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FUZZY SP–IRRESOLUTE MAPPINGS

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Abstract

The aim of this paper is to introduce the fuzzy SP–irresolute continuous, fuzzy SP–irresolute open and fuzzy SP–irresolute closed mappings. Also we establish some of their characteristic properties.

1. Introduction

Continuity and its weaker forms constitute an important area in the field of general topological spaces. Since the fuzzy topological spaces and fuzzy continuity were introduced by Chang [2] early in 1968, the notion of the fuzzy continuity has been proven to be of fundamental importance in the realm of fuzzy topology. According to this, the fact that many workers ([1, 3, 4, 7, 8, 9] etc.) have studied weaker forms of fuzzy continuity, shows that the latter is very important in fuzzy topological spaces.

Mukherjee and Sinha in [4] introduced the concept of irresolute continuous mapping for the class of semiopen sets. Park in [5] first introduced the class of fuzzy semiprecontinuous mappings. Continuing this work, we define and characterize the concept of fuzzy SP–irresolute continuous mappings which is strictly stronger than the concept of fuzzy semiprecontinuous mappings. We introduce and investigate the fuzzy SP–irresolute open mappings, SP–irresolute closed mappings and SP–homeomorphism. Finally the results related to the composition of mappings, and the graph of a mapping are obtained.

2. Preliminaries

We introduce some basic notions and results that are used in the sequel.

In this work by (X, τ) or simply by X will be denoted fuzzy topological space (fts) due to Chang [2]. Int A, cl A and A^c, will denote respectively the interior, closure and complement of a fuzzy set A.

Definition 2.1. *A fuzzy set A of a fts (X, τ) is called*

(1) a fuzzy semiopen set of X if and only if there exists $U \in \tau$ such that $U \leq A \leq clU$ [1];

(2) a fuzzy preopen set of X if and only if $A \leq int(clA)$ [6];

(3) a fuzzy semipreopen set of X if and only there exists preopen set U of X such that $U \leq A \leq clU$ [5].

Definition 2.2. *A fuzzy set A of a fts X is called*

(1) a fuzzy semiclosed set of X if and only if and only if A^c is a fuzzy semiopen set [1];

(2) a fuzzy preclosed set of X if and only if and only if A^c is a fuzzy preopen set [6];

(3) a fuzzy semipreclosed set of X if and only if A^c is a fuzzy semipreopen set [5].

Lemma 2.1. *A fuzzy set A of a fts (X, τ) is*

(1) a fuzzy semiclosed set of X if and only if there exists a $U^c \in \tau$, such that $intU \leq A \leq U$ [1];

(2) a fuzzy preclosed set of X if and only if $A \geq cl(intA)$ [6];

(3) a fuzzy semipreclosed set of X if and only if there exists a fuzzy preclosed set U of X such that $intU \leq A \leq U$ [5]. □

Definition 2.3. *Let A be a fuzzy set of a fts X. Then:*

$pint\,A = \vee B \mid B \leq A$ and B is a fuzzy preopen set of X, is called fuzzy preinterior of A [6];

$pcl\,A = \wedge B \mid B \geq A$ and B is a fuzzy preopen set of X, is called fuzzy preclosure of A [6];

$spint\,A = \vee B \mid B \leq A$ and B is a fuzzy semipreopen set of X, is called fuzzy semipreinterior of A [5];

$spcl\,A = \wedge B \mid B \geq A$ and B is a fuzzy semipreclosed set of X, is called semipreclosure of A [5].

Lemma 2.2. *A fuzzy set A of fts X is a fuzzy semipreopen (semipreclosed) if and only if $A \leq cl(pint\, A)$. $(A \geq int(pcl\, A))$.* □

Lemma 2.3. *Let A be a fuzzy set of a fts X. Then:*

(1) $pcl\, A^c = (pint\, A)^c$ [6];

(2) $pint\, A^c = (pcl\, a)^c$ [6];

(3) $spcl\, A^c = (spint\, A)^c$ [5];

(4) $spint\, A^c = (spcl\, A)^c$ [5]. □

Lemma 2.4. [2] *Let $f: X \to Y$ be a mapping. For any fuzzy sets A and B of X and Y receptively, the following statements hold:*

(1) $ff^{-1}(B) \leq B$;

(2) $f^{-1}f(A) \geq A$;

(3) $f(A^c) \geq f(A)^c$;

