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ONE PROOF FOR THE ANALYTIC REPRESENTATION OF DISTRIBUTIONS

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In this article is given a proof for the analytic representation of distributions with the aid of Fourier transform.

An important operation with the distributions is their analytic representation. Namely, if $T \in D'$ is a distribution then there is a function $f(z)$ analytic in the domain $C \backslash \operatorname{supp} T$, $\operatorname{supp} p$ (support) of the distribution, C is the complexe plane, suh that the regular distributions $f(x+i\varepsilon) - f(x-i\varepsilon)$ converges to T in the space D' as $\varepsilon \to 0^*$. Thus

$$\lim_{x \to 0^*} \int\limits_{-\infty}^{\infty} [f(x+i\varepsilon) - f(x-i\varepsilon)]\varphi(x)dx = T(\varphi) =< T, \varphi >, \varphi \in D. \quad (1)$$

A complete proof is found in ((1). P.76).

Note that the analytic representation $f(z)$ of a distribution T is not uniqe. In fact if $H(z)$ is an entire function then the function $f(z) + H(z)$ is also an analytyc representation for T, because every entire function is an analytic representation of the zero distribution. Further on, if $f(z)$ is an analytic representation for T, then $f(z)$ is an analytic representation for the derivative T' and in general $f^{(m)}(z)$ is an analytic representation, for the m-th derivative $T^{(m)}$ of the distribution T.

If the distribution T has a compact support then the function

$$\hat{T}(z) = \frac{1}{2\pi i}\langle T_l, \ \frac{1}{t-z}\rangle \quad \operatorname{Im}(z) \neq 0 \quad (2)$$

is analytic on $C \setminus \operatorname{supp} T$ and is analytic representation for T.

The function $\hat{T}(z)$ is also called Cauchy representation for T.

For the derivative $T^{(m)}$ the analytic representation is the function

$$\hat{T}^{(m)}(z) = \frac{m!}{2\pi i}\langle T_l, \frac{1}{(t-z)^{m+1}}\rangle \quad \operatorname{Im}(z) \neq 0. \tag{3}$$

In order to represent as many distributions as possible with the Cauchy integral, H. Bremermann in (1) introduced the spaces O_α and dual spaces O'_α, α is a real number. Let the distribution $T \in O'_\alpha$, $\alpha \geq -1$ then the Cauchy integral

$$\hat{T}(z) = \frac{1}{2\pi i}\langle T_l, \frac{1}{t-z}\rangle \quad \operatorname{Im}(z) \neq 0$$

is an analytic representation for T, and it is analytic on the domain $C \setminus \operatorname{supp} T$. A complete proof is found (1.P.73). A proof from the viwpoint of the distributional Plemel relations is found in (2).

Here we give a new proof using the Fourier transform.

Theorem 1. *Let the distribution $T \in O'_\alpha$, $\alpha \geq -1$ then function*

$$\hat{T}(z) = \langle T_l, \frac{1}{t-z}\rangle \quad \operatorname{Im}(z) \neq 0 \tag{4}$$

is analytic representation for T.

Proof.

$$\frac{1}{2\pi i}\int_{-\infty}^{\infty}\left[\hat{T}(x+i\varepsilon) - \hat{T}(x-i\varepsilon)\right]\varphi(x)dx =$$

$$= \frac{1}{2\pi i}\int_{-\infty}^{\infty}\left[\langle T_t, \frac{1}{t-x-i\varepsilon}\rangle - \langle T_t, \frac{1}{t-x+i\varepsilon}\rangle\right]\varphi(x)dx$$

for all $\varphi \in D$.

