EMBEDDINGS OF UNIVERSAL ALGEBRAS IN SEMIGROUPS

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A review of results concerning the problem of embeddings of universal algebras in semigroups is given in this paper.

1. Ω-subalgebras of semigroups. Let $\underline{A}$ be an Ω-algebra and $\underline{S}=(S,\cdot)$ a semigroup such that $A\subseteq S$. We say that $\underline{A}$ is an Ω-subalgebra of $\underline{S}$ if there is a mapping $\omega\mapsto\bar{\omega}$ of Ω into S such that

$$\omega_{\underline{A}}(a_1,\dots,a_n) = \bar{\omega}\, a_1a_2\dots a_n,\ ^{*)}$$

for all $\omega\in\Omega(n)$, $n\geq 0$, $a_1,\dots,a_n\in A$.

If $\mathcal{C}$ is a class of semigroups then $\mathcal{C}(\Omega)$ denotes the class of Ω-algebras that are Ω-subalgebras of semigroups belonging to $\mathcal{C}$. We find it interesting looking for a convenient description of the class $\mathcal{C}(\Omega)$ when a corresponding description of the class $\mathcal{C}$ is given. Thus, the well known Cohn-Rebane's Theorem, [1;p.185], [18;p.78] can be stated in the following way: SEM(Ω) is the class of all Ω-algebras. (By SEM we denote the class of semigroups.)

There are known convenient descriptions of the classes of all Ω-subalgebras of semigroups which belong to the following classes of semigroups:

ABSEM (abelian semigroups) ([23]),

CANSEM (cancellative semigroups) ([24]),

NILSEM (nilpotent semigroups) ([25]),

LZ (left zero semigroups),

RZ (right zero semigroups),

RCB (rectangular bands),

MIN (midle neutral semigroups, i.e. semigroups that satisfy the law xyz=xz),

SL (semilattices) ([11]),

ABPG (abelian periodical groups) ([12]),

$A_{r,m}$ (abelian semigroups that satisfy the law $x^r=x^{r+m}$, where $r\geq 0$, $m\geq 1$; $A_m=A_{o,m}$ is the class of abelian groups the orders of element of which are divisors of m) ([12]),

DISSEM (distributive semigroups, i.e. semigroups which satisfy both left and right distributive law) ([14]),

ABDISSEM (abelian distributive semigroups) ([13]),

LDISSEM (left distributive semigroups) ([15]),

RDISSEM (right distributive semigroups) ([15]).

We notice that there are not known convenient descriptions of $\mathcal{C}(\Omega)$ for a lot of important classes $\mathcal{C}$ of semigroups (bands, inverse semigroups, periodic semigroups, abelian periodical semigroups, periodical groups,...).

There is only one result concerning the class of Ω-subalgebras of finite semigroups. Namely, it is shown in [22] that if $\Omega=\Omega(0)\cup\Omega(1)$ then every finite Ω-algebra is an Ω-subalgebra of a finite semigroup; in the same paper, it is also shown that if $\Omega\neq\Omega(0)\cup\Omega(1)$ then there exist finite Ω-algebras which are not Ω-subalgebras of finite semigroups.

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*) $\omega_{\underline{A}}$ is the interpretation of the n-ary operator $\omega\in\Omega$; further on we write ω instead of $\omega_{\underline{A}}$; Ω(n) is the set of n-ary operators belonging to Ω.

2. Ω-varieties of semigroups. It follows by a more general result ([19;p.274], [16]) that if Q is a quasivariety of semigroups, then $Q(\Omega)$ is a quasivariety of Ω-algebras. Therefore, if V is a variety of semigroups, then $V(\Omega)$ is a quasivariety of Ω-algebras. We say that a variety V of semigroups is an Ω-variety if $V(\Omega)$ is a variety of Ω-algebras. Thus, if Ω is a given operator domain, then the set of varieties of semigroups is divided into two disjoint subsets: Ω-varieties and varieties V of semigroups such that $V(\Omega)$ are proper quasivarieties. Clearly, if $\Omega=\Omega(0)$, then every variety of semigroups is an Ω-variety, as well; and, if $\Omega'=\Omega\setminus\Omega(0)$ then the set of Ω-varieties of semigroups coincides with the set of Ω'-varieties of semigroups. That is why we will assume that $\Omega\neq\emptyset=\Omega(0)$.