(4) $f^{-1}(B^c) = f^{-1}(B)^c$;

(5) *If f is injective, then* $f^{-1}f(A) = A$;

(6) *If f is surjective, then* $ff^{-1}(B) = B$. □

Definition 2.4. *Let $f: (X, \tau_1) \to (Y, \tau_2)$ be a mapping from a fts (X, τ) into a fts (Y, τ_2). The mapping f is called*

(1) *fuzzy semicontinuous if $f^{-1}(B)$ is a fuzzy semipreopen set of X, for each $B \in \tau_2$* [5];

(2) *fuzzy semipreopen (semipreclosed) if $f(A)$ is a fuzzy semipreopen (semipreclosed) set of Y, for each $A \in \tau_1$ $(A^c \in \tau_1)$.*

Definition 2.5. [1] *A* fts *(X, τ_1) is product related to* fts *(Y, τ_2) if for any fuzzy set A of X and B of Y whenever $C^c \not\leq A$ and $D^c \not\leq B$ implies $C^c \times 1 \vee 1 \times D^c \geq A \times B$, where $C \in \tau_1$ and $D \in \tau_2$, there exist $C_1 \in \tau_1$ and $D_1 \in \tau_2$ such that $C_1^c \geq A$ or $D_1^c \geq B$ and*

$$C_1^c \times 1 \vee 1 \times D_1^c = C^c \times 1 \vee 1 \times D^c .$$

Lemma 2.5. [5] *Let X and Y be a fts such that X is a product related to Y. Then, the product $A \times B$ of a fuzzy semipreopen set A of X and a fuzzy semipreopen set B of Y is a fuzzy semipreopen set of the product fuzzy space $X \times Y$.* □

Lemma 2.6. [1] *Let $g: X \to X \times Y$ be the graph of a mapping $f: X \to Y$. If A is a fuzzy set of X and B is a fuzzy set of Y, then*

$$g^{-1}(A \times B) = A \wedge f^{-1}(B). \square$$

3. Fuzzy SP-irresolute continuous mappings

Definition 3.1. *A mapping $f: X \to Y$ from a fts X into a fts Y is called a SP irresolute continuous mapping if $f^{-1}(B)$ is a fuzzy semipreopen set of X for each fuzzy semipreopen set B of Y.*

Theorem 3.1. *Let $f: X \to Y$ be a mapping from a fts X into a fts Y. Then the following statements are equivalent:*

(i) *f is a fuzzy SP irresolute continuous;*

(ii) *$f^{-1}(B)$ is a fuzzy semipreclosed set of X for each fuzzy semipreclosed set B of Y;*

(iii) *$f(spcl\, A) \leq spcl\, f(A)$, for each fuzzy set A of X;*

(iv) *$spcl\, f^{-1}(B) \leq f^{-1}(spcl\, B)$, for each fuzzy set B of Y;*
(v) *$f^{-1}(spint\, B) \leq spint\, f^{-1}(B)$, for each fuzzy set B of Y.*

Proof: (i)⇒(ii) Let B be a fuzzy semipreclosed set of Y. Then B^c is a fuzzy semipreopen set of Y. According to (i) $f^{-1}(B^c)$ is a fuzzy semipreopen set of X. From Lemma 2.4 we obtain that $f^{-1}(B)$ is a fuzzy semipreclosed set of X.

(ii)⇒(iii) Let A be a fuzzy set of X. Then spcl $f(A)$ is a fuzzy semipreclosed set of Y, and from (ii), $f^{-1}(\text{spcl}\, f(A))$ is a fuzzy semipreclosed set of X. Hence

$$\text{spcl}\, A \leq \text{spcl}\, f^{-1} f(A) \leq \text{spcl}\, f^{-1}(\text{spcl}\, f(A)) = f^{-1}(\text{spcl}\, f(A)).$$

Thus $f(\text{spcl}\, A) \leq \text{spcl}\, f(A)$.

(iii)⇒(iv) Let B be a fuzzy set of Y. According to (iii), $f(\text{spcl}\, f^{-1}(B)) \leq \text{spcl}\, f f^{-1}(B)) \leq \text{spcl}\, B$. Thus $\text{spcl}\, f^{-1}(B) \leq f^{-1} f(\text{spcl}\, f^{-1}(B)) \leq f^{-1}(\text{spcl}\, B)$.

