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FUZZY PAIRWISE ALMOST STRONGLY PREOPEN (PRECLOSED) MAPPINGS

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Abstract. The concept of a fuzzy parwise almost strongly preopen (preclosed) mappings has been introduced and studied. Their properties and relationships with other class of early defined types of weaker forms of fuzzy pairwise continuous mappings has been investigated.

1. Introduction

The concept of fuzzy set was introduced by Zadeh in his classic paper [9]. Chang [2] first introduced the fuzzy topological spaces by using the fuzzy sets. Kandil [3] introduced and studied the notion of fuzzy bitopological spaces as a natural generalization of fuzzy topological spaces. Recently, Kumar [7,8] defined the (τ_i, τ_j)-fuzzy semiopen (semiclosed) sets, (τ_i, τ_j)-fuzzy preopen (preclosed) sets and (τ_i, τ_j)-fuzzy strongly semiopen sets. The author [5] introduced the concept of (τ_i, τ_j)-fuzzy strongly preopen (preclosed) sets. Continuity his work, Kumar [7,8] defined the fuzzy pairwise semicontinuous mappings, fuzzy pairwise semiopen (semiclosed) mappings, fuzzy pariwise precontinuous, fuzzy pairwise preopen (preclosed) mappings, fuzzy pairwise strongly semicontinuous mappings and fuzzy pairwise strongly semiopen (semiclosed) mappings sets. The author [5,6] defined the concept of fuzzy pairwise strong precontinuous mappings, fuzzy pairwise strongly preopen (preclosed) mappings and fuzzy pairwise almost strongly precontinuous mappings. In this paper, we will define the concept of fuzzy pairwise almost strongly preopen (preclosed) mappings. We will establish their properties and relationships with other class of early defined types of weaker forms of fuzzy pairwise continuous mappings.

2. Preliminaries

A triple (X, τ_1, τ_2) consisting of a nonempty set X with two fuzzy topologies τ_1 and τ_2 on X is called a fuzzy bitopological space, shortly fbts. Throughout this paper, the indices i and j take values in $\{1, 2\}$ and $i \neq j$. For a fuzzy set A of an fbts (X, τ_1, τ_2) , τ_i-int A and τ_j - cl$A$ means, respectively, the interior and closure of A with respect to the fuzzy topologies τ_i and τ_j.

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Definition 2.1. *[5,7,8] Let A be a fuzzy set of an fbts X. Then A is called*

(1) a (τ_i, τ_j)-fuzzy semiopen set if and only if there exists τ_i-fuzzy open set U such that $U \leq A \leq \tau_j - clU$;

(2) a (τ_i, τ_j)-fuzzy preopen set if and only if $A \leq \tau_i - int\,(\tau_j - clA)$;

(3) a (τ_i, τ_j)-fuzzy strongly semiopen set if and only if

$A \leq \tau_i - int\,(\tau_j - cl(\tau_i - int\,A))$;

(4) a (τ_i, τ_j)-fuzzy strongly preopen set if and only if

$A \leq \tau_i - int((\tau_j, \tau_i) - pclA)$;

(5) a (τ_i, τ_j)-fuzzy regular open set if and only if $A = \tau_i - int\,(\tau_j - clA)$.

Definition 2.2. *[5,7,8] Let A be a fuzzy set of an fbts X. Then A is called*

(1) a (τ_i, τ_j)-fuzzy semiclosed set if and only if A^c is a (τ_i, τ_j)-fuzzy semiopen set;

(2) a (τ_i, τ_j)-fuzzy preclosed set if and only if A^c is a (τ_i, τ_j)-fuzzy preopen set;

(3) a (τ_i, τ_j)-fuzzy strongly semiclosed set if A^c is a (τ_i, τ_j)-fuzzy strongly semiopen set;

(4) a (τ_i, τ_j)-fuzzy strongly preclosed set if and only if A^c is a (τ_i, τ_j)-fuzzy strongly preopen set;

(5) a (τ_i, τ_j)-fuzzy regular closed set if and only if A^c is a (τ_i, τ_j)-fuzzy regular open set.