2.1. If $V\in\{SEM,ABSEM,ABDISSEM,LZ,RZ\}\cup\{A_{1,m} \mid m\geq 1\}$, then V is an Ω-variety for any operator domain Ω ([1], [7], [13], [23]).

2.2. If $V\in\{A_{r,m} \mid r\geq 2, m\geq 1\}$ or V=LDISSEM, then V is an Ω-variety iff $\Omega=\Omega(1)$ ([7], [15]).

2.3. If $m\geq 2$, then A_m is an Ω-variety if $\Omega\setminus\Omega(1)\neq\emptyset$ ([12]).

2.4. Let $V\in\{DISSEM,RCB,MIN\}$. V is an Ω-variety iff $\Omega=\{\omega\}$ consists of only one n-ary operator, with $n\geq 1$ ([14]).

2.5. RDISSEM is an Ω-variety iff $\Omega=\Omega(1)$ and $|\Omega|=1$ ([15]).

We notice that in all mentioned examples of Ω-varieties V of semigroups, convenient axiom systems of the varieties $V(\Omega)$ are known. For example, if ξ,η are Ω-terms then $\xi=\eta$ is an identity equation in $SL(\Omega)$ iff ξ and η have the same content.

The situation is not the same if $V(\Omega)$ is a proper quasivariety. Namely, one proves that a semigroup variety V is not an Ω-variety by finding an algebra $\underline{A}=(A,\Omega)$ which satisfies all the identities of $V(\Omega)$, but $\underline{A}\notin V(\Omega)$. Consider, for example, the variety MIN. Clearly, an Ω-algebra $\underline{A}=(A,\Omega)$ satisfies all the identities of $MIN(\Omega)$ iff it satisfies the following identities:

$$\omega(x_1,\dots,x_{n-1},z) = \omega(y_1,\dots,y_{n-1},z), \qquad (2.1)$$

$$\omega(x_1,\dots,x_{n-1},\tau(y_1,\dots,y_{m-1},z)) = \omega(x_1,\dots,x_{n-1},z), \qquad (2.2)$$

for any $\omega\in\Omega(n)$, $\tau\in\Omega(m)$.

Assume that $\omega\in\Omega(n)$, $\tau\in\Omega(m)$ are two different elements of Ω. Define an Ω-algebra $\underline{A}$ with a carrier $A=\{a,b,c\}$ $(a\neq b\neq c\neq a)$ as follows:

$$\rho(x_1,\dots,x_p) = \begin{cases} b \text{ if } x_p\in\{a,b\},\ \rho\in\Omega(p),\ \rho\neq\omega \\ a \text{ if } x_p\in\{a,b\},\ \rho=\omega \end{cases}$$

$$\rho(x_1,\dots,x_{p-1},c) = c, \text{ for any } \rho\in\Omega(p).$$

It is clear that the obtained algebra $\underline{A}$ satisfies all the identities (2.1) and (2.2). But $\underline{A}\notin MIN(\Omega)$, for if $\underline{A}$ were an Ω-subalgebra of a semigroup $\underline{S}\in MIN$, then we would have:

$$a=\omega(a^n)=\bar{\omega}a^n=\bar{\omega}c^na=\omega(c^n)a=\tau(c^m)a=\bar{\tau}c^ma=\bar{\tau}a^m=\tau(a^m)=b.$$

This shows that $MIN(\Omega)$ is a proper quasivariety.

It can be easily seen that every algebra belonging to $MIN(\Omega)$ satisfies any quasiidentity of the following form:

$$\omega(x_1,\dots,x_n)=\tau(y_1,\dots,y_m)\Rightarrow\omega(z_1,\dots,z_{n-1},z)=\tau(z_1,\dots,z_{m-1},z), \qquad (2.3)$$

for every $\omega\in\Omega(n)$, $\tau\in\Omega(m)$. The converse is also true, i.e. if an Ω-algebra $\underline{A}$ satisfies all the formulas (2.1), (2.2) and (2.3), then $\underline{A}\in MIN(\Omega)$. Let us give a sketch of the proof of the last proposition.