(iv)⇒(v) Let B be a fuzzy set of Y. From (iv) $f^{-1}(\text{spcl}\, B^c) \geq \text{spcl}\, f^{-1}(B^c) = \text{spcl}\, f^{-1}(B)^c$, so $f^{-1}(\text{spint}\, B)^c = f^{-1}(\text{spcl}\, B^c) \geq \text{spcl} f^{-1}(B)^c = (\text{spint}\, f^{-1}(B))^c$. Hence $f^{-1}(\text{spint}\, B) \leq \text{spint}\, f^{-1}(B)$.

(v)⇒(i) Let B be a fuzzy semipreopen set of Y. Then $B = \text{spint}\, B$. From (v) we obtain $f^{-1}(B) = f^{-1}(\text{spint}\, B) \le \text{spint}\, f^{-1}(B) \le f^{-1}(B)$. Thus $f^{-1}(B) = \text{spint}\, f^{-1}(B)$, so $f^{-1}(B)$ is a fuzzy semipreopen set of X. Hence f is a fuzzy SP-irresolute continuous mapping. □

Theorem 3.2. *Let $f: X \to Y$ be a bijective mapping from a fts X into a fts Y. The mapping f is a fuzzy SP irresolute continuous if and only if $\text{spint}\, f(A) \le f(\text{spint}\, A)$, for each fuzzy set A of X.*

Proof: Let f be a fuzzy SP-irresolute continuous. For any fuzzy set A of X, $f^{-1}(\text{spint}\, f(A))$ is a fuzzy semipreopen set of X. Because f is injective, from Theorem 3.1 we obtain that

$$f^{-1}(\text{spint}\, f(A)) \le \text{spint}\, f^{-1} f(A) = \text{spint}\, A\,.$$

Again, since f is surjective, we get

$$\text{spint}\, f(A) = f f^{-1}(\text{spint}\, f(A)) \le f(\text{spint}\, A)\,.$$

Conversely, let B be a fuzzy semipreopen set of Y. Then $\text{spint}\, B = B$. Because f is surjective, from assumption we obtain that
$f(\text{spint}\, f^{-1}(B)) \ge \text{spint}\, f f^{-1}(B) = \text{spint}\, B = B$. This implies that
$f^{-1} f(\text{spint}\, f^{-1}(B) \ge f^{-1}(B)$. Since f is injective we get
$\text{spint}\, f^{-1}(B) = f^{-1} f(\text{spint}\, f^{-1}(B) \ge f^{-1}(B)$. Hence
$\text{spint}\, f^{-1}(B) = f^{-1}(B)$, i.e. $f^{-1}(B)$ is a fuzzy semipreopen set. Thus f is a fuzzy SP-irresolute continuous. □

Theorem 3.3. *Let $f: X \to Y$ be a mapping from a fts X into a fts Y. Then the following statements are equivalent:*

(i) *f is a fuzzy SP irresolute continuous;*

(ii) *$f^{-1}(\text{spint}\, B) \le \text{cl}\,(\text{pint}\, f^{-1}(B))$, for each fuzzy set B of Y;*

(iii) *$f^{-1}(\text{spcl}\, B) \ge \text{int}\,(\text{pcl}\, f^{-1}(B))$, for each fuzzy set B of Y.*

Proof: (i)⇒(ii) Let B be any fuzzy set of Y. Then, according to (i), $f^{-1}(\text{spint}\, B)$ is a fuzzy semipreopen set of X, so

$$f^{-1}(\text{spint}\, B) \le \text{cl}\,(\text{pint}\, f^{-1}(\text{spint}\, B)) \le \text{cl}\,(\text{pint}\, f^{-1}(B))\,.$$

(ii)⇒(iii) Let B be a fuzzy set of Y. From assumption we get
$f^{-1}(\text{spint}\, B^c) \le \text{cl}\,(\text{pint}\, f^{-1}(B^c))$. Hence
$f^{-1}(\text{spcl}\, B)^c \le (\text{int}\,(\text{pcl}\, f^{-1}(B)))^c$ so $f^{-1}(\text{spcl}\, B) \ge \text{int}\,(\text{pcl}\, f^{-1}(B))$.

