we say, respectively, that „·" is *left commutative* (l. c.) — *left associative* (l. a), *commutative* (c.), *simmetric* (s.) — respect to „+".

It is evident that „l. c.", „c." and „s". are simmetric binary relations; the relations „l. c." and „c." are reflexive, but „s" is not reflexive one; „l. a." is not reflexive either simmetric.

At first, we shall give some prelimminary definitions. The element $a \in S$ is *left associative* in the *groupoid* $(S,\cdot)$ if

(5) $(\forall x, y)\ a \cdot xy = ax \cdot y.$

$(S,\cdot)$ is a *left reducible* groupoid if there exists a subset S' of left associative elements such that the mapping „′" defined by

(6) $(\forall x)\, x' \in S' \rightleftarrows x'x = x,$

is a single-valued function of the set S upon S'; we say that S' is a left reduced set. The unary operation (on the set S) φ is a *left translation* of the groupoid $(S,\cdot)$ if

(7) $(\forall x, y)\, \varphi(xy) = \varphi(x)\, y.$

$(S,\cdot^*)$ is dual groupoid of $(S,\cdot)$ if $(\forall x, y)\ x * y = yx$. We say that $(S,\cdot)$ has some „*right*" property, if (S, ·*) has the corresponding „*left*" one.

2. The main results

A. Let a be a right associative element in $(S,\cdot)$. If

(8) $(\forall x, y)\ x \cdot_a y = xy \cdot a,$

then „·" l. c. „$\cdot_a$". Converselly, if $(S,\cdot)$ has an identity e (i. e. $(\forall x)\ ex = xe = x$), and „·" l. c. „+", then „+"=„$\cdot_a$", where $a = e + e$ is a right associative element of $(S,\cdot)$; „$\cdot_a$"$=$„b"$\to a = b$.

More generally, if φ is a right translation of $(S,\cdot)$ and

(9) $(\forall x, y)\ x \dot{\varphi} y = \varphi(xy),$

then „·" l. c. „$\dot{\varphi}$". Converselly ,let $(S,\cdot)$ be left reducible, or idempotent with some left cancelable element (i. e. $(\forall x)\ xx = x$ and $(\exists a)\ (\forall x, y)\ ax = ay \to x = y$); from „·" l. c. „+" it follows „+"=„$\dot{\varphi}$", where $(\forall x)\, \varphi(x) = x' + x$ in the first case, and $(\forall x)\ \varphi(x) = x + x$ in the second one; in these cases we have „$\dot{\varphi}$"=„$\dot{\psi}$"$\to \varphi = \psi$.

Let $(S,\cdot)$ be a group. We have:

$\{$„·" l. c „+"$\rightleftarrows$„$\cdot_a$" l. c. „+"$\} \rightleftarrows (\forall x)\ xa = ax;$

then $(S,\cdot)$ and $(S, \cdot_a)$ are isomorphic.

B. If φ is an endomorphism of $(S,\cdot)$ then „·" c. „$\dot{\varphi}$". Converselly, if $(S,\cdot)$ is a semigroup with a left (or right) identity e, from „·" l. c. „+" it fololws „+"=„$\dot{\varphi}$", where $(\forall x)\, \varphi(x) = e + x$ (or $(\forall x)\, \varphi(x) = x + e$); then φ is an endomorphism of $(S,\cdot)$ and „$\dot{\varphi}$"=„$\dot{\psi}$"$\to \varphi = \psi$.