Now we consider the integrals

$$J_1 = \frac{1}{2\pi i}\int_{-\infty}^{\infty}\langle T_t, \frac{1}{t-x-i\varepsilon}\rangle\varphi(x)dx, \quad J_2 = \frac{1}{2\pi i}\int_{-\infty}^{\infty}\langle T_t, \frac{1}{t-x+i\varepsilon}\rangle\varphi(x)dx.$$

Let us consider the integral J_1. J_1 can be approximate by Riemann sums:

$$J_1 = \lim_{N\to\infty}\sum_{v=1}^{N}\langle T_t, \frac{\varphi(x_v)\Delta x_v}{t-x_v-i\varepsilon}\rangle = \lim_{N\to\infty}\langle T_t, \sum_{v=1}^{N}\frac{\varphi(x_n)\Delta x_v}{t-x_v-i\varepsilon}\rangle,$$

$\varphi \in D$ and let $a > 0$ be such that $\operatorname{supp}\varphi \subset [-a, a]$. The functions

$$\psi_N(t) = \sum_{v=1}^{N} \frac{\psi(x_v)\Delta x_v}{t - x_v - i\varepsilon} \quad \text{for fixed} \quad \varepsilon > 0 \quad \text{are in} \quad O_{-1},$$

$$\lim_{N\to\infty} \psi_N(t) = \int_{-\infty}^{\infty} \frac{\varphi(x)dx}{t - x - i\varepsilon} = \psi_\varepsilon(t) \in O_{-1}.$$

In fact the sequance of functions $\psi_N(t)$ as $N \to \infty$ converge to the function $\psi_\varepsilon(t)$ in the space O_{-1}.

We present brief proof for this:

(a) $|\psi_N(t)| \le \sum_{v=1}^{N} \frac{|\psi(x_v)|\Delta x_v}{|t - x_v - i\varepsilon|}$. $x_v \in [-a, a]$ thas

$\le \frac{\|\psi\|}{\varepsilon} \cdot 2a$, $\|\varphi\| = \sup\limits_{x \in \sup p\varphi} |\varphi(x)|$

(b) $|\psi_N(t^m) - \psi_N(i)] \le \frac{\|\psi\| 2a}{\varepsilon^2}$.

Hence the sequence of functions $\psi_N(t)$ is uniformly bounded and equicontinuous, therefore by using the theorem Arcela-Ascoli, the convergence is uniform on the compact subsets. In the same way on prove for the derivative $\psi_N^{(k)}(t)$, and e.t.c. Therefore we get

$$J_1 = \langle T_t, \frac{1}{2\pi i} \lim_{N\to\infty} \psi_N(t)\rangle = \langle T_t, \frac{1}{2\pi i} \int_{-\infty}^{\infty} \frac{\varphi(x)dx}{t - x - i\varepsilon}\rangle, \quad \varphi \in D.$$

$$\frac{1}{2\pi i} \int_{-\infty}^{\infty} \frac{\varphi(x)dx}{t - x - i\varepsilon} = -\frac{1}{2\pi i} \int_{-\infty}^{\infty} \frac{\varphi(x)dx}{x - t + i\varepsilon}.$$

By using the formula

$$-\frac{1}{2\pi i(x - t - i\varepsilon)} = F^{-1}\big(H(-\omega)e^{i\omega(t - i\varepsilon)}, x\big),$$

where F^{-1} is the inverse Fourier transform, H is the Heaviside function, we obtain

$$\psi_\varepsilon(t) = \int_{-\infty}^{\infty} \varphi(x)F^{-1}\big(H(-\omega)e^{i\omega t}e^{\varepsilon\omega}, x\big)dx = \int_{-\infty}^{\infty} F^{-1}(\varphi, \omega)H(-\omega)e^{i\omega t}e^{\varepsilon\omega}d\omega.$$

By replacing J_1 we get

$$J_1 = \langle T_t, \int_{-\infty}^{\infty} F^{-1}(\varphi,\omega)H(-\omega)e^{i\omega t}e^{\varepsilon\omega}d\omega\rangle$$

In similar way on obtain

$$J_2 = \langle T_t, -\int_{-\infty}^{\infty} F^{-1}(\varphi,\omega)H(\omega)e^{i\omega t}e^{\varepsilon\omega}d\omega\rangle.$$

By substracting we have

$$J_1 - J_2 = \langle T_t, \int_{-\infty}^{\infty} F^{-1}(\varphi,\omega)e^{i\omega t}e^{-\varepsilon|\omega|}d\omega\rangle\,.$$