(iii)⇒(i) Let B be a fuzzy semipreopen set of Y. Then B^c is a fuzzy semipreclosed set of Y. According to (iii)

$$f^{-1}(B) = f^{-1}(\text{spcl } B) \geq \text{int}\,(\text{pcl}\, f^{-1}(B)) .$$

We conclude that $f^{-1}(B)$ is fuzzy semipreclosed set of X. From $f^{-1}(B^c) = f^{-1}(B)^c$ it follows that $f^{-1}(B)$ is a fuzzy semipreopen set of X. □

Theorem 3.4. *Let $f: X \to Y$ be a mapping from a fts X into a fts Y. If f is SP irresolute continuous then $f^{-1}(B) \leq \text{spint}\, f^{-1}(cl\,(\text{pint}\, B))$ for each fuzzy semipreopen set B of Y.*

Proof: Let B be a fuzzy semipreopen set of Y. Then $f^{-1}(B) \leq f^{-1}(\text{cl}\,(\text{pint}\, B))$. Because $f^{-1}(B)$ is fuzzy semipreopen set of X, we get

$$f^{-1}(B) \leq \text{spint}\, f^{-1}(\text{cl}\,(\text{pint}\, B)) . \square$$

Theorem 3.5. *Let $f: X \to Y$ be a from a fts X into a fts Y. If f is a SP irresolute continuous, then f is a fuzzy semiprecontinuous mapping.*

Proof: Every fuzzy open set is a fuzzy semipreopen set. □

Remark. Conversely of Theorem 2.1 may not be true.

Example 3.1. Let $X = \{a, b, c\}$ and A, B, C and D be fuzzy sets of X defined as follows:

$$A(a) = 0.5 \qquad A(b) = 0.3 \qquad A(c) = 0.6;$$

$$B(a) = 0.3 \qquad B(b) = 0.4 \qquad B(c) = 0.3;$$

$$C(a) = 0.2 \qquad C(b) = 0.6 \qquad C(c) = 0.6;$$

$$D(a) = 0.7 \qquad D(b) = 0.7 \qquad D(c) = 0.7 .$$

Let $\tau_1 = \{0, A, B, A \wedge B, A \vee B, 1\}$, $\tau_2 = \{0, C, 1\}$ and $f = \text{id}: (X, \tau_1) \to (X, \tau_2)$. By easy computations it can be seen that f is fuzzy semiprecontinuous, but f is not fuzzy SP irresolute continuous mapping.

Theorem 3.6. *Let $f: X \to Y$ and $g: Y \to Z$ be mappings, where X, Y and Z are fts.*

(1) If f and g are fuzzy SP irresolute continuous, then gf is also SP irresolute continuous.

(2) *If f is fuzzy SP–irresolute continuous and g is fuzzy semiprecontinuous, then gf is fuzzy semiprecontinuous.*

Proof: The results follows from the fact that for any fuzzy set B of Z, $(gf)^{-1}(B) = f^{-1}(g^{-1}(B))$.□

Corollary 3.7. *Let X, X_1 and X_2 be fts and $p_i: X_1 \times X_2 \to X_i$ $(i = 1, 2)$ be the projection of $X_1 \times X_2$ onto X_i. If $f: X \to X \times X$ is a fuzzy SP–irresolute mapping, then $p_i f$ is a fuzzy SP–irresolute continuous.*

Proof: The projections p_i $(i = 1, 2)$ are fuzzy SP–irresolute continuous mappings. □

Theorem 3.8. *Let $f: X \to Y$ be a mapping from a fts X into a fts Y. If the graph $g: X \to X \times Y$ of f is SP–irresolute continuous, then f is also SP–irresolute continuous.*

Proof: From Lemma 2.6, for each fuzzy semipreopen set B of Y, $f^{-1}(B) = 1 \wedge f^{-1}(B) = g^{-1}(1 \times B)$. Since g is SP–irresolute and $1 \times B$ is a fuzzy semipreopen set of $X \times Y$, $f^{-1}(B)$ is a fuzzy semipreopen set of X, and hence f is a fuzzy SP–irresolute continuous mapping. □

4. Fuzzy SP–irresolute open and SP–irresolute closed mappings

Definition 4.1. *Let $f: X \to Y$ be a mapping from a fts X to a fts Y. The mapping f is called a fuzzy SP–irresolute open (closed) mapping if $f(A)$ is a fuzzy semipreopen (semipreclosed) set of Y, for each fuzzy semipreopen (semipreclosed) set A of X.*