Assume, namely, that (A,Ω) satisfies (2.1), (2.2) and (2.3), and let $B=A\cup\Omega$, $A\cap\Omega=\emptyset$. Define an operation $\cdot$ on the set $F=B\times B\cup B$ in the following way:

$$b_1\cdot b_2 = (b_1,b_2),\ b_1\cdot(b_2,b_3) = (b_1,b_3)$$
$$(b_1,b_2)\cdot b_3 = (b_1,b_3),\ (b_1,b_2)\cdot(b_3,b_4) = (b_1,b_4),$$

$b_i\in B$. In such a way we get a semigroup $\underline{F}\in MIN$. Let $\approx$ be the minimal congruence on $\underline{F}$ such that:

$$a = \omega(a_1,\dots,a_n) \text{ in } \underline{A} \Rightarrow a \approx (\omega,a_n) \text{ in } F.$$

Then, it can be shown that:

$$a,b\in A \Rightarrow (a \approx b \Rightarrow a = b),$$

and this implies that $\underline{A}$ can be embedded as an Ω-subalgebra into the semigroup $\underline{S}=\underline{F}/_{\approx}$. Thus, $\underline{A}\in MIN(\Omega)$.

Clearly, if $\Omega=\{\omega\}$ consists of only one element, then (2.3) is always satisfied, and therefore in this case $MIN(\Omega)=MIN(\omega)$ is a variety.

We point out that $MIN(\Omega)=RCB(\Omega)$, for every operator domain Ω.

We notice that in all other known examples of semigroup varieties $\mathcal{V}$ such that $\mathcal{V}(\Omega)$ is a proper quasivariety (and $\mathcal{V}\neq\mathcal{A}_m$) we do not know a convenient axiom system of the quasivariety $\mathcal{V}(\Omega)$.

3. Semigroups of finitary operations. Let A be a nonempty set, n be a positive integer, and $O_n(A)$ be the set of all n-ary operations on A. Then $O(A)=\bigcup_{n\geq 1} O_n(A)$ is the set of all non-nullary operations on A. If a binary operation $\bullet$ is defined on $O(A)$ in the usual way, i.e. by:

$$(\omega\bullet\tau)(x_1,\dots,x_{m+n-1}) = \omega(\tau(x_1,\dots,x_m),\dots,x_{m+n-1}),$$

where $\omega\in O_n(A)$, $\tau\in O_m(A)$, then we get a semigroup $(O(A),\bullet)$. Any subsemigroup Γ of this semigroup is said to be a semigroup of operations on A, and then we have a Γ-algebra $\underline{A}=(A,\Gamma)$. By Cohn-Rebane's Theorem if Γ is a semigroup of operations on A, then the corresponding Γ-algebra (A,Γ) is a Γ-subalgebra of a semigroup. And, if $\mathcal{C}$ is a class of semigroups, then we denote by $\mathcal{C}^{(1)}$ the class of semigroups of operations Γ such that the corresponding Γ-algebras are Γ-subalgebras of semigroups belonging to $\mathcal{C}$. If, moreover, Γ is a semigroup of operations on A such that the corresponding Γ-algebra $\underline{A}=(A,\Gamma)$ can be embedded as a Γ-subalgebra in a semigroup $\underline{S}\in\mathcal{C}$ in such a way that the mapping $\omega\to\overline{\omega}$ is a homomorphism (an injective homomorphism), then we say that Γ belongs to the class $\mathcal{C}^{(2)}$ ($\mathcal{C}^{(3)}$).

It is clear that:

3.1. $\mathcal{C}^{(3)}\subseteq\mathcal{C}^{(2)}\subseteq\mathcal{C}^{(1)}$, for any class $\mathcal{C}$ of semigroups.

We do not know any class of semigroups $\mathcal{C}$, such that $\mathcal{C}^{(2)}$ is a proper subclass of $\mathcal{C}^{(1)}$. Below we state some known results.

3.2. If $\mathcal{C}\in\{SEM,ABSEM,CANSEM,ABCANSEM,ABGP\}$, then $\mathcal{C}^{(1)}=\mathcal{C}^{(2)}=\mathcal{C}^{(3)}$ ([8]).

3.3. If $\mathcal{C}\in\{NILSEM,ABNILSEM,MIN,RCB\}\cup\{\mathcal{A}_m \mid m\geq 2\}$ then $\mathcal{C}^{(3)}\subset\mathcal{C}^{(2)}=\mathcal{C}^{(1)}$, and the inclusion is strict ([8]).