The function

$$h_\varepsilon(t) = \int_{-\infty}^{\infty} F^{-1}(\varphi,\omega)e^{i\omega t}e^{-\varepsilon|\omega|}d\omega \in O_{-1}$$

and converges to the functions

$$\int_{-\infty}^{\infty} F^{-1}(\varphi,\omega)e^{i\omega t}d\omega = \varphi(t) \quad \text{in} \quad O_{-1}$$

Finally we get

$$\lim_{\varepsilon\to 0^+}\int_{-\infty}^{\infty}\left[\hat{T}(x+i\varepsilon) - \hat{T}(x-i\varepsilon)\varphi(x)\right]dx = \lim_{\varepsilon\to 0}(J_1 - J_2) = \langle T,\varphi\rangle\,.$$

II. General case for the space O_α

If $\alpha < -1, T \in O_\alpha$ and the number k is so that $\dfrac{1}{(t-x-i\varepsilon)^{k+1}} \in O_\alpha$, then we have

Theorem 2. *The function defined by*

$$\hat{S}(z) = \frac{k!}{2\pi i}\langle T_t, \frac{1}{(t-z)^{k+1}}\rangle, \quad \mathrm{Im}\,(z)k \neq 0$$

is an analytic representation for the derivative $T^{(k)}$.

Proof. As in the proof of theorem 1 we consider the integrals:

$$J_{1,k} = \frac{k!}{2\pi i} \int_{-\infty}^{\infty} \langle T_t, \frac{1}{(t-x+i\varepsilon)^{k+1}} \rangle \varphi(x) dx$$

$$J_{2,k} = \frac{k!}{2\pi i} \langle T_t, \frac{1}{(t-x+i\varepsilon)^{k+1}} \rangle \varphi(x) dx$$

$$J_{1,k} = \lim_{N\to\infty} \frac{k!}{2\pi i} \sum_{v=1}^{N} T_t, \langle \frac{\varphi(x_v)\Delta x_v}{(t-x_v-i\varepsilon)^{k+1}} \rangle$$

$$= \lim_{N\to\infty} \frac{k!}{2\pi i} \langle T_t, \sum_{v=1}^{N} \frac{\varphi(x_v)\Delta x_v}{(t-x-i\varepsilon)^{k+1}} \rangle$$

The function $\psi_N(t) = \sum_{v=1}^{N} \frac{\varphi(x_v)\Delta x_v}{(t-x-i\varepsilon)^{k+1}} \in O_\alpha$ converges in O_α as $N \to \infty$ to the function: $\psi_{\varepsilon,k}(t) = \frac{k!}{2\pi i} \int_{-\infty}^{\infty} \frac{\varphi(x)dx}{(t-x-i\varepsilon)^{k+1}}$. The proof is the same as in the teorem 1. Thus

$$J_{1,k} = \langle T_t, \frac{k!}{2\pi i} \int_{-\infty}^{\infty} \frac{\varphi(x)dx}{(t-x-i\varepsilon)^{k+1}} \rangle \quad J_{2,k} = \langle T_t, \frac{k!}{2\pi i} \int_{-\infty}^{\infty} \frac{\varphi(x)dx}{(t-x-i\varepsilon)^{k+1}} \rangle$$

$$\int_{-\infty}^{\infty} \frac{\varphi(x)dx}{(t-x-i\varepsilon)^{k+1}} = (-1)^{k+1} \int_{-\infty}^{\infty} \frac{\varphi(x)dx}{(x-t+i\varepsilon)^{k+1}}$$

$$\int_{-\infty}^{\infty} \frac{\varphi(x)dx}{(t-x-i\varepsilon)^{k+1}} (-1)^{k+1} \int_{-\infty}^{\infty} \frac{\varphi(x)dx}{(x-t+i\varepsilon)^{k+1}}.$$