Theorem 4.1. *Let $f: X \to Y$ and $g: Y \to Z$ be mappings, where X, Y and Z are fts.*

(1) *If f and g are fuzzy SP–irresolute open (closed), then gf is also SP–irresolute open (closed).*

(2) *If f is fuzzy semipreopen (semipreclosed) and g is fuzzy SP–irresolute open, (closed) then gf is fuzzy semipreopen (semipreclosed) mapping.*

Proof: The results follows from the fact that for any fuzzy set A of X, $(gf)(A) = g(f(A))$.□

Theorem 4.2. *Let $f: X \to Y$ be a mapping from a fts X into a fts Y. Then the following statements are equivalent:*

(i) *f is an SP irresolute open mapping;*

(ii) *$f(\text{spint } A) \leq \text{spint } f(A)$, for each fuzzy set A of X;*

(iii) *$\text{spint } f^{-1}(B) \leq f^{-1}(\text{spint } B)$, for each fuzzy set B of Y;*

(iv) *$f^{-1}(\text{spcl } B) \leq \text{spcl } f^{-1}(B)$, for each fuzzy set B of Y.*

Proof: (i)⇒(ii) Let A be any fuzzy set of X. Then $f(\text{spint } A) = \text{spint } f(\text{spint } A) \leq \text{spint } f(A)$.

(ii)⇒(iii) Let B be any fuzzy set of Y. Therefore, according to (ii) $f(\text{spint } f^{-1}(B)) \leq \text{spint } ff^{-1}(B) \leq \text{spint } B$. Hence $\text{spint } f^{-1}(B) \leq f^{-1}f(\text{spint } (f^{-1}(B)) \leq f^{-1}(\text{spint } B)$.

(iii)⇒(iv) Let B be any fuzzy set of Y. From assumption we get $\text{spint } f^{-1}(B^c) \leq f^{-1}(\text{spint } B^c)$, so $f^{-1}(\text{spcl } B)) \leq \text{spcl } f^{-1}(B)$.

(iv)⇒(iii) Let B be any fuzzy set of Y. From (iv) we obtain that $f^{-1}(\text{spcl } B^c) \leq \text{spcl } f^{-1}(B^c)$, so $\text{spint } f^{-1}(B) \leq f^{-1}(\text{spint } B)$.

(iii)⇒(i) Let A be any fuzzy semipreopen set of X. Then $A = \text{spint } A$. According to assumption $A = \text{spint } A \leq \text{spint } f^{-1}f(A) \leq f^{-1}(\text{spint } f(A))$, so $f(A) \leq ff^{-1}(\text{spint } f(A)) \leq \text{spint } f(A)$. Hence $f(A) = \text{spint } f(A)$, i.e. $f(A)$ is a fuzzy semipreopen set of Y. □

Theorem 4.3. *Let $f: X \to Y$ is a mapping where X and Y are fuzzy spaces. Then the following statements are equivalent:*

(i) *f is a fuzzy SP irresolute open mapping;*

(ii) *$f(\text{spint } A) \leq \text{cl}\,(\text{pint } f(A))$ for every fuzzy set A of X.*

Proof: (i)⇒(ii) Let A be a fuzzy set of X. Then $f(\text{spint } A)$ is fuzzy semipreopen set of Y, so $f(\text{spint } A) \leq \text{cl}\,(\text{pint } f(\text{spint } A)) \leq \text{cl}\,(\text{pint } f(A))$.

(ii)⇒(i) Let A be a fuzzy semipreopen set of X. From $f(A) = f(\text{spint } A) \leq \text{cl}\,(\text{pint } f(A))$, it follows that $f(A)$ is fuzzy semipreopen set of Y, so f is fuzzy SP irresolute open mapping. □

Theorem 4.4. *Let $f: X \to Y$ be a mapping from a fts X to a fts Y. Then the following statements are equivalent:*

(i) *f is a fuzzy SP–irresolute closed mapping;*

(ii) *spcl $f(A) \leq f(\text{spcl}\, A)$ for every fuzzy set A of X.*

Proof: (i)⇒(ii) Let f be a fuzzy SP–irresolute closed mapping and A any fuzzy set of X. Then $f(\text{spcl}\, A)$ is a fuzzy semipreclosed set of Y. From $f(A) \leq f(\text{spcl}\, A)$ follows that $\text{spcl}\, f(A) \leq f(\text{spcl}\, A)$.