Definition 2.3. *[5,6,7,8] A mapping $f : (X, \tau_1, \tau_2) \to (Y, \eta_1, \eta_2)$ from an fbts X into an fbts Y is called*

(1) a fuzzy pairwise semicontinuous if $f^{-1}(B)$ is a (τ_i, τ_j)-fuzzy semiopen set of X, for each η_i-fuzzy open set B of Y;

(2) a fuzzy pairwise precontinuous if $f^{-1}(B)$ is a (τ_i, τ_j)-fuzzy preopen set of X, for each η_i-fuzzy open set B of Y;

(3) a fuzzy pairwise strongly semicontinuous if $f^{-1}(B)$ is a (τ_i, τ_j)-fuzzy strongly semiopen set of X, for each η_i-fuzzy open set B of Y ;

(4) a fuzzy pairwise strongly precontinuous if $f^{-1}(B)$ is a (τ_i, τ_j)-fuzzy strongly preopen set of X, for each η_i-fuzzy open set B of Y ;

(5) a fuzzy pairwise almost strongly precontinuuous if $f^{-1}(B)$ is a

(τ_i, τ_j)-fuzzy strongly preopen set of X, for each (η_i, η_j)-fuzzy regular open set B of Y ;

Definition 2.4. *[6,7,8] A mapping $f : (X, \tau_1, \tau_2) \to (Y, \eta_1, \eta_2)$ from an fbts X into an fbts Y is called*

(1) a fuzzy pairwise semiopen (semiclosed) if $f(A)$ *is a* (η_i, η_j)*-fuzzy semiopen set of* Y *(* (η_i, η_j)*-fuzzy semiclosed set of* Y *), for each* τ_i *-fuzzy open set* A *of* X *(*τ_i *-fuzzy closed set* A *of* X *);*

(2) a fuzzy pairwise preopen (preclosed) if $f(A)$ *is a* (η_i, η_j)*- fuzzy preopen set of* Y *(* (η_i, η_j)*-fuzzy preclosed set of* Y *), for each* τ_i *-fuzzy open set* A *of* X *(* τ_i *-fuzzy closed set* A *of* X *);*

(3) a fuzzy pairwise strongly semiopen (strongly semiclosed) if $f(A)$ *is a* (η_i, η_j)*-fuzzy strongly semiopen set of* Y *(* (η_i, η_j)*-fuzzy strongly semiclosed set of* Y *), for each* τ_i *-fuzzy open set* A *of* X *(* τ_i *-fuzzy closed set* A *of* X *);*

(4) a fuzzy pairwise strongly preopen (preclosed) if $f(A)$ *is a* (η_i, η_j)*-fuzzy preopen set of* Y *(*(η_i, η_j)*-fuzzy strongly preclosed set of* Y *), for each*

τ_i *-fuzzy open set* A *of* X *(*τ_i *-fuzzy closed set* A *of* X *).*

(5) a fuzzy pairwise semiopen (semiclosed) irresolution if $f(A)$ *is a* (η_i, η_j)*-fuzzy preopen set of* Y *(*(η_i, η_j)*-fuzzy strongly preclosed set of* Y *), for each*

(τ_i, τ_j)*- fuzzy semiopen set* A *of* X *(*(τ_i, τ_j)*-fuzzy semiclosed set* A *of* X *)).*

(6) a fuzzy pairwise regular open (closed) irresolution if $f(A)$ *is a* (η_i, η_j)*-fuzzy regular open set of* Y *(*(η_i, η_j)*-fuzzy regular closed set of* Y *), for each*

(τ_i, τ_j)*- fuzzy regular open set* A *of* X *(*(τ_i, τ_j)*-fuzzy regular closed set* A *of* X *)).*

(7) a fuzzy pairwise almost semiopen (semiclosed) if $f(A)$ *is* (η_i, η_j)*-fuzzy semiopen set of* Y *(*(η_i, η_j)*-fuzzy semiclosed set of* Y *), for each*

(τ_i, τ_j)*- fuzzy regular open set* A *of* X *(*(τ_i, τ_j)*-fuzzy regular closed set* A *of* X *).*

