Math. Maced.
Vol. 1 (2003)
89-99

# FUZZY ALMOST STRONG PRECONTINUITY

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**Abstract.** The concept of fuzzy almost strong continuous mapping and fuzzy almost strongly preopen (preclosed) mappings has been introduced and studied. A characterization of fuzzy open mappings by using those mappings and some weaker forms of fuzzy continuous mappings has been established.

## 1. Introduction

The concept of fuzzy set was introduced by Zadeh in his classic paper [12] . Azad [1] has introduced the concept of fuzzy semiopen and fuzzy semiclosed sets. The fuzzy preopen and fuzzy preclosed sets were introduced by Singal and Prakash [9]. Bay Shi Zhong [3] introduced the fuzzy strongly semiopen and fuzzy strongly semiclosed sets. The author [6] defined the class of fuzzy strongly preopen and fuzzy strongly preclosed sets. After that, many authors have used these concepts to define and study weaker forms of fuzzy continuous mapping between fuzzy topological spaces.

In the Section 3 new weaker form of fuzzy continuity called fuzzy almost strongly precontinuity are introduced. Characterization theorems for that mapping are produced. The relation between this weaker form and some other form of continuity defined early are discussed.

In the Section 4, in a similar manner we continue the investigation for fuzzy almost strongly preopen and fuzzy strongly preclosed mappings.

## 2. Preliminaries

Some notions and results that will be needed in this paper are recalled here.

By $(X,\tau)$ or simply by $X$ we will denote a fuzzy topological spaces due to Chang [4]. The interior, closure and the complement of a fuzzy set $A$ we will denote by $\text{int}A$, $clA$ and $A^c$, respectively.

**Definition 2.1.** *Let $A$ be a fuzzy set of an fts $X$. Then $A$ is called*
1. *a fuzzy semiopen set if and only if there exists a fuzzy open set $U$ such that $U \leq A \leq clU$* [1]
2. *a fuzzy preopen set if and only if $A \leq \text{int}(clA)$* [9].
3. *a fuzzy strongly semiopen set if and only if $A \leq \text{int}(cl(\text{int } A))$* [7].

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*2000 Mathematics Subject Classification.* 54A40.

*Key words and phrases.* fuzzy topology, fuzzy almost strong precontinuous mapping, fuzzy almost strongly preopen mapping, fuzzy almost strongly preclosed mapping.

4. *a fuzzy semipreopen set if and only if* $A \leq cl(\mathrm{int}(clA))$ [8].
5. *a fuzzy regular open set if and only if* $A = \mathrm{int}(clA)$ [1].

The family of all fuzzy semiopen sets, fuzzy preopen sets, fuzzy strongly semiopen sets, fuzzy semipreopen sets and fuzzy regular open sets of an fts $(X, \tau)$ will be denote by $FSO(\tau)$, $FPO(\tau)$, $FSSO(\tau)$, $FSEPO(\tau)$ and $FRO(\tau)$ respectively.

**Definition 2.2.** *Let $A$ be a fuzzy set of an fts $X$. Then $A$ is called*
1. *a fuzzy semiclosed set if and only if $A^c$ is a fuzzy semiopen set* [1].
2. *a fuzzy preclosed set if and only if $A^c$ is a fuzzy preopen set* [9].
3. *a fuzzy strongly semiclosed set if and only if $A^c$ is a fuzzy strongly semiopen set* [3].
4. *a fuzzy semipreclosed set if and only if $A^c$ is a fuzzy semopreopen set* [8].
5. *a fuzzy regular closed set if and only if $A^c$ is a fuzzy regular open* [1].

The family of all fuzzy semiclosed sets, fuzzy preclosed sets, fuzzy strongly semiclosed sets, fuzzy semipreclosed sets and fuzzy regularly closed sets of an fts $(X, \tau)$ will be denote by $FSC(\tau)$, $FPC(\tau)$, $FSSC(\tau)$, $FSEPC(\tau)$ and $FRC(\tau)$ respectively.

**Definition 2.3.** *Let $A$ be a fuzzy set of an fts $(X, \tau)$. Then,*
$p\ \mathrm{int}A = \{B | B \leq A, B \in FPO(\tau)\}$, *is called the fuzzy preinterior of $A$* [9].
$p\ \mathrm{cl}A = \{B | B \geq A, B \in FPC(\tau)\}$,*is called the fuzzy preclosure of $A$* [9].

**Definition 2.4.** *A fuzzy set $A$ of an fts $X$ is called a fuzzy strongly preopen set if and only if* $A \leq \mathrm{int}(pclA)$ [6].

**Definition 2.5.** *A fuzzy $A$ of an fts $X$ is called a fuzzy strongly preclosed set if and only if $A^c$ is a fuzzy strongly preopen set* [6].

The family of all fuzzy strongly preopen sets and fuzzy strongly preclosed sets of an fts $(X, \tau)$ will be denoting by $FSPO(\tau)$ and $FSPC(\tau)$ respectively.

**Definition 2.6.** *Let $A$ be a fuzzy set of an fts $(X, \tau)$. Then,*
$sp\ \mathrm{int}A = \{B | B \leq A, B \in FSPO(\tau)\}$, *is called the fuzzy strong preinterior of $A$* [6].
$sp\mathrm{cl}A = \{B | B \geq A, B \in FSPC(\tau)\}$ *is called the fuzzy strong preclosure of $A$* [6].