(ii)⇒(i) Let A be a fuzzy semipreclosed set of X. From $f(A) = f(\text{spcl}\, A) \geq \text{spcl}\, f(A)$, we obtain $\text{spcl}\, f(A) = f(A)$, so f is a fuzzy SP–irresolute closed mapping. □

Theorem 4.5. *Let $f: X \to Y$ be a mapping from a fts X into a fts Y.*

(1) *If $f(cl\,(pint\, A)) \leq cl\,(pint\, f(A))$, for every fuzzy semipreopen set A of X, then f is a fuzzy SP–irresolute open mapping.*

(2) *If $f(int\,(pcl\, A)) \geq int\,(pcl\, f(A))$, for every fuzzy semipreclosed set A of X, then f is a fuzzy SP–irresolute closed mapping.*

Proof We prove only (1). Let A be any fuzzy semipreopen set of X. Then $A \leq \text{cl}(\text{pint}\, A)$. According to assumption $f(A) \leq f(\text{cl}(\text{pint}\, A)) \leq \text{cl}\,(\text{pint}\,(f(A)))$, so $f(A)$ is fuzzy semipreopen set of Y, i.e. f is SP–irresolute open mapping. □

Theorem 4.6. *Let $f: X \to Y$ is a bijective mapping, where X and Y are fts. Then the following statements are equivalent:*

(i) *f is a fuzzy SP–irresolute closed mapping;*

(ii) *$f^{-1}(\text{spcl}\, B) \leq (\text{spcl}\, f^{-1}(B))$ for each fuzzy set B of Y;*

(iii) *$\text{spint}\, f^{-1}(B) \leq f^{-1}(\text{spint}\, B))$ for each fuzzy set B of Y.*

Proof: (i)⇒(ii) Let f be a fuzzy SP–irresolute closed. For any fuzzy set B of Y, $f(\text{spcl}\, f^{-1}(B))$ is a fuzzy semipreclosed set of Y. Because f is surjective, we obtain that $f(\text{spcl}\, f^{-1}(B)) \geq \text{spcl}\, ff^{-1}(B) = \text{spcl}\, B$. Again, since f is injective, $\text{spcl}\, f^{-1}(B) = f^{-1}f(\text{spcl}\, f^{-1}(B)) \geq f^{-1}(\text{spcl}\, B)$.

(ii)⇒(iii) Let B be any fuzzy set of Y. From (ii) we obtain

$$f^{-1}(\text{spcl}\, B^c) \leq \text{spcl}\, f^{-1}(B^c), \quad \text{so} \quad (\text{spint}\, f^{-1}(B)) \leq f^{-1}(\text{spint}\, B).$$

(iii)⇒(ii) Let B be any fuzzy set of Y. According to assumption $\text{spint}\, f^{-1}(B^c) \leq f^{-1}(\text{spint}\, B^c)$, so $f^{-1}(\text{spcl}\, B) \leq \text{spcl}\, f^{-1}(B)$.

(ii)⇒(i) Let A be a fuzzy semipreclosed set of X. Then $\text{spcl}\, A = A$. Because f is injective, from assumption we obtain,

$f^{-1}(\text{spcl}\, f(A)) \le \text{spcl}\, f^{-1}f(A) = \text{spcl}\, A = A$. This implies that $ff^{-1}(\text{spcl}\, f(A)) \le f(A)$. Since f is surjective, we have $\text{spcl}\, f(A) = f(A)$, so $f(A)$ is a fuzzy semipreclosed set. Thus f is a fuzzy SP irresolute closed. □

Theorem 4.7. *Let $f: X \to Y$ is a mapping from fts X into a fts Y. Then the following statements are equivalent:*

(i) *f is a fuzzy SP irresolute closed mapping;*

(ii) *$\text{int}(\text{pcl}\, f(A)) \le f(\text{spcl}\, A)$ for every fuzzy set A of X.*

Proof: (i)⇒(ii) Let A be a fuzzy set of X. Then $f(\text{spcl}\, A)$ is fuzzy semipreclosed set of Y, so $f(\text{spcl}\, A) \ge \text{int}\left(\text{pcl}\, f(\text{spcl}\, A)\right) \ge \text{int}\left(\text{pcl}\, f(A)\right)$.

