Математички Билтен
24 (L)
2000 (23-36)
Скопје, Македонија

ISSN 0351-336X

# NEW COUNTERPARTS OF SOME INEQUALITIES FOR ENTROPY AND MUTUAL INFORMATION

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## Abstract

In this paper, by the use of a new analytic inequality which counterparts arithmetic mean-geometric mean's classical inequality, we point out some new results for entropy and mutual information.

## 1. Introduction

Suppose that $X$ is a discrete random variable whose range $R = \{x_1, \ldots, x_r\}$ is finite. Let $p_i = P\{X = x_i\}$, $i = 1, \ldots, r$ and assume that $p_i > 0$ for all $i \in \{1, \ldots, r\}$. Define the $b$-entropy of $X$ by

$$H_b(X) := \sum_{i=1}^{r} p_i \log_b \frac{1}{p_i}, \tag{1.1}$$

where $b > 1$. The following theorem is well known in the literature and concerns the maximum possible value for $H_b(X)$ in terms of the size of the range $R$ [3, p. 17]:

**Theorem 1.1.** *Let $X$ have values in $R = \{x_1, \ldots, x_r\}$. Then*

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AMS Subject Classification 26D15, 99A20

$$0 \le H_b(X) \le \log_b r \,. \tag{1.2}$$

*Furthermore,* $H_b(X) = 0$ *iff* $p_i = 1$ *for some* $i$ *and* $H_b(X) = \log_b r$ *iff* $p_i = 1/r$ *for all* $i \in \{1, \ldots, r\}$.

In the recent paper [1], S. S. Dragomir and C. J. Goh proved the following inequality for $\log_b$-mapping [1, Proposition 6.3]:

**Lemma 1.2.** *Let* $\xi_k \in (0, \infty)$ *and* $p_k > 0$ *with* $\sum_{k=1}^{n} p_k = 1$. *Then*

$$0 \le \log_b \left( \sum_{k=1}^{n} p_k \xi_k \right) - \sum_{k=1}^{n} p_k \log_b \xi_k \le \frac{1}{2 \ln b} \sum_{i,j=1}^{n} \frac{p_i p_j}{\xi_i \xi_j} (\xi_i - \xi_j)^2 \,. \tag{1.3}$$

*The equality holds simultaneously in both inequalities iff* $\xi_1 = \cdots = \xi_n$.

They have applied this lemma for the $b$-entropy mapping and obtained the following counterpart of (1.2):

**Theorem 1.3.** *Let* $X$ *be defined as above. Then*

$$0 \le \log_b r - H_b(X) \le \frac{1}{\ln b} \left[ r \sum_{i=1}^{r} p_i^2 - 1 \right] = \frac{1}{\ln b} \sum_{1 \le i < j \le r} (p_i - p_j)^2 \,. \tag{1.4}$$

*Furthermore, the equality holds simultaneously in all the above ineqalities iff* $p_i = 1/r$ *for all* $i = 1, 2, \ldots, r$.

The following corollary gives a tighter lower bound to the entropy when the outcomes of $X$ are close to uniformity.

**Corollary 1.4.** *Let* $\varepsilon > 0$ *be given and* $X$ *be as above with*

$$\max_{1 \le i < j \le r} |p_i - p_j| \le \left[ \frac{2\varepsilon \ln b}{r(r-1)} \right]^{1/2} .$$

*Then we have*

$$0 \le \log_b r - H_b(X) \le \varepsilon \,. \tag{1.5}$$

For other results concerning conditional entropy, mutual information, conditional mutual information etc., see the recent papers [1, 2] where further references are given.