**Definition 2.7.** [11] *A fuzzy singleton $x_\alpha$ of an fts $X$ is called a fuzzy set of $X$ defined by:*

$$x_\alpha = \begin{cases} \alpha, & \text{for } y = x \\ 0, & \text{otherwise} \end{cases} \qquad 0 < \alpha \leq 1.$$

**Definition 2.8.** [5] *An fts $(X, \tau)$ is called extremely disconnected if and only if $ClU \in \tau$, for each $U \in \tau$.*

**Lemma 2.1.** [6] *An fts $(X, \tau)$ is extremely disconnected if and only if* $FSO(\tau) \subseteq FSPO(\tau)$

**Definition 2.9.** [10] *A fuzzy set $A$ of an fts $X$ is called fuzzy $\delta-$ open if and only if there exists fuzzy regular open sets $A_i$, $i \in I$, such that $A = \vee_{i \in I} A_i$.*

**Definition 2.10.** [10] *A fuzzy set $A$ of an fts $X$ is called fuzzy $\delta-$ closed if and only if $A^c$ is a fuzzy $\delta-$ open set.*

**Definition 2.11.** [10] *Let $A$ be a fuzzy set of an fts $(X, \tau)$. Then,*
$\text{int}_\delta A = \{B | B \leq A, B \in FRO(\tau)\}$, *is called the fuzzy $\delta-$ interior of $A$.*
$cl_\delta A = \{B | B \geq A, B \in FRC(\tau)\}$ *is called the fuzzy $\delta-$ closure of $A$.*

Let $(X, \tau)$ be an fts. Since the intersection of two fuzzy regular open sets is a regular open set, the family $FRO(\tau)$ forms a base for a smallest topology $S(X)$ on $X$, called semiregularization of $X$.

**Definition 2.12.** [10] *An fts $(X, \tau)$ is called fuzzy semiregular if the family of all fuzzy regular open sets of $X$ forms a base for the fuzzy topology $\tau$.*

**Definition 2.13.** *A mapping $f : (X, \tau_1) \to (Y, \tau_2)$ from an fts $X$ into an fts $Y$ is called*

1. *a fuzzy strong precontinuous if $f^{-1}(B) \in FSPO(\tau_1)$ for each $B \in \tau_2$.* [7]
2. *a fuzzy weak strong precontinuuous if $f^{-1}(B) \leq \text{spint} f^{-1}(clB)$, for each $B \in \tau_2$.* [7]
3. *a fuzzy strong precontinuous irresolution if $f^{-1}(B) \in FSPO(\tau_1)$, for each $B \in FSPO(\tau_2)$.* [7]
4. *a fuzzy semicontinuous irresolution if $f^{-1}(B) \in FSO(\tau_1)$ for each $B \in FSPO(\tau_2)$.* [2]
5. *a fuzzy regular continuous irresolution if $f^{-1}(B) \in FRO(\tau_1)$ for each $B \in FRPO(\tau_2)$.* [7]

**Definition 2.14.** *A mapping $f : (X, \tau_1) \to (Y, \tau_2)$ from an fts $X$ into an fts $Y$ is called*

1. *a fuzzy semiopen (semiclosed) if $f(A) \in FSO(\tau_2)$, $(f(A) \in FSC(\tau_2),)$ for each $A \in \tau_1 (A^c \in \tau_1)$.* [2].
2. *a fuzzy strongly semiopen (semiclosed) if $f(A) \in FSSO(\tau_2)$, $(f(A) \in FSSC(\tau_2),)$ for each $A \in \tau_1 (A^c \in \tau_1)$.* [3].
3. *a fuzzy strongly preopen (preclosed) if $f(A) \in FSPO(\tau_2)$, $(f(A) \in FSPC(\tau_2),)$ for each $A \in \tau_1 (A^c \in \tau_1)$.* [6].
4. *a fuzzy semiopen (semiclosed) irresolution if $f(A) \in FSO(\tau_2)$, $(f(A) \in FSC(\tau_2),)$ for each $A \in FSO(\tau_1)$ $(A \in FSC(\tau_1))$*[2].
5. *a fuzzy regularly open (closed) irresolution if $f(A) \in FRO(\tau_2)$, $(f(A) \in FRC(\tau_2),)$ for each $A \in FRO(\tau_1)$ $(A \in FRC(\tau_1))$*[7].

**Lemma 2.2.** [1] *Let $g : X \to X \times Y$ be a graph of a mapping $f : X \to Y$. If $A$ is a fuzzy set of $X$ and $B$ be a fuzzy set of $Y$, then $g^{-1}(A \times B) = A \wedge f^{-1}(B)$.*

## 3. Fuzzy almost strong precontinuity

**Definition 3.1.** *A mapping $f : (X, \tau_1) \to (X, \tau_2)$ from an fts $X$ into an fts $Y$ is called fuzzy almost strong precontinuous if $f^{-1}(B) \in FSPO(\tau_1)$ for each $B \in FRO(\tau_2)$.*

**Remark 3.1.** *Let be a mapping from an fts into an fts . If is fuzzy strong precontinuous, then is a fuzzy almost strong precontinuous mapping. The following example shows that the converse statement may not be true.*