(ii)⇒(i). Let A be a fuzzy semipreclosed set of X. From $f(A) = f(\text{spcl}\, A) \ge \text{int}\, \text{pcl}\, f(A)$, follows that $f(A)$ is fuzzy semipreclosed, so f is fuzzy SP irresolute closed mapping. □

Theorem 4.8. *Let $f: X \to Y$ be a mapping from a fts X into a fts Y. The mapping f is fuzzy SP irresolute open if and only if for each fuzzy set B of Y and each fuzzy semipreclosed set A of X, $f^{-1}(B) \le A$, there exists a fuzzy semipreclosed set C of Y such that $B \le C$ and $f^{-1}(C) \le A$.*

Proof: Let B be a fuzzy set of Y and A be a fuzzy semipreclosed set of X such that $f^{-1}(B) \le A$. Then $A^c \le f^{-1}(B^c)$, or $f(A^c) \le ff^{-1}(B^c) \le B^c$. Since A^c is a fuzzy semipreopen set of X we obtain that $f(A^c)$ is a fuzzy semipreopen set of Y and hence $f(A^c) \le \text{spint}\, B^c$. So $A^c \le f^{-1}f(A^c) \le f^{-1}(\text{spint}\, B^c)$. Hence $A \ge f^{-1}(\text{spint}\, B^c)^c = f^{-1}(\text{spcl}\, B)$. The result follows for $C = \text{spcl}\, B$.

Conversely, let U be a fuzzy semipreopen set of X. We want to show that $f(U)$ is a fuzzy semipreopen set of Y. From $U \le f^{-1}f(U)$ follows that $U^c \ge f^{-1}f(U)^c$ where U^c is a fuzzy semipreclosed set of X. Hence there is a fuzzy semipreclosed set B of Y such that $B \ge f(U)^c$ and $f^{-1}(B) \le U^c$. Hence from $B \ge f(U)^c$ follows $B \ge \text{spcl}\, f(U)^c$ or $B^c \le (\text{spcl}\, f(U)^c)^c = \text{spint}\, f(U)$. From $f^{-1}(B) \le U^c$ we obtain $f^{-1}(B^c) \ge U$ or $B^c \ge ff^{-1}(B^c) \ge f(U)$. Since $f(U) \le B^c \le \text{spint}\, f(U)$, we get $f(U) = \text{spint}\, f(U)$. Thus $f(U)$ is a fuzzy semipreopen set, so f is a fuzzy SP irresolute open mapping. □

Theorem 4.9. *Let $f: X \to Y$ be a mapping from a fts X into a fts Y. Then f is fuzzy SP irresolute closed if and only if for each fuzzy set A of Y and each fuzzy semipreopen set U of X, $f(A) \le U$, there exists a fuzzy semipreopen set B of Y such that $A \le B$ and $f^{-1}(B) \le U$.*

Proof: Let A be a fuzzy set of Y and U be a fuzzy semipreopen set of X such that $f^{-1}(A) \leq U$. Then $f(U^c)$ is a fuzzy semipreclosed set of Y. Put $B = f(U^c)^c$. Then B is a semipreopen set of Y, $A \leq B$ and

$$f^{-1}(B) = f^{-1}(f(U^c)^c) = f^{-1}f(U^c)^c \leq (U^c)^c = U.$$

Conversely, let U be a fuzzy semipreclosed set of X. Then U^c is fuzzy semipreopen set of X and $U^c \leq f^{-1}f(U^c)$. According to assumption there exists fuzzy semipreopen set B of Y such that $f(U^c) \leq B$ and $f^{-1}(B) \leq U^c$. Hence $f(U) = B^c$ is fuzzy semipreclosed set of Y. □

Theorem 4.10. *Let $f: X \to Y$ and $g: Y \to Z$ be mappings, where X, Y and Z are fts and gf be a fuzzy SP-irresolute open (closed) mapping.*

(1) *If g is a fuzzy SP-irresolute continuous and injective, then f is fuzzy SP-irresolute open (closed).*

(2) *If f is a fuzzy SP-irresolute continuous and surjective, then g is fuzzy SP-irresolute open (closed).*

Proof: (1) For any fuzzy semipreopen (semipreclosed) set A of X, according to assumption $g^{-1}(gf)(A) = f(A)$ is a fuzzy semipreopen (semipreclosed) set of Y. Hence f is fuzzy SP–irresolute open (closed). □