By using formulas

$$\begin{cases} \dfrac{1}{(x-t+i\varepsilon)^{k+1}} = -F^{-1}\left(2\pi i^{k+1} \dfrac{\omega^k}{k!} e^{i\omega t} e^{\varepsilon\omega} H(-\omega), x\right) \\ \dfrac{1}{(x-t+i\varepsilon)^{k+1}} = F^{-1}\left(2\pi i^{k+1} \dfrac{\omega^k}{k!} e^{i\omega t} e^{-\varepsilon\omega} H(\omega), x\right) \end{cases} \qquad (*)$$

and the Parseval relation, on obtain

$$\psi_{\varepsilon,k}(t) = -(-1)^{k+!} \int_{-\infty}^{\infty} F^{-1}(\varphi,\omega)(i\omega)^k e^{i\omega t+\varepsilon\omega} H(-\omega)d\omega$$

thus

$$J_{1,k} = \langle T_t, (-1)^k \int_{-\infty}^{\infty} F^{-1}(\varphi,\omega)(i\omega)^k e^{i\omega t+\varepsilon\omega} H(-\omega)d\omega\rangle\,.$$

Completly similary

$$J_{2,k} = \langle T_t, (-1)^{k+1} \int_{-\infty}^{\infty} F^{-1}(\varphi,\omega)(i\omega)^k e^{i\omega t-\varepsilon\omega} H(\omega)d\omega\rangle\,.$$

Hence by substration we get

$$J_{1,k} - J_{2,k} = \langle T_t, (-1)^k \int_{-\infty}^{\infty} F^{-1}(\varphi,\omega)(i\omega)^k e^{i\omega t-\varepsilon\omega} d\omega\rangle\,.$$

As in Theorem 1 the functions

$$h_{\varepsilon,k}(t) = (-1)^k \int_{-\infty}^{\infty} F^{-1}(\varphi,\omega)(i\omega)^k e^{i\omega t-\varepsilon\omega} d\omega \in O_\alpha$$

and converge to the function $(-1)^k \int_{-\infty}^{\infty} F^{-1}(\varphi,\omega)(i\omega)^k e^{i\omega t} d\omega$ in these space O_α.

By passing in the limmes on obtain

$$\lim_{\varepsilon\to 0} \int_{-\infty}^{\infty} \left[\hat{S}(x+i\varepsilon) - \hat{S}(x-i\varepsilon)\right]\varphi(x)dx \lim_{\varepsilon\to 0}(J_{1,k} - -J_{2,k}) =$$

$$= \langle T_1, (-1)^k \lim_{\varepsilon\to 0} \int_{-\infty}^{\infty} F^{-1}(\varphi,\omega)(i\omega)^k e^{i\omega t-\varepsilon|\omega|} d\omega\rangle$$

$$= \langle T_t, (-1)^k \int_{-\infty}^{\infty} F^{-1}(\varphi,\omega)(i\omega)(i\omega)^k e^{i\omega t} d\omega\rangle$$

$$\langle T_t, (-1)^k \frac{d^k}{dt^k} \int_{-\infty}^{\infty} F^{-1}(\varphi,\omega) e^{i\omega t} d\omega\rangle$$

$$= \langle T_t, (-1)^k \frac{d^k}{dt^k}\varphi(t)\rangle = \langle T^{(k)}, \varphi\rangle\,.$$

The proof of theorem 2 is complete.

At aim to complete the theorem 2 in following we give a theorem for the analytic representation of primitive distribution for a given distribution $T \in D'$

By D'_+ we denote all the distributions with supports in a interval $[c, \infty)$. Similarry by D'_- we denote the distributions with supports in a interval $(-\infty, c]$. Every distribution $T \in D'$ can be decomposed in the form $T = T^+ + T^=$, where $T^+ \in D'_+$, $T^- \in D'_-$.

A primitive distribution S for a given distribution T can be found with the relation

$$\langle S, \varphi \rangle = -\langle T, \int_{-\infty}^{x} \varphi^*(t)dt \rangle, \quad \varphi \in D \quad \text{and} \quad \varphi(t) = \alpha p(t) + \varphi^*(t),$$

$\alpha = \int_{-\infty}^{\infty} \varphi(t)dt, p(t) \in D$ is fixed function with $\int_{-\infty}^{\infty} p(t)dt = 1$.