**Example 3.1.** *Let $X = \{a, b, c\}$ and $A$, $B$, $C$ be fuzzy sets of $X$ defined as follows:*

$$A(a) = 0,5 \qquad A(b) = 0,3 \qquad A(c) = 0,6;$$
$$B(a) = 0,3 \qquad B(b) = 0,4 \qquad B(c) = 0,3;$$
$$C(a) = 0,5 \qquad C(b) = 0,5 \qquad C(c) = 0,6;$$

*If we put $\tau_1 = \{0, B, A \vee B, 1\}$, $\tau_2 = \{0, A, B, A \wedge B, A \vee B, 1\}$ and $f = id : (X, \tau_1) \to (Y, \sigma_2)$ we conclude that $f$ is fuzzy almost strong precontinuous but $f$ is not fuzzy strong precontinuous mapping.*

**Theorem 3.1.** *Let $f : X \to Y$ be a mapping from an fts $X$ into an fts $Y$. Then the following statements are equivalent:*
*(i) $f$ is a fuzzy almost precontinuous mapping.*
*(ii) $f^{-1}(B)$ is a fuzzy strongly preclosed set of $X$, for each fuzzy regular closed set $B$ of $Y$.*
*(iii) $spclf^{-1}(cl(\text{int}B)) \leq f^{-1}(B)$, for each fuzzy closed set $B$ of $Y$.*
*(iv) $f^{-1}(B) \leq \text{spint}f^{-1}(\text{int}(clB))$, for each fuzzy open set $B$ of $Y$.*
*(v) $f^{-1}(B) \leq \text{spint}f^{-1}(\text{int}(cl(\text{int}B)))$ for each fuzzy strongly semiopen set $B$ of $Y$.*
*(vi) $spclf^{-1}(cl(\text{int}(clB))) \leq f^{-1}(B)$ for each fuzzy strongly semiclosed set $B$ of $Y$.*
*(vii) $spclf^{-1}(cl(\text{int}B)) \leq f^{-1}(B)$ for each fuzzy preclosed set $B$ of $Y$.*
*(viii) $f^{-1}(B) \leq \text{spint}f^{-1}(\text{int}(clB))$ for each fuzzy preopen set $B$ of $Y$.*
*(ix) $spclf^{-1}(cl(\text{int}(clB))) \leq f^{-1}(clB)$ for each fuzzy set $B$ of $Y$.*
*(x) $f^{-1}(\text{int}B) \leq \text{spint}f^{-1}(\text{int}(cl(\text{int}B)))$, for each fuzzy set $B$ of $Y$.*

*Proof.* The proof is standard and therefore is omitted. □

**Theorem 3.2.** *Let $f : X \to Y$ be a mapping from an fts $X$ into an fts $Y$. Then the following statements are equivalent:*
*(i) $f$ is a fuzzy almost precontinuous mapping.*
*(ii) $spclf^{-1}(B) \leq f^{-1}(clB)$, for each fuzzy semiopen set $B$ of $Y$.*
*(iii) $f^{-1}(\text{int}B) \leq \text{spint}f^{-1}B$, for each fuzzy semiclosed set $B$ of $Y$.*
*(iv) $f^{-1}(\text{int}B) \leq \text{spint}f^{-1}B$, for each fuzzy semipreclosed set $B$ of $Y$.*
*(v) $spclf^{-1}(B) \leq f^{-1}(clB)$ for each fuzzy semipreopen set $B$ of $Y$.*

*Proof.* The proof is standard and therefore is omitted. □

**Corollary 3.2.1.** *Let $f : X \to Y$ be a fuzzy almost strong precontinuous mapping from an fts $X$ into an fts $Y$. Then the following statements holds:*
*(i) $spclf^{-1}B \leq f^{-1}(clB)$, for each fuzzy open set $B$ of $Y$.*
*(ii) $f^{-1}(\text{int}B) \leq \text{spint}f^{-1}B$, for each fuzzy closed set $B$ of $Y$.*

The following theorem gives some local characterizations of the fuzzy almost strong precontinuous mappings.

**Theorem 3.3.** *Let $f : X \to Y$ be a mapping from an fts $X$ into an fts $Y$. Then the following statements are equivalent:*
*(i) $f$ is a fuzzy almost precontinuous mapping.*
*(ii) for each fuzzy singleton $x_\alpha$ of $X$ and fuzzy open set $B$ containing $f(x_\alpha)$ there exists fuzzy strongly preopen set $A$ of $X$ containing $x_\alpha$ such that $f(A) \leq \mathrm{int}(clB)$.*
*(iii) for each fuzzy singleton $x_\alpha$ of $X$ and fuzzy regularly open set $B$ containing $f(x_\alpha)$ there exists fuzzy strongly preopen set $A$ of $X$ containing $x_\alpha$ such that $f(A) \leq B$ .*

*Proof.* $(i) \Rightarrow (ii)$ Let $f$ be fuzzy almost strong precontinuous, $x_\alpha$ be a fuzzy singleton of $X$ and let $B$ be a fuzzy open set of $Y$ such that $f(x_\alpha) \leq B$. Then $x_\alpha \leq f^{-1}(B) \leq sp(\mathrm{int} f^{-1}(\mathrm{int}(clB))$. Let $A = sp(\mathrm{int} f^{-1}(\mathrm{int}(clB))$. Then $A$ is a fuzzy strongly preopen set and $f(A) = f(sp\mathrm{int} f^{-1}(\mathrm{int}(clB)) \leq ff^{-1}(\mathrm{int}(clB)) \leq$ $\leq (\mathrm{int}(clB))$.