(2) For any fuzzy semipreopen (semipreclosed) set B of Y, $f^{-1}(B)$ is a fuzzy semipreopen (semipreclosed) set of X, since f is SP-irresolute continuous. Since gf is fuzzy SP-irresolute open (closed) and f is surjective, $(gf)(f(B)) = g(B)$ is a fuzzy semipreopen (semipreclosed) set of Z. Hence g is fuzzy SP–irresolute open (closed). □

5. Fuzzy SP–homeomorphism

Definition 5.1. *A bijective mapping $f: X \to Y$ is called a fuzzy SP-homeomorphism if both f and f^{-1} are fuzzy SP-irresolute continuous.*

Theorem 5.1. *Let $f: X \to Y$ be a mapping from a fts X into a fts Y. The following statements are equivalent:*

(i) *f is a fuzzy SP-homeomorphism;*

(ii) *f^{-1} is a fuzzy SP-homeomorphism;*

(iii) *f is a fuzzy SP-irresolute continuous and fuzzy SP-irresolute open;*

(iv) *f is a fuzzy SP-irresolute continuous and fuzzy SP-irresolute closed;*

(v) *$f(spcl\, A) = spcl\, f(A)$, for every fuzzy set A of X.*

Proof: (i) $\Rightarrow$ (ii) Follows immediately from definition of SP homeomorphism.

(i) $\Rightarrow$ (iii) Let A be a fuzzy set of X. Since f^{-1} is SP irresolute continuous we get $(f^{-1})^{-1}(\text{spint } A) \leq \text{spint } (f^{-1})^{-1}(A)$ and hence $f(\text{spint } A) \leq \text{spint } f(A)$.

(iii)$\Rightarrow$(iv) Let A be a fuzzy set of X. Since f is SP irresolute open and bijective we obtain that $f(\text{spint } A^c) \leq \text{spint } f(A)$ and hence $f(\text{spcl } A)^{\leq} (\text{spcl } f(A))^c$, i.e. $f(\text{spcl } A) \geq \text{spcl } f(A)$.

(iv)$\Rightarrow$(v) Let A be any fuzzy set of X. Since f is fuzzy SP irresolute continuous we get $f(\text{spcl } A) \leq \text{spcl } f(A)$. Again since f is fuzzy SP irresolute closed we obtain $\text{spcl } f(A) \leq f(\text{spcl } A)$.

(v)$\Rightarrow$(i) Let A be any fuzzy set of X. From $(f^{-1})^{-1}(\text{spcl } A) \geq \text{spcl}(f^{-1})^{-1}(A)$ it follows that f^{-1} is a fuzzy SP irresolute continuous. Relation $f(\text{spcl } A) \leq \text{spcl } f(A)$ implies that f is SP irresolute continuous. □

Theorem 5.2. *For a fuzzy SP homeomrphism f between fts X and fts Y the following properties hold:*

(1) *$f^{-1}(B)$ is a fuzzy semipreopen set of X, for each fuzzy semipreopen set B of Y ;*

(2) *$f(A)$ is a fuzzy semipreopen set of X, for each fuzzy semipreopen set A of X;*

(3) *$f^{-1}(B)$ is a fuzzy semipreclosed set of X, for each fuzzy semipreclosed set B of Y ;*

(4) *$f(A)$ is a fuzzy semipreclosed set of X, for each fuzzy semipreclosed set A of X;*

(5) *$\text{spcl } f^{-1}(B) = f^{-1}(\text{spcl } B)$, for each fuzzy set B of Y ;*

(6) *$f(\text{spcl } A) = \text{spcl } f(A)$, for each fuzzy set A of X;*

(7) *$f^{-1}(\text{spint } B) = \text{spint } f^{-1}(B)$, for each fuzzy set B of Y ;*

(8) *$f(\text{spint } A) = f(\text{spint } A)$, for each fuzzy set A of X.*

Proof: It follows easily from Theorem 3.1, Theorem 3.2, Theorem 4.2, Theorem 4.4 and Theorem 4.6. □

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ФАЗИ СП–ИРЕЗОЛУТОРНИ ПРЕСЛИКУВАЊА

Билјана Крстеска

Р е з и м е

Во трудот [5] е воведен поимот за фази семинепрекинато пресликување. Како продолжение во оваа работа е воведен концептот на фази ирезолуција во класата на фази семипреотворени множества.

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