Also if $T \in D_+$ the convolutions $H * T$ is well defined and is a primitive distribution for T. $\operatorname{supp} H * T \subset \operatorname{supp} H + \operatorname{supp} T$, thus $H * T \in D'_+$. The primitive distribution T for a given distribution T is unique up to a constant distribution c, i.e. every distribution $S + c$ is primitive for T, that is why the analytic representation of S is uniqe to the analytic representation of the constant c, whose representation id $c > 0$ for $\operatorname{Im}(z) > 0$ and for $\operatorname{Im}(z) < 0$ or $\frac{c}{2}$ for $\operatorname{Im}(z) > 0$, $-\frac{c}{2}$ for $\operatorname{Im}(z) < 0$.

From here it follows that to compute an analytic representation of a primitive distribution for a given distribution will be obtain by an appropriate chose of the constant.

Theorem 3. *Let the functionf(z) is the analytic representation for the distribution $T \in D'_+$. Then for the primitive distribution S as deffined in (3.P.96), the primitive function $F(z)$ of $f(z)$ is an analytic representation for the distribution S.*

Proof. Let $\operatorname{supp} T \subset [c, \infty)$ and the function $\rho(t) \in D$ with $\operatorname{supp} \rho \subset (-\infty, c)$ and $\int_{-\infty}^{\infty} \rho(t)dt = 1$. In the simply connected domian $\Omega = C \setminus [c, \infty)$ the function $f(z)$ has a primitive function $F(z)$. With integration by parts we obtain

$$\int_{-\infty}^{\infty} [F(x + i\varepsilon) - F(x - i\varepsilon)]\varphi^*(x)dx = [F(x + i\varepsilon) - F(x - i\varepsilon)] \times$$

$$\times \int_{-\infty}^{x} \varphi^*(t)dt \Big|_{-\infty}^{\infty} - \int_{-\infty}^{\infty} [f(x + i\varepsilon) - f(x - i\varepsilon)] \int_{-\infty}^{x} \varphi^*(t)dt =$$

$$= -\int_{-\infty}^{\infty} \big[f(x+i\varepsilon) - f(x-i\varepsilon)\big]dx \int_{-\infty}^{x} \varphi^*(t)dt\,.$$

Because $\varphi^*(t) = \varphi(t) - \alpha\rho(t)$, we have

$$\int_{-\infty}^{\infty} \big[F(x+i\varepsilon) - F(x-i\varepsilon)\big]\,\big[\varphi(x) - \alpha\rho(x)\big]dx =$$

$$= \int_{-\infty}^{\infty} \big[F(x+i\varepsilon) - F(x-i\varepsilon)\big]\varphi(x) - \alpha \int_{-\infty}^{\infty} \big[F(x+i\varepsilon) - F(x-i\varepsilon)\big]\rho(x)dx.$$

Thus

$$\lim_{\varepsilon\to 0} \int_{-\infty}^{\infty} \big[F(x+i\varepsilon) - F(x-i\varepsilon)\big]\varphi(x)dx -$$

$$- \lim_{\varepsilon\to 0} \alpha \int_{-\infty}^{\infty} \big[F(x+i\varepsilon) - F(x-i\varepsilon)\big]\rho(x)dx =$$

$$= -\lim_{\varepsilon\to 0} \int_{-\infty}^{\infty} \big[f(x+i\varepsilon) - f(x-i\varepsilon)\big]\Big(\int_{-\infty}^{x} \varphi^*(t)dt\Big)dx$$

$$\lim_{\varepsilon\to 0} \int_{-\infty}^{\infty} \big[F(x+i\varepsilon) - F(x-i\varepsilon)\big]\rho(x)dx =$$

$$= \lim_{\varepsilon\to 0} \int_{\sup .\rho} \big[F(x+i\varepsilon) - F(x-i\varepsilon)\big]\rho(x)dx = 0.$$

Finally on obtain

$$\lim_{\varepsilon\to 0} \int_{-\infty}^{\infty} \big[F(x+i\varepsilon) - F(x-i\varepsilon)\big]\varphi(x)dx =$$

$$= -\lim_{\varepsilon\to 0} \int_{-\infty}^{\infty} \big[f(x+i\varepsilon) - f(x-i\varepsilon)\big] \int_{-\infty}^{x} \varphi^*(t)dt\,dx = -\langle T, \int_{-\infty}^{x} \varphi^*(t)dt\rangle$$

$$= \langle S, \varphi\rangle\,.$$

The proof is complete.

