E. Let φ and ψ be two endomorphisms of $(S,\cdot)$ and

(16) $(\forall x, y)\ x\,\varphi\cdot\psi\,y = \varphi(x)\,\psi(y)$.

We have

„$\cdot$" s. „$\cdot$" $\to$ „$\cdot$" s. „$\varphi\cdot\psi$",

and

[„$\cdot$" l. a. „$\cdot$", „$\varphi\cdot\psi$" = „φ*ψ"] $\to$ „$\cdot$" s. „$\varphi\cdot\psi$".

Converselly, if $(S,\cdot)$ has an identity e, from „$\cdot$" s. „$+$" it follows „$+$" = „$\varphi\cdot\psi$" = „φ*ψ", where φ and ψ are two endomorphisms of $(S,\cdot)$ defined by $(\forall x)\,\varphi(x) = e + x$, $\psi(x) = x + e$; if there is not idempotent elements $(\neq e)$ in $(S,\cdot)$, the corresponding pair of endomorphisms φ, ψ are uniquelly determined by „$+$", i. e.

„$\varphi\cdot\psi$" = „$\xi\cdot\eta$" $\to \varphi = \xi,\ \psi = \eta$.

Let e, o be the identity element of $(S,\cdot)$ and $(S,+)$ respectively. From „$\cdot$" s. „$+$" it follows „$\cdot$" = „$+$"; in this case $(S,\cdot)\ (= (S,+))$ is a commutative semigroup.

A semigroup with identity e is commutative if and only if there exists an operation „$+$" such that „$\cdot$" s. „$+$" and $(\forall x)(\exists u, v)\ u + e = e + v = x$.

F. Let $\prod\limits_{r}^{r} x_r = \sum\limits_{r}^{r} x_r = x_r$ and $\prod\limits_{i=1}^{n} x_i \left\{ \sum\limits_{i=1}^{n} x_i \right\}$ be an arbitrary product (sum) of the elements $x_1, x_2, \cdots x_n \in S$. We have

$$„\cdot" \text{ s. } „+" \to (\forall x_{i,j}) \prod_{i=1}^{n}\Big(\sum_{j=1}^{m} x_{ij}\Big) = \sum_{j=1}^{m}\Big(\prod_{i=1}^{n} x_{ij}\Big).$$

3. Some notes

A. The fact that we have supposed some known properties of the groupoid $(S,\cdot)$, and not of $(S,+)$, is not essential, because „l. c.", „c." and „s." are simetric relations, and „$\cdot$" l. a. „$+$" $\rightleftarrows$ „$+$*" l. a. „$\cdot$*".

B. The all results which we have got above can be transfered to five other relations:

(17) $(\forall u, v, x, y)\ u(x+y) = yx + u$

(18) $= x + uy$

(19) $(u+x)y = y + xu$

(20) $= ux + y$

(21) $(u+v)(x+y) = yx + vu$.