## 2. A New Counterpart Inequality

We shall start with the following lemma which is interesting in itself too:

**Lemma 2.1.** *Let $\xi_k \in [m,\infty)$ where $m>0$ and $p_k \geq 0$ $(k=1,\dots,n)$ with $\sum_{k=1}^{n} p_k = 1$. Then we have the inequality:*

$$0 \leq \log_b\left(\sum_{k=1}^{n} p_k\xi_k\right) - \sum_{k=1}^{n} p_k \log_b \xi_k \leq \frac{1}{m\ln b}\sum_{i=1}^{n} p_i \left|\xi_i - \sum_{j=1}^{n} p_j\xi_j\right| . \quad (2.1)$$

*Furthermore, the equality holds in all inequalities simultaneously iff $\xi_1 = \cdots = \xi_n$.*

**Proof.** By Lagrange's theorem we find we find for every $x, y \in [m,\infty)$ an element $\eta$ between $x$ and $y$ so that

$$\log_b x - \log_b y = \frac{1}{\eta \ln b}(x-y).$$

As $\eta \geq m$, we deduce

$$|\log_b x - \log_b y| \leq \frac{1}{m\ln b}|x-y| \quad (2.2)$$

for all $x, y \in [m,\infty)$. If we choose $y = \xi_i$ and $x = \sum_{j=1}^{n} p_j\xi_j$ in the above inequality (2.2), then we have

$$\left|\log_b\left(\sum_{j=1}^{n} p_j\xi_j\right) - \log_b \xi_i\right| \leq \frac{1}{m\ln b}\left|\sum_{j=1}^{n} p_j\xi_j - \xi_i\right| \quad (2.3)$$

for all $i \in \{1,\dots,n\}$. If we multiply (2.3) with $p_i \geq 0$ and summing over $i$ to 1 at $n$, then we derive:

$$\sum_{i=1}^{n} p_i\left|\log_b\left(\sum_{j=1}^{n} p_j\xi_j\right) - \log_b \xi_i\right| \leq \frac{1}{m\ln b}\sum_{i=1}^{n} p_i\left|\xi_i - \sum_{j=1}^{n} p_j\xi_j\right| .$$

Using the triangle inequality, we have

$$\sum_{i=1}^{n} p_i \left| \log_b \left( \sum_{j=1}^{n} p_j \xi_j \right) - \log_b \xi_i \right| \geq \left| \log_b \left( \sum_{j=1}^{n} p_j \xi_j \right) - \sum_{i=1}^{n} p_i \log_b \xi_i \right| =$$

$$= \log_b \left( \sum_{j=1}^{n} p_j \xi_j \right) - \sum_{i=1}^{n} p_i \log_b \xi_i \geq 0$$

and the inequality (2.1) is proved. The case of equality is obvious.

**Corollary 2.2.** *With the above assumptions over $\xi_k$, we have the inequality:*

$$0 \leq \log_b \left( \frac{1}{n} \sum_{k=1}^{n} \xi_k \right) - \frac{1}{n} \sum_{k=1}^{n} \log_b \xi_k \leq \frac{1}{m \ln b} \frac{1}{n} \sum_{i=1}^{n} \left| \xi_i - \frac{1}{n} \sum_{j=1}^{n} \xi_j \right| . \quad (2.4)$$

**Remark 2.3.** It is well known in theory of inequalities that the following inequality between the arithmetic mean and geometric mean holds:

$$A_n(p,x) \geq G_n(p,x) \quad (\mathrm{A-G})$$

where

$$A_n(p,x) := \sum_{i=1}^{n} p_i x_i \quad \text{(: arithmetic mean)},$$

$$G_n(p,x) := \prod_{i=1}^{n} x_i^{p_i} \quad \text{(: geometric mean)}$$

and $p_i \geq 0$ with $\sum_{i=1}^{n} p_i = 1$ and $x_i > 0$ $(i = 1, \ldots, n)$. Using inequality (2.1), we obtain

$$0 \leq \ln \left( \frac{\sum_{k=1}^{n} p_k x_k}{\prod_{i=1}^{n} x_i^{p_i}} \right) \leq \frac{1}{m} \sum_{i=1}^{n} p_i \left| x_i - \sum_{j=1}^{n} p_j x_j \right| ,$$

which is equivalent with

$$1 \leq \frac{A_n(o,x)}{G_n(p,x)} \leq \exp \left[ \frac{1}{m} A_n \left( p, x - A_n(p,x) \right) \right] , \quad (2.5)$$

which is a counterpart result for (A-G).

