By symmetry, $x \neq y \Rightarrow x^2 \neq yx$.

Let $xy = xz$ (or $yx = zx$). If $x \neq y$, $x \neq z$, then by (iii): $y = z$. If $x = y$, then: $x^2 = xz \Rightarrow x = z$. Thus:

$$xy = xz \quad \text{or} \quad yx = zx \ \Rightarrow \ y = z\,, \tag{2.4}$$

i.e. **H** is cancellative. □

Below we assume that **H** is a given $\mathcal{V}$–free groupoid.

As a concequence of (i) we obtain:

Corollary 2.6. For every $k \geq 0$, the mapping $x \mapsto x^{(k)}$ is injective. □

As in (0.2), the equality $x = y^{(-k)}$ is equivalent with $y = x^{(k)}$, where $k \geq 0$. Thus, for every $x \in H$, there exists the largest nonnegative integer m such that $x^{(-m)} \in H$; it will be denoted by $[x]$. Therefore, we have a mapping $x \mapsto [x]$ from H into the set of nonnegative integers. It can be easily seen that if we replace F by H in (1.2)–(1.5) we obtain relations which hold in a $\mathcal{V}$–free groupoid **H**. Moreover, we obtain the following property which is an "extension" of *L.* 1.2).

Proposition 2.7. If $x, y \in H$ and m are such that $[x]+m \geq 0$, $[y]+m \geq 0$, then $[xy] + m \geq 0$ and:

$$(xy)^{(m)} = x^{(m)}y^{(m)}. \quad \square$$

We note that:

$$x, y \in H \ \Rightarrow \ [xy] = \min\{[x], [y]\}. \tag{2.5}$$

Remark. *Th.* 2 could be stated in a weaker form, i.e. without the assumption $\mathbf{H} \in \mathcal{V}$, replacing (iv) by:

(iva) $x^2 = yz,\ y \neq z \Leftrightarrow (\exists u, v)\ \ x = uv,\ y = u^2,\ z = v^2,\ u \neq v.$

3. Subgroupoids of $\mathcal{V}$-free groupoids

Below we assume that **H** is a given $\mathcal{V}$–free groupoid with the basis B, and **Q** is a subroupoid of **H** with the carrier Q.

From *Th.* 2 one obtains:

Proposition 3.1. $\mathbf{Q} \in \mathcal{V}$ and it satisfies the conditions (i), (ii) and (iii) in *Th.* 2. □

Proposition 3.2. The set of primes in **Q** is nonempty and it is the least generating subset of **Q**. □