$(ii) \Rightarrow (iii)$ Let $x_\alpha$ be a fuzzy singleton of $X$ and let $B$ be a fuzzy regularly open set of $Y$ containing $f(x_\alpha)$. Then $B$ is a fuzzy open set. According to the assumption there exists a strongly preopen set $A$ of $X$ containing $x_\alpha$ such that $f(A) \leq \mathrm{int}(clB) = B$

$(iii) \Rightarrow (i)$ Let $B$ be a fuzzy regularly open set of $Y$ and let $x_\alpha$ be a fuzzy singleton of $X$ such that $x_\alpha \leq f^{-1}(B)$. According to the assumption there exists a strongly preopen set $A$ of $X$ such that $x_\alpha \leq A$ and $f(A) \leq B$. Hence $x_\alpha \leq A \leq f^{-1}f(A) \leq f^{-1}(B)$ and $x_\alpha \leq A = sp\mathrm{int}A \leq sp\mathrm{int} f^{-1}(B)$. Since $x_\alpha$ is arbitrary and $f^{-1}(B)$ is the union of all fuzzy singletons of $f^{-1}(B)$, $f^{-1}(B) \leq sp\mathrm{int} f^{-1}(B)$. Thus $f$ is fuzzy almost precontinuous mapping. □

**Theorem 3.4.** *Let $f : X \to Y$ be a mapping from an fts $X$ into an fts $Y$. Then the following statements are equivalent:*
*(i) $f$ is a fuzzy almost precontinuous mapping.*
*(ii) $f^{-1}(B)$ is a fuzzy strongly preopen set of $X$ for each fuzzy $\delta-$ open set $B$ of $Y$.*
*(iii) $f^{-1}(B)$ is a fuzzy strongly preclosed set of $X$ for each fuzzy $\delta-$ closed set $B$ of $Y$.*
*(iv) $f(splcA) \leq cl_\delta f(A)$, for each fuzzy set $A$ of $X$.*
*(v) $spclf^{-1}(B) \leq f^{-1}(cl_\delta B)$, for each fuzzy set $B$ of $Y$.*
*(vi) $f^{-1}(\mathrm{int}_\delta B) \leq sp\mathrm{int} f^{-1}(B)$, for each fuzzy set $B$ of $Y$.*

*Proof.* $(i) \Rightarrow (ii)$ Let $B$ be a fuzzy $\delta-$ open set of $Y$. Then $B = \vee_{\alpha \in I} B_\alpha$, where $B_\alpha$ is a fuzzy regularly open sets of $Y$, for each $\alpha \in I$. From $f^{-1}(B) = f^{-1}(\vee_{\alpha\in I} B_\alpha) =$ $= \vee_{\alpha \in I} f^{-1}(B_\alpha)$ it follows that $f^{-1}(B)$ is a fuzzy strongly preopen set as a union of fuzzy strongly preopen sets.

$(ii) \Rightarrow (iii)$ Can be proved by using the complement.

$(iii) \Rightarrow (iv)$ Let $A$ be a fuzzy set of $X$. Then $cl_\delta f(A)$ is a fuzzy $\delta-$ closed set of $Y$. According to the assumption $f^{-1}(cl_\delta f(A))$ is a fuzzy strongly preclosed set of $X$. Hence $splcA \leq spclf^{-1}(f(A)) \leq spclf^{-1}(cl_\delta f(A)) = f^{-1}(cl_\delta f(A))$, so $f(splcA) \leq cl_\delta f(A)$.

$(iv) \Rightarrow (v)$ Let $B$ be a fuzzy set of $Y$. From the assumption it follows that $f(spclf^{-1}(B)) \leq cl_\delta f(f^{-1}(B)) \leq cl_\delta B$. Thus $spclf^{-1}(B) \leq f^{-1}f(spclf^{-1}(B)) \leq$ $\leq f^{-1}(cl_\delta B)$.

$(v) \Rightarrow (vi)$ Can be proved by using the complement.

$(vi) \Rightarrow (i)$ Let $B$ be a fuzzy regular open set of $Y$. Then $B = \text{int}_\delta B$. According to the assumption $f^{-1}(B) = f^{-1}(\text{int}_\delta B) \leq sp\text{int}f^{-1}(B) \leq f^{-1}(B)$. Hence $f^{-1}(B) = sp\text{int}f^{-1}(B)$ so $f^{-1}(B)$ is a fuzzy strongly preopen set. Thus $f$ is a fuzzy almost precontinuous mapping. □

**Corollary 3.4.1.** *A mapping $f : X \to Y$ from an ftp $X$ into an ftp $Y$ is a fuzzy almost strong precontinuous if and only if the mapping $f : X \to (Y, S(X))$ is a fuzzy strong precontinuous.*

**Corollary 3.4.2.** *Let $f : X \to Y$ be a mapping from an fts $X$ into a semiregular fts $Y$. The mapping $f$ is a fuzzy almost strong precontinuous if and only if $f$ is a fuzzy strong precontinuous.*

