

If „ $\cdot$ ” l. a. „ $+$ ” or „ $\cdot$ ” c. „ $+$ ” then

$$\{(\forall x)(\exists y, z) x = yz = y + z\} \rightarrow „\cdot” = „+”.$$

C. Let  $(\forall x)\varphi(x)$  is a right associative element in  $(S, \cdot)$ . From

$$(10) (\forall x, y) x \cdot \varphi y = x \varphi (y),$$

it follows „ $\cdot$ ” l. a. „ $\varphi$ ”. Conversely, if  $(S, \cdot)$  has an identity  $e$ , from „ $\cdot$ ” l. a. „ $+$ ” it follows „ $+$ ” = „ $\varphi$ ”, where  $(\forall x)\varphi(x) = e + x$  is a right associative element in  $(S, \cdot)$ .

More generally, let „ $\circ$ ” be a binary operation on  $S$  such that

$$(11) (\forall u, x, y) ux \circ y = x \circ y.$$

If

$$(12) (\forall x, y) x + y = x(x \circ y),$$

then „ $\cdot$ ” l. a. „ $+$ ”. Conversely, if  $(S, \cdot)$  is a right reducible groupoid and „ $\cdot$ ” l. a. „ $+$ ” then there exists a binary operation „ $\circ$ ” such that (11) and (12) are satisfied.

D. Let  $(S, \cdot)$  be a semigroup.

If the binary operation „ $\circ$ ” satisfies the relation

$$(13) (\forall u, v, x, y) ux \circ y = x \circ yv = xy,$$

and „ $+$ ” is defined by

$$(14) (\forall x, y) x + y = x(x \circ y)y,$$

then „ $\cdot$ ” l. a. „ $+$ ”, „ $+$ ” l. a. „ $\cdot$ ”, and  $(S, +)$  is a semigroup. If both „ $\circ_1$ ” and „ $\circ_2$ ” satisfy (13) and „ $+$ ”, „ $+$ ” are their corresponding operations by (14), then „ $+$ ” l. a. „ $+$ ” and „ $+$ ” l. a. „ $+$ ”. Conversely, if the semigroup  $(S, \cdot)$  is reducible both left and right, from „ $\cdot$ ” l. a. „ $+$ ”, „ $+$ ” l. a. „ $\cdot$ ” it follows that there exists an operation „ $\circ$ ” such that (13) and (14) are satisfied.

If „ $\circ$ ” is a constant  $a$ , i. e.  $(\forall x, y) x \circ y = a$ , (13) is satisfied; hence it follows „ $\cdot$ ” l. a. „ $\hat{a}$ ” and „ $\hat{a}$ ” l. a. „ $\cdot$ ”, where

$$(15) (\forall x, y) x \hat{a} y = x a y.$$

Conversely, let the semigroup  $(S, \cdot)$  has an identity  $e$ . From „ $\cdot$ ” l. a. „ $+$ ”, and „ $+$ ” l. a. „ $\cdot$ ” it follows „ $+$ ” = „ $\hat{a}$ ”, where  $e + e = a$ . In this case we have

$$\begin{aligned} \{[„\cdot” \text{ l. a. } „+”, „+” \text{ l. a. } „\cdot”] \Leftrightarrow [„\hat{a}” \text{ l. a. } „+”, „+” \text{ l. a. } „\hat{a}”]\} &\Leftrightarrow \\ \Leftrightarrow \{(\forall x)(\exists y) x = aya\} \Leftrightarrow \{(\exists a) aa' = a'a = e\} &\Leftrightarrow \\ \Leftrightarrow \{\text{The semigroups } (S, \cdot) \text{ and } (S, \hat{a}) \text{ are isomorphic}\}. & \end{aligned}$$