## 3. Some Counterpart Inequalities for Entrory and Joint Entrory

Suppose that $X$ is a discrete random variable whose range $R = \{x_1, \ldots, x_r\}$ is finite. Let $p_i = P\{X = x_i\}$, $i = 1, \ldots, r$, and assume that $p_i > 0$ for all $i \in \{1, \ldots, r\}$.

The following counterpart result for (1.2) holds:

**Theorem 3.1.** *Let $p_M := \max\{p_i \mid i = 1, \ldots, r\}$. Then we have the inequality:*

$$0 \le \log_b r - H_b(X) \le \frac{r p_M}{\ln b} \sum_{i=1}^{r} \left| p_i - \frac{1}{r} \right| . \tag{3.1}$$

*Furthermore, the equality holds simultaneously is both inequalities iff $p_i = 1/r$ for all $i = \{1, \ldots, r\}$.*

**Proof.** If we choose in Lemma 1.1, $m = 1/p_M$ and $\xi_i = 1/p_i$ $(i = 1, \ldots, r)$, we have

$$0 \le \log_b r - H_b(X) \le \frac{p_M}{\ln b} \sum_{i=1}^{r} p_i \left| \frac{1}{p_i} - r \right| = \frac{r p_M}{\ln b} \sum_{i=1}^{r} \left| p_i - \frac{1}{r} \right|$$

and the inequality (3.1) is proved.

**Remark 3.2.** If we don't know the maximal probability $p_M$, then we can only state that

$$0 \le \log_b r - H_b(X) \le \frac{r}{\ln b} \sum_{i=1}^{r} \left| p_i - \frac{1}{r} \right| , \tag{3.2}$$

which is weaker than (3.1).

**Corollary 3.3.** *Let $\varepsilon > 0$ be given. If we assume that*

$$\left| p_i - \frac{1}{r} \right| \le \frac{\varepsilon \ln b}{r^2 p_M}$$

*for all $i \in \{1, \ldots, r\}$, then we have the estimation:*

$$0 \le \log_b r - H_b(X) \le \varepsilon . \tag{3.3}$$

For a pair of random variables $X$ and $Y$ with the ranges $\{x_1, \ldots, x_r\}$ and $\{y_1, \ldots, y_s\}$, respectively, the joint entropy of $X$ and $Y$ is defined by [3, p. 25]:

$$H_b(X,Y) = \sum_{x,y} p(x,y) \log_b \frac{1}{p(x,y)}$$

where

$$p(x,y) := \text{Prob}\{X = x,\ Y = y\}, \quad x \in \{1,\dots,r\}, \quad y \in \{1,\dots,s\}.$$

In the recent paper [2] it is proved the following result:

**Theorem 3.4.** *With the above assumptions, we have*

$$0 \le \log_b(rs) - H_b(X,Y) \le \frac{1}{2\ln b}\sum_{x,y}\sum_{u,v}\left(p(x,y) - p(u,v)\right)^2. \tag{3.4}$$

*The eqality holds in both inequalities simultaneously iff* $p(x,y) = 1/rs$ *for all* $x, y$.

