^n = x^ny^n$, are considered in the last part of the paper.

0. Introduction

First we state some necessary preliminaries.

Let $\mathbf{G} = (G, \cdot)$ be a **groupoid**, i.e. an algebra with a binary operation: $(x, y) \mapsto xy$ on G. If $a, b, c \in G$ are such that $a = bc$, then we say that b and c are **divisors** of a in $\mathbf{G}$. A sequence $a_1, a_2, \ldots$ of elements of G is a **divisor chain** in $\mathbf{G}$ if a_{i+1} is a divisor of a_i. We say that $a \in G$ is a **prime** in $\mathbf{G}$ if the set of divisors of a in $\mathbf{G}$ is empty. Thus, primes in $\mathbf{G}$ can be only the last members of divisor chains in $\mathbf{G}$.

Throughout the paper we will always write "a free groupoid" instead of "a free groupoid in the variety of all groupoids". It will be denoted by $\mathbf{F} = (F, \cdot)$, and its basis by B. (We write $\mathbf{F} = F(B)$ when it is necessary to emphasize the basis B.) It is well known (see, for example, [1], I.1) that the following properties characterize $\mathbf{F}$:

a) $ab = cd \Rightarrow a = c,\ b = d$, i.e. the mapping $(a, b) \mapsto ab$ is injective.

b) Every divisor chain in $\mathbf{F}$ is finite.

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[1] As usual: $x^2 = xx$.