4. n-subsemigroups of semigroups. Let $\underline{S}=(S,\cdot)$ be a semigroup and Q a subset of S such that:

$$x_1,\dots,x_n\in Q \Rightarrow x_1\cdot x_2\cdot\ldots\cdot x_n\in Q,$$

where $n\geq 2$. Then we say that Q is an n-subsemigroup of $\underline{S}$. If we define an n-ary operation [ ] on Q by:

$$[x_1 \ldots x_n] = x_1 \ldots x_n,$$

then we get that [ ] is an associative operation, i.e. $(Q,[\ ])$ is an n-semigroup.

If $\mathcal{C}$ is a class of semigroups, then we denote by $\mathcal{C}(n)$ the class of n-semigroups which are n-subsemigroups of semigroups belonging to $\mathcal{C}$. A variety of semigroups $\mathcal{V}$ is said to be an n-variety of semigroups iff $\mathcal{V}(n)$ is a variety of n-semigroups.

4.1. Each of the varieties of semigroups SEM, ABSEM, LZ, RZ, DISSEM, RCB, MIN, $\mathcal{A}_{r,m}$ is an n-variety for any $n \geq 2$ ([4], [5], [20], [21]).

4.2. Let $r \geq 0$ and $m \geq 1$. The variety $\mathcal{P}_{r,m}$ of semigroups satisfying the law $x^r = x^{r+m}$ is an n-variety iff $r \in \{0,1\}$ or $n-1$ is a divisor of m ([4]).

4.3. All members of the set of varieties:

$$\{\mathcal{V} \mid \mathrm{ABSEM} \subseteq \mathcal{V}\} \cup \{\mathcal{V} \mid \mathcal{V} \subseteq \mathcal{P}_{1,m}\}$$

are n-varieties.

4.4. LDISSEM(n) and RDISSEM(n) are proper quasivarieties, for any $n \geq 3$ ([20]).

4.5. Let $n \geq 3$. There exist infinite many varieties of abelian semigroups which are not n-varieties ([6]).

The results stated in this section and in the section 2 suggests the conjecture that if a variety of semigroups is an ω-variety, where ω is an n-ary operator, then it is an n-variety as well.

4.6. An n-semigroup $(Q,[\ ])$ belongs to the quasivariety CAN(n) iff it is cancellative. Every cancellative n-semigroup $(Q,[\ ])$ is an n-subsemigroup of a universal cancellative semigroup $(Q^{\sim},\cdot)=\underline{Q}^{\sim}$, and then:

$$(Q,[\ ]) \in GP(n) \text{ iff } \underline{Q}^{\sim} \in GP(2) \quad ([17]).$$

The last result can be used for obtaining an axiom system of the quasivariety GP(n). Namely, we can take some of the known axiom system of GP(2) (see, for example, [1;VII.3]) and then we can transform it into an axiom system of GP(n).

5. Ω-subsemigroups of semigroups. The class of Ω-subsemigroups of semigroups is an extension of the class of n-subsemigroups of semigroups. Assume first that $\Omega(0)=\emptyset$. An Ω-algebra $\underline{A}=(A,\Omega)$ is an Ω-subsemigroup of a semigroup $\underline{S}=(S,\cdot)$ if $A \subseteq S$ and

$$\omega(a_1,\ldots,a_n) = a_1 \ldots a_n,$$

for every $\omega \in \Omega(n) \subseteq \Omega$ and $a_1,\ldots,a_n \in A$. If $\mathcal{C}$ is a class of semigroups, then we denoted by $\mathcal{C}[\Omega]$ the class of Ω-subsemigroups of semigroups belonging to $\mathcal{C}$.

Clearly, if $\omega \in \Omega(1)$ and if $\underline{A}=(A,\Omega)$ is an Ω-subsemigroup of a semigroup, then we have $\omega(a)=a$, for every $a \in A$, and that is why below we assume that $\Omega(1)=\emptyset$ and $\Omega \neq \emptyset$.

A convenient description of the quasivariety SEM$[\Omega]$ is given in [2]. Almost all known results concerning Ω-subsemigroups of semigroups could be found in [5], and we will state only a few of them.

5.1. Let $J_\Omega=\{n-1 \mid \Omega(n) \neq \emptyset\}$, and let d_Ω be the greatest common divisor of numbers belonging to J_Ω. Then, SEM$[\Omega]$ is a variety iff $d_\Omega \in J_\Omega$.