**Corollary 3.5.** *Let* $\varepsilon > 0$ *be given. If*

$$\max_{(x,y),(u,v)} |p(x,y) - p(u,v)| \le \sqrt{\frac{2\varepsilon \ln b}{rs}}.$$

*then we have the estimation:*

$$0 \le \log_b(rs) - H_b(X,Y) \le \varepsilon.$$

Using Lemma 2.1, we can state and prove the following theorem:

**Theorem 3.6.** *Suppose that*

$$p_M = \max\{p(x,y) \mid x \in \{1,\dots,r\},\ y \in \{1,\dots,s\}\}.$$

*Then we have the counterpart inequality:*

$$0 \le \log_b(rs) - H_b(X,Y) \le \frac{P_M rs}{\ln b}\sum_{x,y}\left|p(x,y) - \frac{1}{rs}\right|. \tag{3.5}$$

*Furthermore, the equality holds simultaneously in both inequalities iff* $p(x,y) = 1/rs$ *for all* $(x,y)$..

**Proof.** If, in Lemma 2.1, we choose $m = 1/p_M$, $\xi_i = 1/p(x,y)$, $p_i = p(x,y)$, we deduce

$$0 \le \log_b(rs) - H_b(X,Y) \le \frac{p_M}{\ln b}\sum_{x,y} p(x,y)\left|\frac{1}{p(x,y)} - rs\right| \le$$

$$\le \frac{p_M rs}{\ln b}\sum_{x,y}\left|p(x,y) - \frac{1}{rs}\right|$$

and the estimation (3.5) is obtained.

**Corollary 3.7.** *Let $\varepsilon > 0$ be given. If we have*

$$\left|p(x,y) - \frac{1}{rs}\right| \leq \frac{\varepsilon \ln b}{p_M r^2 s^2}$$

*for all $(x,y)$, then we have the estimation:*

$$0 \leq \log_b(rs) - H_b(X,Y) \leq \varepsilon .$$

## 4. Some Counterpart Inequalities for Conditional Entropy

For a pair random variables $X$ and $Y$, the conditional entropy of $X$ given $Y$ is defined by [3, p. 22]:

$$H_b(X \mid Y) = \sum_{x,y} p(x,y) \log_b \frac{1}{p(x \mid y)},$$

where

$$p(x,y) := \operatorname{Prob}\{X = x\ Y = y\}$$

and

$$p(x \mid y) := \operatorname{Prob}\{X = x \mid Y = y\} = \frac{p(x,y)}{p(y)} .$$

One can interpret the conditional entropy as the amount of uncertainty remaining about $X$ after $Y$ has been observed.

In the paper [2], S. S. Dragomir and C. J. Goh have proved the following theorem:

**Theorem 4.1.** *Let $X$ and $Y$ be two discrete random variables and the range of $X$ has $r$ elements. Then we have the inequality:*

$$\begin{aligned} 0 &\leq \log_b r - H_b(X \mid Y) \leq \\ &\leq \frac{1}{2 \ln b} \sum_{x,y} \sum_{u,v} p(y) p(v) \big(p(x \mid y) - p(u \mid v)\big)^2 . \end{aligned} \tag{4.1}$$

The following corollary also holds:

**Corollary 4.2.** *With the above assumptions and if*

$$\max_{(x,y),(u,v)} |p(x \mid y) - p(u \mid v)| < \frac{\sqrt{2\varepsilon \ln b}}{r} .$$

*for every* $\varepsilon > 0$*, then we have the estimation:*

$$0 \leq \log_b r - H_b(X \mid Y) \leq \varepsilon .$$

Using Lemma 2.1, we can prove the following different counterpart result:

**Theorem 4.3.** *Under the assumptions of Theorem* 4.1 *and if* $\overline{p}_M$: $\max\{p(x \mid y) \mid x \in \{1, \ldots, r\},\ y \in \{1, \ldots, s\}\}$*, then we have the estimation:*

$$0 \leq \log_b r - H_b(X \mid Y) \leq \frac{r\overline{p}_M}{\ln b} \sum_{x,y} p(y) \left| p(x \mid y) - \frac{1}{r} \right| . \tag{4.2}$$

**Proof.** We have

$$H_b(X \mid Y) = \sum_{x,y} p(x,y) \log_b \frac{1}{p(x \mid y)} \leq \sum_{x,y} p(x,y) \log_b \frac{p(y)}{p(x,y)} .$$