**Theorem 3.5.** *Let $f : X \to Y$ be a bijective mapping from an fts $X$ into an fts $Y$. The mapping $f$ is a fuzzy almost strong precontnuous if and only if* $\text{int}_\delta f(A) \leq f(sp\text{int}A)$, *for each fuzzy set $A$ of $X$.*

*Proof.* We suppose that $f$ is fuzzy almost strong precontinuous. Then $f^{-1}(\text{int}_\delta f(A))$ is a fuzzy strongly preopen set of $X$ for any fuzzy set $A$ of $X$. Since $f$ is injective, from Theorem 3.4 it follows that $f^{-1}(\text{int}_\delta f(A)) = sp\text{int}f^{-1}(\text{int}_\delta f(A)) \leq$ $\leq sp\text{int}f^{-1}f(A) = sp\text{int}A$. Again, since $f$ is surjective, we obtain $\text{int}_\delta f(A) = ff^{-1}(\text{int}_\delta f(A)) \leq f(sp\text{int}A)$.

Conversely, let $B$ be a fuzzy $\delta-$ open set of $Y$. Then $\text{int}_\delta B = B$. According to the assumption, $f(sp\text{int}f^{-1}(B)) \geq \text{int}_\delta ff^{-1}(B) = \text{int}_\delta B = B$. Thus implies that $f^{-1}f(sp\text{int}f^{-1}(B)) \geq f^{-1}(B)$. Since $f$ is injective we obtain $(sp\text{int}f^{-1}(B)) =$ $= f^{-1}f(sp\text{int}f^{-1}(B)) \geq f^{-1}(B)$. Hence $sp\text{int}f^{-1}(B) = f^{-1}(B)$, so $f^{-1}(B)$ is a fuzzy strongly preopen set. Thus $f$ is a fuzzy almost precontinuous mapping. □

**Definition 3.2.** *A mapping $f : (X, \tau_1) \to (Y, \tau_2)$ from an fts $X$ into an fts $Y$ is called:*

1. *fuzzy almost precontinuity if $f^{-1}(B) \in FPO(\tau_1)$, for each $B \in FRO(\tau_2)$*
2. *fuzzy almost strong semicontinuity if $f^{-1}(B) \in FSSO(\tau_1)$, for each $B \in FRO(\tau_2)$*
3. *fuzzy almost semicontinuity if $f^{-1}(B) \in FSO(\tau_1)$ for each $B \in FRO(\tau_2)$.*

**Theorem 3.6.** *Let $X_1$, $X_2$, $Y_1$ and $Y_2$ be fts's such that $X_1$ is a product related to $X_2$. Then the product $f_1 \times f_2 : X_1 \times X_2 \to Y_1 \times Y_2$ of fuzzy almost strong precontinuous mappings $f_1 : X_1 \to Y_1$ and $f_2 : X_2 \to Y_2$ is a fuzzy almost precontinuous mapping.*

*Proof.* Let $B = \vee(U_\alpha \times V_\beta)$, where $U_\alpha$ and $V_\beta$ are fuzzy open sets of $Y_1$ and $Y_2$ respectively.

$$\text{From}(f_1 \times f_2)^{-1}(B) = \vee(f^{-1}(U_\alpha) \times f_2^{-1}(V_\beta)) \leq$$
$$\leq \vee(sp\text{int}f_1^{-1}(\text{int}(clU_\alpha)) \times sp\text{int}f_2^{-1}(\text{int}(cl(V_\beta)) \leq$$

$$\leq \vee(\text{pint} f_1^{-1}(\text{int}(clU_\alpha)) \times \text{pint} f_2^{-1}(\text{int}(cl(V_\beta)) \leq$$
$$\leq \vee(\text{pint}(f_1^{-1}(\text{int}(clU_\alpha)) \times f_2^{-1}(\text{int}(cl(V_\beta)))) \leq$$
$$\leq \text{pint}(\vee(f_1^{-1}(\text{int}(clU_\alpha)) \times f_2^{-1}(\text{int}(cl(V_\beta)) \leq$$
$$\leq \text{pint}(f_1 \times f_2)^{-1}(\vee(\text{int}(clU_\alpha) \times \text{int}(clV_\beta))) =$$
$$\leq \text{pint}(f_1 \times f_2)^{-1}(\vee(\text{int}(cl(U_\alpha \times V_\beta))) =$$
$$\leq \text{pint}(f_1 \times f_2)^{-1}((\text{int}(cl(\vee(U_\alpha \times V_\beta))) =$$
$$= \text{pint}(f_1 \times f_2)^{-1}(\text{int}(clB)),$$

it follows that $(f_1 \times f_2)^{-1}(B)$ is fuzzy almost precontinuous mapping. □

**Theorem 3.7.** *Let $X$, $X_1$ and $X_2$ be fts's and $p_i : X \to X_1 \times X_2$ $(i = 1, 2)$ are the projections of $X_1 \times X_2$ onto $X_i$. If $f : X \to X_1 \times X_2$ is a fuzzy almost strong precontinuous, then $p_i f$ are also fuzzy almost precontinuous mappings.*

