node b iff b is a divisor of a. From the given conditions it follows that every node of the graph is a "source" of finitely many edges and that every (directed) path in the graph is finite. Then, by König's Lemma (for example, [3,4]), one obtains that the set of path lengths, of which the origin is a given node, is bounded. $\square$

To complete the proof of *Th.* 2 we have to show the following

Proposition 2.3. If a $\mathcal{V}$–groupoid $\mathbf{H}$ satisfies the conditions (i)–(iv), then $\mathbf{H}$ is $\mathcal{V}$–free, and the set B of primes in $\mathbf{H}$ is the basis of $\mathbf{H}$.

Proof. First, (i) implies that the set B of primes in $\mathbf{H}$ is nonempty. By (ii), (iii) and (iv), for each $a \in H$, div(a) consists of at most 3 elements; thus the conclusion of *L.* 2.2 holds. By induction on $L(a)$ we obtain that B is the least generating subset of $\mathbf{H}$.

Let $\mathbf{G} = (G, \cdot) \in \mathcal{V}$, and $\lambda: B \to G$ be an arbitrary mapping. Again by induction on $L(a)$ we will show that there is a (unique) homomorphism $\varphi: \mathbf{H} \to \mathbf{G}$ which is an extension of λ. First we put $\varphi(b) = \lambda(b)$ if $b \in B$. Assume that, for any $x \in H$ such that $L(x) \leq k$, $\varphi(x) \in G$ is well defined, and if $x = uv$, then $\varphi(x) = \varphi(u)\,\varphi(v)$. Let $t \in H$ be such that $L(t) = k+1$. Then t is a product, $t = uv$, where $L(u)$, $L(v) \leq k$; and, there exist at most two distinct such pairs. Then we can put $\varphi(t) = \varphi(u)\,\varphi(v)$. If $t = x^2 = yz$, where $y \neq z$, then, by (iv), there exist u, v such that: $x = uv$, $y = u^2$, $z = v^2$, and thus:

$$\begin{aligned}\varphi(x^2) &= \varphi(x)\,\varphi(x) = \big(\varphi(u)\,\varphi(v)\big)^2 = \varphi(u)^2\,\varphi(v)^2 = \varphi(u^2)\,\varphi(v^2) = \\ &= \varphi(y)\,\varphi(z).\end{aligned}$$

Thus $\varphi(t) \in G$ is well defined. Moreover, we have $\varphi(uv) = \varphi(u)\,\varphi(v)$, for each u, v such that $L(uv) \leq k+1$. So, there exits a homomophism $\varphi: \mathbf{H} \to \mathbf{G}$ which is an extension of λ. $\square$

The additive groupoid of positive integers belongs to $\mathcal{V}$, and this implies:

Proposition 2.4. If $\mathbf{H}$ is a $\mathcal{V}$–free groupoid with the basic B, then there exists a (unique) mapping $x \mapsto |x|$ from $\mathbf{H}$ into the set of positive integers such that:

$$|b| = 1, \qquad |xy| = |x| + |y|\,, \tag{2.2}$$

for each $b \in B$, x, $y \in H$. (We say that $|x|$ is the **norm** of x.) $\square$

Now we will show the following:

Proposition 2.5. Every $\mathcal{V}$–free groupoid $\mathbf{H}$ is a cancellative groupoid.

Proof. First we will show that:

$$x \neq y \Rightarrow x^2 \neq xy\,, x^2 \neq yx\,. \tag{2.3}$$

Namely, (2.3) is clear if x, $y \in B$. Assume that x, $y \in H$ are such that $x \neq y$, $x^2 = xy$ and $|x|$ is the least possible. Then, by (iv), there exist u, $v \in H$ such that $x = uv$, $x = u^2$, $y = v^2$, $u \neq v$. Therefore $u^2 = uv$, $u \neq v$ and $|u| < |x|$.