Simillary on prove for a distribution $T \in D'_-$. Clearly every distribution T with compacte support is in $D'_+ \cap D'_-$.

Example. The Heaviside distribution H is a primitive for the δ distribution with $\sup p\delta = \{0\}$. Let the $\sup p\rho(t) \subset (0, \infty)$. Then the analytic representation $-\frac{1}{2\pi i z}$ of δ has a primitive function:

$$F(z) = -\frac{1}{2\pi i} \log z, \arg z \in (-\pi, \pi] \quad \text{in} \quad C \setminus ((-\infty, 0] \cdot S = H + c.$$

$$-\frac{1}{2\pi i} \lim_{\varepsilon \to 0} \int_{-\infty}^{\infty} \left[\log(x + i\varepsilon) - \log(x - i\varepsilon)\right] \varphi(x) dx = \int_{0}^{\infty} \varphi(x) dx + c \int_{-\infty}^{\infty} \varphi(x) dx$$

$$-\frac{1}{2\pi i} \int_{-\infty}^{0} 2\pi i \varphi(x) dx = \int_{0}^{\infty} \varphi(x) dx + c \int_{-\infty}^{\infty} \varphi(x) dx, \quad \text{thus} \quad c = -1$$

$$H = S + 1, \hat{H}(z) = \frac{1}{2\pi i} \log z + \hat{1}(z) = -\frac{1}{2\pi i} \log z + \begin{cases} \frac{1}{2} \operatorname{Im} z > 0 \\ -\frac{1}{2} \operatorname{Im} z < 0 \end{cases} = \frac{1}{2\pi i} \log(-z).$$

Remark. From theorem 1 we have:

(i) The regular distributions $\hat{T}(x + i\varepsilon)$ converge to the distribution T^+ in D' as $\varepsilon \to 0^+$ where

$$\langle T^+, \varphi \rangle = \langle T, \int_{0}^{\infty} F^{-1}(\varphi, \omega) e^{i\omega t} d\omega \rangle, \quad \varphi \in D$$

(ii) The regular distributions $\hat{T}(x - i\varepsilon)$ convergs to the distribution T^-, where

$$\langle T^-, \varphi \rangle = \langle T, -\int_{-\infty}^{0} F^{-1}(\varphi, \omega) e^{i\omega t} d\omega \rangle, \quad \varphi \in D$$

(iii) $T = T^+ - T^-$

Remark. Operating formally in the integrals $\int_{-\infty}^{\infty} \hat{T}(x + i\varepsilon) \varphi(x) dx$ and

$\int\limits_{-\infty}^{\infty} \hat{T}(x - i\varepsilon)\varphi(x)dx$ on obtain

$$\int\limits_{-\infty}^{\infty} \hat{T}(x+i\varepsilon)\varphi(x)dx = \frac{1}{2\pi i}\int\limits_{-\infty}^{\infty} \langle T_t, \frac{1}{t-x-i\varepsilon}\rangle\varphi(x)dx$$

$$= \langle T_t, \frac{1}{2\pi i}\int\limits_{-\infty}^{\infty} \frac{\varphi(x)dx}{t-x-i\varepsilon}\rangle = T_t, \frac{1}{2\pi i}\int\limits_{-\infty}^{\infty} \frac{\varphi(t-x)}{x-i\varepsilon}\rangle$$

$$\lim_{\varepsilon\to 0}\int\limits_{-\infty}^{\infty} \frac{\varphi(t-x)}{x-i\varepsilon}dx = 2\pi i\langle\delta^-, \varphi(t-x)\rangle.$$

Thus we have

$$\lim_{\varepsilon\to 0}\int\limits_{-\infty}^{\infty} \hat{T}(x+i\varepsilon)\varphi(x)dx = \langle T_t, \langle\delta^-, \varphi(t-x)\rangle\rangle = \langle T_t, \delta^- * \varphi(t)\rangle\,.$$