3. Fuzzy pairwise almost strongly preopen (preclosed) mappings

Definition 3.1. *A mapping* $f : (X, \tau_1, \tau_2) \to (Y, \eta_1, \eta_2)$ *from an fbts* X *into an fbts* Y *is called*

(1) a fuzzy pairwise almost strongly preopen if $f(A)$ *is* (η_i, η_j)*-fuzzy strongly preopen set of* Y*, for each* (τ_i, τ_j)*-fuzzy regular open set* A *of* X*.*

(2) a fuzzy pairwise almost strongly preclosed if $f(A)$ *is* (η_i, η_j)*-fuzzy strongly preclosed set of* Y*, for each* (τ_i, τ_j)*-fuzzy regular closed set* A *of* X*.*

Remark 3.1. Let $f : X \to Y$ be a mapping from an fbts X into an fbts Y. If f is a fuzzy pairwise strongly preopen (preclosed), then f is a fuzzy pairwise almost strongly preopen (preclosed) mapping. The following example shows that the converse statement may not be true.

Example 3.1. Let $X = \{a, b, c\}$ and A, B, C and be fuzzy sets of X defined as follows:

$$A(a) = 0,5 \qquad A(b) = 0,3 \qquad A(c) = 0,6;$$
$$B(a) = 0,3 \qquad B(b) = 0,4 \qquad B(c) = 0,3;$$

$$C(a) = 0,5 \qquad C(b) = 0,5 \qquad C(c) = 0,6.$$

If we put $\tau_1 = \tau_2 = \{0, B, A \vee B, 1\}$, $\eta_1 = \eta_2 = \{0, A, B, A \wedge B, A \vee B, 1\}$ and $f = id : (X, \tau_1, \tau_2) \to (Y, \eta_1, \eta_2)$ we conclude that f is a fuzzy pairwise almost strong precontinuous but f is not a fuzzy strong precontinuous mapping.

Theorem 3.1. *Let $f : X \to Y$ be a bijective mapping from an fbts X into an fbts Y. Then f is a fuzzy pairwise almost strongly preopen (preclosed) if and only if it is a fuzzy almost strongly preclosed (preopen).*

Proof. It can be proved by using the complement. □

Theorem 3.2. *Let $f : (X, \tau_1, \tau_2) \to (Y, \eta_1, \eta_2)$ be a bijective mapping from an fbts X into an fbts Y. Then f is a fuzzy pairwise almost strongly preopen (preclosed) if and only if f^{-1} is a fuzzy pairwise almost strong precontinuous.*

Proof. It follows from the relation $\left(f^{-1}\right)^{-1}(A) = f(A)$, for each (τ_i, τ_j)-fuzzy regular open (closed) set A of X. □

Theorem 3.3. *Let $f : (X, \tau_1, \tau_2) \to (Y, \eta_1, \eta_2)$ be a mapping from an fbts X into an fbts Y. Then f is a fuzzy pairwise almost strongly preopen if and only if $f(\tau_i - intA) \leq (\eta_i, \eta_j) - spintf(A)$, for each fuzzy (τ_i, τ_j)-semiclosed set A of X.*

Proof. Let f be a fuzzy pairwise almost strongly preopen mapping and let A be any fuzzy semiclosed set of X. Then $\tau_i - intA = \tau_i - int(\tau_j - clA)$. According to the assumption we have

$$f(\tau_i - intA) = f(\tau_i - int(\tau_j - clA)) = (\eta_i, \eta_j) - spintf(\tau_i - int(\tau_j - clA)) =$$
$$= (\eta_i, \eta_j) - spintf(\tau_i - intA) \leq (\eta_i, \eta_j) - spintf(A).$$

Conversely, let A be any (τ_i, τ_j)-fuzzy regular open set of X. Then A is a fuzzy semiclosed set of X. According to the assumption we have
$f(A) = f(\tau_i - intA) \leq (\eta_i, \eta_j) - spintf(A)$.
Thus $f(A)$ is a fuzzy strongly preopen set of Y, so f is a fuzzy pairwise almost strongly preopen mapping. □