*Proof.* Since the projections are fuzzy continuous and fuzzy open mappings we have $clp_i(B) \leq p_i^{-1}(clB)$ and $\text{int}p_i(B) \leq p_i^{-1}(\text{int}B)$, for each fuzzy set $B$ of $X_i$. Hence $(p_i f)^{-1}(B) = f^{-1}(p_i^{-1}(B)) \leq spint f^{-1}(\text{int}(cl(p_i^{-1}(B))) \leq$
$\leq spint f^{-1}(p_i^{-1}(\text{int}(clB))) \leq spint(p_i f)^{-1}(\text{int}(cl(B))$, for each fuzzy open set $B$ of $X$. □

**Theorem 3.8.** *Let $f : X \to Y$ be a mapping from an fts $X$ into an fts $Y$. If the graph $g : X \to X \times Y$ of $f$ is fuzzy almost strong precontinuous, then is fuzzy almost strong precontinuous.*

*Proof.* By Lemma 2.2 $f^{-1}(B) = 1 \wedge f^{-1}(B) = g^{-1}(1 \times B)$, for each fuzzy regular open set $B$ of $Y$. Since $g$ is fuzzy almost precontinuous and $1 \times B$ is a fuzzy regular open set of $X \times Y$, $f^{-1}(B)$ is a fuzzy strongly preopen set of $X$, so $f$ is fuzzy almost strong precontinuous. □

**Theorem 3.9.** *Fuzzy almost strong continuity implies fuzzy weak strong precontinuity.*

*Proof.* Let $f : X \to Y$ be a fuzzy almost strong precontinuous mapping from an fts $X$ into an fts $Y$, and let $B$ be a fuzzy open set of $Y$. Then $\text{int}(clB)$ is a fuzzy regular open set of $Y$. Therefore $f^{-1}(B) \leq spint f^{-1}(\text{int}(cl(B)) \leq spint f^{-1}(clB)$, so $f$ is fuzzy weak strong precontinuous mapping. □

**Theorem 3.10.** *Let $f : X \to Y$ be a fuzzy strong preopen and fuzzy strong precontinuous irresolution from an fts $X$ onto an fts $Y$ and let $g : Y \to Z$ be a mapping from fts $Y$ into fts $Z$. The mapping $gf$ is a fuzzy almost strong precontinuous if and only if $g$ is a fuzzy almost strong precontinuous.*

*Proof.* Let $gf$ be a fuzzy almost strong precontinuous. Then $g^{-1}(C) = f(gf)^{-1}(C)$ is a fuzzy strongly preopen set of $Y$, for each fuzzy regular open set $C$ of $Z$. Hence $g$ is a fuzzy almost strong precontinuous mapping.

Conversely, let $g$ be a fuzzy almost strong precontinuous mapping and let $C$ be a fuzzy regular open set of $Z$. From $(gf)^{-1}(C) = f^{-1}(g^{-1}(C))$ it follows that $gf$ is a fuzzy almost strong precontinuous mapping. □

**Theorem 3.11.** *Let $f : X \to Y$ be a mapping from an fts $X$ into an fts $Y$. The mapping $f$ is a fuzzy almost strong semicontinuous if and only if it is both fuzzy almost semicontinuous and fuzzy almost strong precontinuous.*

*Proof.* It can be proved in a similar manner as Theorem 3.9. □

**Theorem 3.12.** *A fuzzy almost semicontinuous mapping $f : X \to Y$ from an extremely disconnected fts $X$ into an fts $Y$ is a fuzzy almost strong precontinuity.*

*Proof.* It follows from the Lemma 2.1. □

**Theorem 3.13.** *Let $f : X \to Y$ be a mapping from an extremely disconnected fts $X$ into an fts $Y$. The mapping $f$ is a fuzzy almost strong semicontinuous if and only if it is fuzzy almost semicontinuous.*

*Proof.* It follows from the Theorem 3.11 and Theorem 3.12. □

**Theorem 3.14.** *Let $f : X \to Y$ be a mapping from an fts $X$ into an fts $Y$. If the mapping $f$ is both fuzzy almost strong precontinuous and fuzzy semicontinuous irresolution then $f$ is fuzzy regular continuous irresolution.*

*Proof.* It can be proved in a similar manner as Theorem 3.15. □

**Corollary 3.14.1.** *Let $f : X \to Y$ be a mapping from an fts $X$ into a semiregular fts $Y$. If the mapping $f$ is both fuzzy almost strong precontinuous and fuzzy semicontinuous irresolution then $f$ is fuzzy continuous mapping.*

## 4. Fuzzy almost strongly preopen and fuzzy almost strongly preclosed mappings

**Definition 4.1.** *A mapping $f : (X, \tau_1) \to (Y, \tau_2)$ from an fts $X$ into an fts $Y$ is called*

1. *fuzzy almost strongly preopen if $f(A) \in FSPO(\tau_2)$ for each $A \in FRO(\tau_1)$.*
2. *fuzzy almost strongly preclosed if $f(A) \in FSPC(\tau_2)$ for each $A \in FRC(\tau_1)$.*

**Remark 4.1.** *Let be a mapping from an fts $X$ into an fts $Y$. If $f$ is fuzzy strongly preopen (preclosed), then $f$ is a fuzzy almost strongly preopen (preclosed) mapping. The following example shows that the converse statement may not be true.*