This means

$$\langle T_t^+ = T_t, \delta * \varphi(t)\rangle\,.$$

Simillary from integral $\int\limits_{-\infty}^{\infty} \hat{T}(x - i\varepsilon)\varphi(x)dx$ on get

$$\langle T_t^-, \varphi(t)\rangle = \langle T_t, -\delta^+ * \varphi(t)\rangle\,.$$

Because $\delta^- = \frac{1}{2}\delta + \frac{1}{2\pi i}\nu p\frac{1}{t}$, $\delta^+ = \frac{1}{2}\delta - \frac{1}{2\pi i}\nu p\frac{1}{2}$ on obtain

$$\begin{aligned}\langle T^+, \varphi &= \langle T_t, \frac{1}{2}\delta * \varphi(t) + \frac{1}{2\pi i}\nu p\frac{1}{t} * \varphi(t)\rangle \\ &= \langle T_t, \frac{1}{2}\varphi(t)\rangle + \langle T_t, \frac{1}{2\pi i}\nu p\frac{1}{t} * \varphi(t)\rangle\,.\end{aligned}$$

From the fact that $T(\varphi) = T * \breve{\varphi}(0)$ and $\breve{\nu p}\frac{1}{t} = -\nu p\frac{1}{t}$ on resulte

$$\begin{aligned}\langle T_t, \frac{1}{2\pi i}\nu p\frac{1}{t} * \varphi(t)\rangle &= T * \left(\nu p\frac{1}{t} * \varphi\right)^{\breve{}}(0) = T * \left(\breve{\nu p}\frac{1}{t} * \breve{\varphi}\right)(0) \\ &= -T * \left(\nu p\frac{1}{t} * \breve{\varphi}\right)(0) = -\left(T * \nu p\frac{1}{t}\right) * \breve{\varphi}(0) = -\langle T * \nu p\frac{1}{t}, \varphi\rangle\,.\end{aligned}$$

Thus on get

(iv) $T^+ = \frac{1}{2}T - \frac{1}{2.\ i}T * \nu p\frac{1}{t}.$

Similar' from the integral $\int_{-\infty}^{\infty} \hat{T}(x - i\varepsilon)\varphi(x)dx$ we have

(v) $T^- = -\frac{1}{2}T - \frac{1}{2\pi i}T * \nu p\frac{1}{t}.$

Finally from (i), (iii), (iv), (v) on get

$$\langle\frac{1}{2}T - \frac{1}{2\pi i}T * \nu p\frac{1}{t}, \varphi\rangle = \langle T, \int_0^\infty F^{-1}(\varphi,\omega)e^{i\omega t}d\omega\rangle$$

and

$$\langle -\frac{1}{2}T - \frac{1}{2\pi i}T * \nu p\frac{1}{t}, \varphi\rangle = \langle T, -\int_{-\infty}^0 F^{-1}(\varphi,\omega)e^{i\omega t}d\omega\rangle\,.$$

Other

(vi) $\langle\frac{1}{2}T - \frac{1}{2\pi i}T * \nu p\frac{1}{t}, \varphi\rangle = \langle T, \int_0^\infty F^{-1}(\varphi,\omega)e^{i\omega t}d\omega\rangle$

(vii) $\langle\frac{1}{2}T + \frac{1}{2\pi i}T * \nu p\frac{1}{t}, \varphi\rangle = \langle T, \int_{-\infty}^0 F^{-1}(\varphi,\omega)e^{i\omega t}d\omega\rangle.$

By adding vi) and (vii) on obtain the distribution T.
By substracting (vi) and (vii) on obtain

$$\frac{1}{\pi i}\langle T * \nu p\frac{1}{t}, \varphi\rangle = \langle T, \int_{-\infty}^0 F^{-1}(\varphi,\omega)e^{i\omega t}d\omega - \int_0^\infty F^{-1}(\varphi,\omega)e^{i\omega t}d\omega\rangle\,.$$

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ЕДЕН ДОКАЗ ЗА АНАЛИТИЧНА РЕПРЕЗЕНТАЦИЈА НА ДИСТРИБУЦИИ

Никола Речкоски

Р е з и м е

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