Theorem 3.4. *Let $(X, \tau_1, \tau_2) \to (Y, \eta_1, \eta_2)$ be a mapping from an fbts X into an fbts Y. Then f is a fuzzy pairwise almost strongly preclosed if and only if (η_i, η_j)-$spclf(A) \leq f(\tau_i - clA)$, for each (τ_i, τ_j)-fuzzy semiopen set A of X.*

Proof. It can be proved in a similar manner as Theorem 3.3. □

Theorem 3.5. *Let $(X, \tau_1, \tau_2) \to (Y, \eta_1, \eta_2)$ be a bijective mapping from an fbts X into an fbts Y. Then the following statements are equivalent:*

(i) f is a fuzzy pairwise almost strongly preopen (preclosed) mapping;

(ii) $f(\tau_i - intA) \leq (\eta_i, \eta_j)$-$spintf(A)$, for each (τ_i, τ_j)-fuzzy semiclosed set A of X;

(iii) (η_i, η_j)-$spclf(A) \leq f(\tau_i - clA)$, for each (τ_i, τ_j)-fuzzy semiopen set A of X.

Proof. It follows from the Theorem 3.1, Theorem 3.3 and Theorem 3.4. □

Theorem 3.6. *Let $(X, \tau_1, \tau_2) \to (Y, \eta_1, \eta_2)$ be a mapping from an fbts X into an fbts Y. Then the following statements holds:*

(1) *f is a fuzzy pairwise almost strongly preopen mapping if and only if $f(\tau_i - intA) \leq \eta_i - int((\eta_j, \eta_i) - pclf(A))$, for each (τ_i, τ_j)-fuzzy semiclosed set A of X.*

(2) *f is a fuzzy pairwise almost strongly preclosed mapping if and only if $\eta_i - cl((\eta_j, \eta_i) - pintf(A)) \leq f(\tau_i - clA)$, for each (τ_i, τ_j)-fuzzy semiopen set A of X.*

Proof. We will prove the statements (1) only. Let f be a fuzzy pairwise almost strongly preopen mapping. Then, for any (τ_i, τ_j)-fuzzy semiclosed set A of X we have $\tau_i - intA = \tau_i - int(\tau_j - clA)$, so $f(\tau_i - intA)$ is a (η_i, η_j)-fuzzy strongly preopen set of Y. Thus

$$f(\tau_i - intA) \leq \eta_i - int((\eta_j, \eta_i) - pclf(\tau_i - intA)) \leq \eta_i - int((\eta_j, \eta_i) - pclf(A)).$$

Conversely, let A be any (τ_i, τ_j)-fuzzy regular open set of X. Then A is a (τ_i, τ_j)-fuzzy semiclosed set of X.

From $f(\tau_i - intA) \leq \eta_i - int((\eta_j, \eta_i) - pclf(A))$, it follows that $f(A)$ is a (η_i, η_j)-fuzzy strongly preopen set, so f is a fuzzy pairwise almost strongly preopen mapping. □

Theorem 3.7. *Let $(X, \tau_1, \tau_2) \to (Y, \eta_1, \eta_2)$ be a mapping from an fbts X into an fbts Y. Then f is a fuzzy pairwise almost strongly preopen if and only if for each fuzzy set B of Y and each (τ_i, τ_j)-fuzzy regular closed set A of X, $f^{-1}(B) \leq A$, there exists an (η_i, η_j)-fuzzy strongly preclosed set C of Y such that $B \leq C$ and $f^{-1}(C) \leq A$.*

Proof. Let B be any fuzzy set of Y and let A be a (τ_i, τ_j)-fuzzy regular closed set of X such that $f^{-1}(B) \leq A$. Then $A^c \leq f^{-1}(B^c)$, so $f(A^c) \leq \leq ff^{-1}(B^c) \leq B^c$. Since A^c is a (τ_i, τ_j)-fuzzy regular open set, $f(A^c)$ is an (η_i, η_j)-fuzzy strongly preopen set, so $f(A^c) \leq (\eta_i, \eta_j) - spintB^c$. Hence $A^c \leq f^{-1}f(A^c) \leq \leq f^{-1}((\eta_i, \eta_j) - spintB^c)$. The result follows for $C = (\eta_i, \eta_j) - spclB$.