**Example 4.1.** *We consider the Example 3.1. The mapping $f \to id : (X, \tau_2) \to (Y, \tau_1)$ is fuzzy almost strongly preopen (preclosed) but $f$ is not fuzzy strongly preopen (preclosed).*

**Theorem 4.1.** *Let $f : X \to Y$ be a bijective mapping from an fts $x$ into an fts $Y$. Then $f$ is a fuzzy almost strongly preopen (preclosed) if and only if it is a fuzzy almost strongly preclosed (preopen).*

*Proof.* Using the complement can prove it. □

**Theorem 4.2.** *Let $f : X \to Y$ be a bijective mapping from an fts $X$ into an fts $Y$. Then $f$ is a fuzzy almost strongly preopen (preclosed) if and only if $f^{-1}$ is a fuzzy almost strong precontinuous.*

*Proof.* It follows from the relation $(f^{-1})^{-1}(A) = f(A)$, for each fuzzy regular open (closed) set $A$ of $X$. □

**Theorem 4.3.** *Let $f : X \to Y$ be a mapping from an fts $X$ into an fts $Y$. Then is a fuzzy almost strongly preopen (preclosed) if and only if $f : (X, S(X)) \to (Y, S(Y))$ is a fuzzy strongly preopen (preclosed) mapping.*

*Proof.* Let $A$ be an arbitrary fuzzy open set of $Y$. Then $A = \vee_{\alpha \in I} A_\alpha$ where $A_\alpha$ is a fuzzy regular open sets of $X$ for each $\alpha \in I$. From $f(A) = f(\vee_{\alpha \in I} A_\alpha) = = \vee_{\alpha \in I} f(A_\alpha)$ it follows that $f(A)$ is a fuzzy strongly preopen set as a union of fuzzy strongly preopen sets.

Conversely statement follows from the condition that each fuzzy regular open set of $X$ is a fuzzy open set of $S(X)$. □

**Corollary 4.3.1.** *Let $f : X \to Y$ be a mapping from a fuzzy semiregular fts $X$ into an fts $Y$. Then $f$ is a fuzzy almost strongly preopen (preclosed) if and only if it is fuzzy strongly preopen (preclosed) mapping.*

**Theorem 4.4.** *Let $f : X \to Y$ be a mapping from an fts $X$ into an fts $Y$. Then $f$ is a fuzzy almost strongly preopen if and only if $f(\text{int}A) \leq spint f(A)$, for each fuzzy semiclosed set $A$ of $X$.*

*Proof.* Let $f$ be a fuzzy almost strongly preopen mapping and let $A$ be any fuzzy semiclosed set of $X$. Then $\text{int}A = \text{int}(clA)$. According to the assumption we have $f(\text{int}A) = f(\text{int}(clA)) = spint f(\text{int}(clA)) = spint f(\text{int}A) \leq spint f(A)$.

Conversely, let $A$ be any fuzzy regular open set of $X$. Then $A$ is a fuzzy semiclosed set of $X$. According to the assumption we have $f(A) = f(\text{int}A) \leq \leq spint f(A)$. Thus $f(A)$ is a fuzzy strongly preopen set of $X$. □

**Theorem 4.5.** *Let $f : X \to Y$ be a mapping from an fts $X$ into an fts $Y$. Then $f$ is a fuzzy almost strongly preclosed if and only if $spcl f(A) \leq f(clA)$, for each fuzzy semiopen set $A$ of $X$.*

*Proof.* It can be proved in a similar manner as previous theorem. □

**Theorem 4.6.** *Let $f : X \to Y$ be a bijective mapping from an fts $X$ into an fts $Y$. Then the following statements are equivalent:*

*(i) $f$ is a fuzzy almost strongly preopen (preclosed) mapping.*

*(ii) $f(\text{int}A) \leq spint f(A)$, for each fuzzy semiclosed set $A$ of $X$.*

*(iii) $spcl f(A) \leq f(clA)$, for each fuzzy semiopen set $A$ of $X$.*

*Proof.* It follows from the Theorem 4.1, Theorem 4.4 and Theorem 4.5. □

**Theorem 4.7.** *Let $f : X \to Y$ be a mapping from an fts $X$ into an fts $Y$. Then the following statements holds:*

*(1) $f$ is a fuzzy almost strongly preopen mapping if and only if $f(\text{int}A) \leq \leq \text{int}(pcl f(A))$, for each fuzzy semiclosed set $A$ of $X$.*

*(2) $f$ is a fuzzy almost strongly preclosed mapping if and only if $cl(\text{pint} f(A)) \leq \leq f(clA))$, for each fuzzy semiopen set $A$ of $X$.*

*Proof.* We will prove only the statement (1). Let $f$ be a fuzzy almost strongly preopen mapping. Then, for any fuzzy semiclosed set $A$ of $X$, $\text{int}A = \text{int}(clA)$, so $f(\text{int}A)$ is a fuzzy strongly preopen set of $Y$. Thus $f(\text{int}A) = \text{int}(pclf(\text{int}A)) \leq$ $\leq \text{int}(pclf(A))$.