Applying Lemma 2.1 for

$$p_k = p(x,y), \quad \xi_k = \frac{p(y)}{p(x,y)} = \frac{1}{p(x \mid y)} \geq \frac{1}{\overline{p}_M},$$

we have

$$0 \leq \log_b \left( \sum_{x,y} p(x,y) \frac{p(y)}{p(x,y)} \right) - \sum_{x,y} p(x,y) \log_b \left( \frac{p(y)}{p(x,y)} \right) \leq$$

$$\leq \frac{\overline{p}_M}{\ln b} \sum_{x,y} p(x,y) \left| \frac{p(y)}{p(x,y)} - \sum_{u,v} p(u,v) \frac{p(v)}{p(u,v)} \right| =$$

$$= \frac{\overline{p}_M}{\ln b} \sum_{x,y} |p(y) - p(x,y) r| =$$

$$= \frac{r\overline{p}_M}{\ln b} \sum_{x,y} p(y) \left| p(x \mid y) - \frac{1}{r} \right|$$

and the estimation (4.2) is obtained.

The following corollary also holds:

**Corollary 4.4.** *With the above assumptions and if*

$$\max_{(x,y)}\left|p(x \mid y) - \frac{1}{r}\right| \leq \frac{\varepsilon \ln b}{r^2 \overline{p}_M},$$

*then w have the estimation:*

$$0 \leq \log_b r - H_b(X \mid Y) \lesssim \varepsilon .$$

The following result concerning conditional entropy is well known in the literature and can be obtained by applying Jensen's inequality:

**Theorem 4.5.** *Let $X$, $Y$ and $Z$ be discrete random variables with finite ranges. Then we have*

$$H_b(X \mid Y) \leq H_b(Z) + (\log_b A), \tag{4.3}$$

*where $H_b(Z) := \sum_z p(z)\log_b \frac{1}{p(z)}$ is the usual entropy of $Z$, and we have*

$$A(Z) := \sum_{x,y} \alpha_{xy}(z),$$

*where*

$$\alpha_{xy} := p(y)p(z \mid x,y) := \frac{p(x,y,z)}{p(x \mid y)}.$$

In the paper [1], S. S. Dragomir and C. J. Goh proved the following counterpart of (4.3):

**Theorem 4.6.** *With the above assumtions we have:*

$$\begin{aligned} 0 &\leq H_b(Z) + E(\log_b A) - H_b(X \mid Y) \leq \\ &\leq \frac{1}{2\ln b}\sum_z \frac{1}{p(z)} \sum_{x,y}\sum_{u,v} \alpha_{xy}(z)\alpha_{uv}(z)\big(p(x \mid y) - p(u \mid v)\big)^2 . \end{aligned} \tag{4.4}$$

The following corollary is important in applications:

**Corollary 4.7.** *Under the same assumptions, let $\varepsilon > 0$ be given. If, in addition,*

$$\max\left|p(x \mid y) - p(u \mid v)\right| \leq \sqrt{\frac{2\varepsilon \ln b}{M}},$$

*then we have*

$$0 \leq H_b(Z) + E(\log_b A) - H_b(X \mid Y) \leq \varepsilon,$$

*where* $M := \sum_z \frac{[A(z)]^2}{p(z)}$.

Furthermore, we shall apply Lemma 2.1 to obtain another type of the converse for the inequality (4.3).