Conversely, let U be any (τ_i, τ_j)-fuzzy regular open set of X. We will show that $f(U)$ is a fuzzy strongly preopen set of Y. From $U \leq f^{-1}f(U)$ follows that $U^c \geq (f^{-1}f(U))^c \geq f^{-1}f(U)^c$ where U^c is a (τ_i, τ_j)-fuzzy regular closed set of X. Hence there is an (η_i, η_j)-fuzzy strongly precolsed B of Y such that $B \geq f(U)^c$ and $f^{-1}(B) \leq U^c$. From $B \geq f(U)^c$ follows that $B \geq (\eta_i, \eta_j)$-$spclf(U)^c$, so $B^c \geq ((\eta_i, \eta_j) - spclf(U)^c)^c \leq ((\eta_i, \eta_j) - spintf(U)$. From $f^{-1}(B) \leq U^c$ we have $B \geq f^{-1}(B^c) \geq U$, so $B^c \geq ff^{-1}(B^c) \geq f(U)$. Hence $f(U) = (\eta_i, \eta_j) - spintf(U)$. Thus $f(U)$ is an (η_i, η_j)-fuzzy strongly preopen set, so f is a fuzzy pairwise almost strongly preopen mapping. □

Theorem 3.8. *Let $(X, \tau_1, \tau_2) \to (Y, \eta_1, \eta_2)$ be a mapping from an fbts X into an fbts Y. Then f is a fuzzy pairwise almost strongly preclosed mapping if and only if for each fuzzy set B of Y and each (τ_i, τ_j)-fuzzy regular open set A of X,*

$f^{-1}(B) \leq A$ there exists an (η_i, η_j)-fuzzy strongly preopen set C of Y such that $B \leq C$ and $f^{-1}(C) = A$.

Proof. It can be proved in a similar manner as Theorem 3.7. □

Theorem 3.9. *Let $(X, \tau_1, \tau_2) \rightarrow (Y, \eta_1, \eta_2)$ be a mapping from an fbts X into an fbts Y. The mapping f is fuzzy pairwise almost strongly semiopen (semiclosed) if and only if it is both fuzzy pairwise almost semiopen (semiclosed) and fuzzy pairwise almost strongly preopen (preclosed).*

Proof. Let A be any (τ_i, τ_j)-fuzzy regular open set of X. Then the mapping f is both fuzzy pairwise almost semiopen and fuzzy pairwise almost strongly preopen if and only if $f(A)$ is an (η_i, η_j)-fuzzy semiopen set of Y and $f(A)$ is an (η_i, η_j)-fuzzy strongly preopen set of Y. The results follows from the fact that any fuzzy set of Y is an (η_i, η_j)-fuzzy strongly semiopen set if and only if it is an (η_i, η_j)-fuzzy semiopen set and (η_i, η_j)-fuzzy strongly preopen set. □

Theorem 3.10. *Let $(X, \tau_1, \tau_2) \rightarrow (Y, \eta_1, \eta_2)$ be a mapping from an fbts X into an fbts Y. If the mapping f is both fuzzy pairwise almost strongly preopen (preclosed) and fuzzy pairwise semiclosed (semiopen) irresolution, then f is fuzzy regular (closed) open irresolution.*

Proof. Let A be any (τ_i, τ_j)-fuzzy regular open set of X. On the one hand , since f is fuzzy pairwise almost strongly preopen(preclosed) mapping it follows that $f(A)$ is an (η_i, η_j)-fuzzy strongly preopen (preclosed) set of Y. On the other hand f is fuzzy pairwise semiclosed (semiopen) irresolution, so $f(A)$ is an (η_i, η_j)-fuzzy semiclosed (semiopen) set of Y. This means that $f(A)$ is an (η_i, η_j)-fuzzy strongly preopen (preclosed) set and (η_i, η_j)-fuzzy semiclosed (semiopen) set, so $f(A)$ is an (η_i, η_j)-fuzzy regular open (closed) set of Y. Hence f is a fuzzy pairwise regular open (closed) irresolution. □

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