Conversely, let $A$ be a fuzzy regular open set of $X$. From $f(A) = f(\text{int}A) \leq$ $\leq \text{int}(pclf(A))$, we conclude that $f(A)$ is a fuzzy strongly preopen set, so $f$ is a fuzzy almost strongly preopen set. □

**Theorem 4.8.** *Let $f : X \to Y$ be a mapping from an fts $X$ into an fts $Y$. Then $f$ is fuzzy almost strongly preopen if and only if for each fuzzy set $B$ of $Y$ and each fuzzy regularly closed set $A$ of $X$, $f^{-1}(B) \leq A$, there exists a fuzzy strongly preclosed set $C$ of $Y$ such that $B \leq C$ and $f^{-1}(C) \leq A$.*

*Proof.* Let $B$ be any fuzzy set of $Y$ and let $A$ be a fuzzy regularly closed set of $X$ such that $f^{-1}(B) \leq A$. Then $A^c \leq f^{-1}(B^c)$, so $f(A^c) \leq ff^{-1}(B^c) \leq B^c$. Since $A^c$ is a fuzzy regularly open set, $f(A^c)$ is a fuzzy strongly preopen set, so $f(A^c) \leq spintB^c$. Hence $A^c \leq f^{-1}f(A^c) \leq f^{-1}(spintB^c)$.Thus result follows for $C = spclB$.

Conversely, let $U$ be a fuzzy regularly open set of $X$. We will show that $f(U)$ is a fuzzy strongly preopen set of $Y$. From $U \leq f^{-1}f(U)$ follows that $U^c \geq (f^{-1}f(U))^c$ where $U^c$ is fuzzy regularly closed set of $X$. Hence there is a fuzzy strongly precolsed $B$ of $Y$ such that $B \geq f(U)^c$ and $f^{-1}(B) \leq U^c$. From $B \geq f(U)^c$ follows that $B \geq spclf(U)^c \geq U$, so $B^c \leq (spclf(U)^c)^c \leq spintf(U)$. From $f^{-1}(B) \leq U^c$ we have $B \geq f^{-1}(B^c) \geq U$, so $B^c \geq ff^{-1}(b^c) \geq f(U)$. Hence $f(U) = spintf(U)$. Thus $f(U)$ is a fuzzy strongly preopen set, so $f$ is a fuzzy almost strongly preopen mapping. □

**Theorem 4.9.** *Let $f : X \to Y$ be a mapping from an fts $X$ into an fts $Y$. Then $f$ is fuzzy almost strongly preclosed if and only if for each fuzzy set $B$ of $Y$ and each fuzzy regularly open set $A$ of $X$, $f^{-1}(B) \leq A$, there exists a fuzzy strongly preopen set $C$ of $Y$ such that $B \leq C$ and $f^{-1}(C) \leq A$.*

*Proof.* It can be proved in a similar manner as Theorem 4.8. □

**Theorem 4.10.** *Let $f : X \to Y$ be a mapping from an fts $X$ into an fts $Y$. The mapping $f$ is a fuzzy almost strong semiopen (semiclosed) if and only if it is both fuzzy almost semiopen (semiclosed) and fuzzy almost strongly preopen (preclosed).*

*Proof.* Let $A$ be any fuzzy regularly open set of $X$. Then the mapping $f$ is both fuzzy almost semiopen and fuzzy almost strongly preopen if and only if $f(A) \in FSO(\sigma_2)$ and $f(A) \in FSPO(\sigma_2)$. This means that
$f(A) \in FSO(\sigma_2) \cap f(A) \in FSPO(\sigma_2)$. Since $FSO(\sigma_2) \supseteq FSPO(\sigma_2)$, from the last relations we get $f(A) \in FSSO(\sigma_2)$. □

**Theorem 4.11.** *Let $f : X \to Y$ be a mapping from an fts $X$ into an extremely disconnected fts $Y$. If the mapping $f$ is fuzzy almost semiopen (semiclosed) then $f$ is fuzzy almost strongly preopen (preclosed).*

*Proof.* Each fuzzy semiopen set in extremely disconnected fts is a fuzzy strongly semiopen set. □

**Corollary 4.11.1.** *Let $f : X \to Y$ be a mapping from an fts $X$ into an extremely disconnected fts $Y$. The mapping $f$ is fuzzy strongly semiopen (semiclosed) if and only if it is fuzzy semiopen (semiclosed).*

*Proof.* It follows from the Theorem 4.10 and Theorem 4.11. □

**Theorem 4.12.** *Let $f : X \to Y$ be a mapping from an fts $X$ into an fts $Y$. If the mapping $f$ is both fuzzy almost strongly preopen (preclosed) and fuzzy semiclosed (semiopen) irresolution, then $f$ is fuzzy regularly (closed) open irresolution.*

*Proof.* Let $A$ be any fuzzy regular open set of $X$. Since $f$ is fuzzy almost strongly preopen mapping it follows that $f(A) \in FSPO(\sigma_2)$. On the other hand $f$ is fuzzy semiclosed irresolution, so $f(A) \in FSC(\sigma_2)$. This means that
$f(A) \in FSPO(\sigma_2) \cap FSC(\sigma_2)$, so $f(A) \in FRO(\sigma_2)$. □

**Corollary 4.12.1.** *Let $f : X \to Y$ be a mapping from a semiregular fts $X$ into an fts $Y$. If the mapping $f$ is both fuzzy almost strongly preopen (preclosed) and fuzzy semiclosed (semiopen) irresolution, then $f$ is fuzzy (closed) open mapping.*

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