**Theorem 4.8.** *With the assumptions of Theorem 4.5 and if* $\overline{p}_M = \max\{(x \mid y) \mid x \in \{1,\ldots,r\},\ y \in \{1,\ldots,s\}\}$, *then we have the inequality:*

$$0 \le H_b(z) + E(\log_b A) - H_b(Z \mid Y) \le$$

$$\le \frac{\overline{p}_M}{\ln b} \sum_{x,y,z} \frac{p(x,y,z)}{p(z)} \left| \frac{p(z)}{p(x \mid y)} - A(z) \right|. \tag{4.5}$$

**Proof.** If in Lemma 2.1, we repalce $p_k$ by $p(x,y,z)/p(z)$ and $\xi_k$ by $1/p(x \mid y) \ge 1/\overline{p}_M$, we have

$$0 \le \log_b \left( \sum_{x,y} \frac{p(x,y,z)}{p(z)} \frac{1}{p(x \mid y)} \right) - \sum_{x,y} \frac{p(x,y,z)}{p(z)} \log_b \left( \frac{1}{p(x \mid y)} \right) \le$$

$$\le \frac{\overline{p}_M}{\ln b} \sum_{x,y} \frac{p(x,y,z)}{p(z)} \left| \frac{1}{p(x \mid y)} - \sum_{u,v} \frac{p(u,v,z)}{p(z)} \frac{1}{p(u \mid v)} \right|$$

for each $z$. Multiplying the above inequality with $p(z)$ and summing over $z$, we have

$$0 \le \sum_z p(z) \log_b \frac{1}{p(z)} + \sum_z p(z) \log_b \left( \sum_{x,y} \frac{p(x,y,z)}{p(x,y)} \right) -$$

$$- \sum_{x,y,z} p(x,y,z) \log_b \left( \frac{1}{p(x \mid y)} \right) \le$$

$$\le \frac{\overline{p}_M}{\ln b} \sum_{x,y,z} \frac{p(x,y,z)}{p(z)} \left| \frac{p(z)}{p(x \mid y)} - \sum_{u,v} \frac{p(u,v,z)}{p(u \mid v)} \right| =$$

$$= \frac{\overline{p}_M}{\ln b} \sum_{x,y,z} \frac{p(x,y,z)}{p(z)} \left| \frac{p(z)}{p(x \mid y)} - A(z) \right|$$

and the inequality (4.5) is obtained.

**Corollary 4.9.** *If the range of $z$ has $n$ elements and*

$$\max_{x,y,z} \left| \frac{p(z)}{p(x \mid y)} - A(z) \right| \le \frac{\varepsilon \ln b}{n \, \overline{p}_M}$$

*for every* $\varepsilon > 0$, *then we have the estimation*

$$0 \leq H_b(Z) + E(\log_b A) - H_b(X \mid Y) \leq \varepsilon . \tag{4.6}$$

**Proof.** As, for all $z$, we have

$$\sum_{x,y} \frac{p(x,y,z)}{p(z)} = 1$$

then

$$\frac{\overline{p}_M}{\ln b} \sum_{x,y,z} \frac{p(x,y,z)}{p(z)} \left| \frac{p(z)}{p(x \mid y)} - A(z) \right| \leq \frac{\varepsilon}{n\, \overline{p}_M} \overline{p}_M \sum_{x,y,z} \frac{p(x,y,z)}{p(z)} = \varepsilon$$

and the estimation (4.6) is proved.

## 5. Some New Counterparts Results for Mutual Information

Consider the mutual information between two random variables $X$ and $Y$ defined by

$$\begin{aligned} I_b(X,Y) &:= \sum_{x,y} p(x,y) \log_b \left[ \frac{p(x,y)}{p(x)\, p(y)} \right] = \\ &= H_b(X) - H_b(X \mid Y) = \\ &= H_b(X) + H_b(Y) - H_b(X,Y) . \end{aligned}$$

The following theorem concerning the mutual information is known in the literature [3, p. 25]:

**Theorem 5.1.** *For any pair of discrete random variables* $X$ *and* $Y$, *we have* $I_b(X;Y) \geq 0$. *Moreover,* $I_b(X;Y) = 0$ *iff* $X$ *and* $Y$ *are independent.*

The following converse of the above inequality and its corollary have been established in [1]:

**Theorem 5.2.** *Given a pair of discrete random variables* $X$ *and* $Y$, *we have*

$$\begin{aligned} &0 \leq I_b(X;Y) \leq \\ &\leq \frac{1}{2 \ln b} \sum_{x,y} \sum_{u,v} p(x)p(y)p(u)p(v) \left( \frac{p(u,v)}{p(u)p(v)} - \frac{p(x,y)}{p(x)p(y)} \right)^2 . \end{aligned} \tag{5.1}$$

*The identities hold in both inequalities iff $X$ and $Y$ are independent.*

**Corollary 5.3.** *With the above assumptions and if*

$$\max_{(u,v),(x,y)} \left| \frac{p(u,v)}{p(u)p(v)} - \frac{p(x,y)}{p(x)(y)} \right| \leq \sqrt{2\varepsilon \ln b},$$

*then*

$$0 \leq I_b(X,Y) \leq \varepsilon$$

*for every $\varepsilon > 0$.*

Now, we are able to prove some new counterpart inequalities by the use of Lemma 2.1 proved above.

**Theorem 5.4.** *With the above assumptions and if*

$$\overline{I}_M := \max \left\{ \frac{p(x,y)}{p(x)p(y)} \;\middle|\; x \in \{1,\ldots,r\},\ y \in \{1,\ldots,s\} \right\},$$

*then we have the inequality:*

$$0 \leq I_b(X;Y) \leq \frac{\overline{I}_M}{\ln b} \sum_{x,y} p(x)p(y) \left| \frac{p(x,y)}{p(x)p(y)} - 1 \right|. \tag{5.2}$$

*The equality holds simultaneously in both inequalities iff $X$ and $Y$ are independent.*

**Proof.** We choose, in Lemma 2.1, $p_k = p(x,y)$ and $\xi_k = p(x)p(y)/p(x,y) \geq 1/\overline{I}_M$ for all $x, y$. Thus we have

$$0 \leq= log_b \left( \sum_{x,y} p(x,y) \frac{p(x)p(y)}{p(x,y)} \right) - \sum_{x,y} p(x,y) \log_b \frac{p(x)p(y)}{p(x,y)} \leq$$

$$\leq \frac{\overline{I}_M}{\ln b} \sum_{x,y} p(x,y) \left| \frac{p(x)p(y)}{p(x,y)} - \sum_{u,v} p(u,v) \frac{p(u)p(v)}{p(u,v)} \right| =$$

$$= \frac{\overline{I}_M}{\ln b} \sum_{x,y} p(x,y) \left| \frac{p(x)p(y)}{p(x,y)} - 1 \right| =$$

$$= \frac{\overline{I}_M}{\ln b} \sum_{x,y} p(x)p(y) \left| \frac{p(x,y)}{p(x)p(y)} - 1 \right|.$$

Finally, the following corollary holds:

**Corollary 5.5.** *With the above assumptions and if*

$$\max_{(x,y)} \left| \frac{p(x,y)}{p(x)p(y)} - 1 \right| \leq \frac{\varepsilon \ln b}{\overline{I}_M}, \tag{5.3}$$

*then*

$$0 \leq I_b(X,Y) \leq \varepsilon,$$

*for every* $\varepsilon > 0$.

**Remark 5.6.** A sufficient condition for (5.3) to be fulfilled is

$$\left| \frac{p(x,y)}{p(x)p(y)} - 1 \right| \leq \varepsilon \ln b$$

for all $x, y \in \{1, \ldots, r\} \times \{1, \ldots, s\}$.

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# НОВИ СОЗНАНИЈА ЗА НЕКОИ НЕРАВЕНСТВА НА ЕНТРОПИЈА И ВЗАЕМНА ИНФОРМАЦИЈА

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**Р е з и м е**

Во оваа работа употребувајќи аналитички неравенства, коишто се нови сознанија за класичните неравенства меѓу аритметичката и геометриската средина, добиваме нови резултати за ентропијата и взаемната информација.

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