5.2. If $\mathcal{V} \in \{LZ, RZ, MIN, RCB, SL\}$, then $\mathcal{V}[\Omega]$ is a variety.

There are also known corresponding description of CAN$[\Omega]$, GP$[\Omega]$, and of some subclasses of SEM$[\Omega]$.

6. Generalized subalgebras of semigroups. The classes of Ω-subalgebras of semigroups and Ω-subsemigroups of semigroups are special kinds of Δ-subalgebras of semigroups defined in the following way.

Let C be a set and $C_o=C \cup \{e\}$, where $e \notin C$. Assume now that Ω is an operator domain and $\Delta: \omega \mapsto \omega^\Delta$ a mapping from Ω into the set $\bigcup_{k \geq 1} C_o^k$ such that:

(i) $\omega \in \Omega(n) \Rightarrow \omega^\Delta = (\omega_o,\omega_1,\ldots,\omega_n) \in C_o^{n+1}$

(ii) if $c \in C$ then there exists an $\omega \in \Omega(n)$ and an $i \in \{0,\ldots,n\}$ such that $c=\omega_i$.

An Ω-algebra $\underline{A}=(A,\Omega)$ is said to be a Δ-subalgebra of a semigroup $\underline{S}=(S,\cdot)$ if $A\subseteq S$ and there is a mapping $d\mapsto\bar{d}$ from C_o into S^1 *) such that:

(iii$_1$) $\bar{e}=1$, (iii$_2$) $c\in C_o \Rightarrow \bar{c}\in S$, and

(iv) $\omega(a_1,\dots,a_n)=\bar{\omega}_o a_1\bar{\omega}_1 a_2\dots\bar{\omega}_{n-1}a_n\bar{\omega}_n$,

for any $\omega\in\Omega(n)$, $n\geq 0$, $a_\nu\in A$.

We say that Δ is a semigroup representation of Ω.

If $\mathcal{C}$ is a class of semigroups, then we denote by $\mathcal{C}(\Delta)$ the class of Δ-subalgebras of semigroups belonging to $\mathcal{C}$.

6.1. If $C=\Omega$, and $\Omega^\Delta=(\omega,e,\dots,e)$ for any $\omega\in\Omega$, then $\mathcal{C}(\Delta)=\mathcal{C}(\Omega)$.

6.2. If $C=\emptyset$, then $\mathcal{C}(\Delta)=\mathcal{C}[\Omega]$.

6.3. Let $\Omega(0)\cup\Omega(1)=\emptyset$, $C=\Omega$, and $\omega^\Delta=(e,\omega,e,\dots,e)$ for any $\omega\in\Omega$. Then SEM(Δ) is a variety of Ω-algebras ([3]).

6.4. Let $\{\hat{\omega}\mid\omega\in\Omega\}$ be a collection of disjoint sets such that:

$$\omega\in\Omega(n) \Rightarrow \hat{\omega}=\{\omega_o,\omega_1,\dots,\omega_n\},\ \omega_i\neq\omega_j \text{ if } i\neq j,$$

and let $C=\bigcup_{\omega\in\Omega}\hat{\omega}$. Define a semigroup representation Δ by:

$\omega^\Delta=(\omega_o,\omega_1,\dots,\omega_n)$. Then SEM($\Delta$) is the class of all Ω-algebras ([10]).

It follows from 6.2 and 5.1 that there exists a semigroup representation Δ such that SEM(Δ) is not a variety. This result suggests the question of giving a description of semigroup representations Δ such that SEM(Δ) is a variety.

Before giving an answer to this question we have to define a mapping $\xi\mapsto\xi^\Delta$ of Ω-terms into $C\cup\{\cdot\}$-terms which is induced by a semigroup representation Δ. This mapping is defined, namely, in the following way.

(v) If x is a variable, then $x^\Delta=x$.

(vi) If $\omega\in\Omega(n)$ and $\xi_1,\dots,\xi_n$ are Ω-terms, then

$$(\omega(\xi_1,\dots,\xi_n))^\Delta=\omega_o\xi_1^\Delta\omega_1\xi_2^\Delta\dots\omega_{n-1}\xi_n^\Delta\omega_n.$$

(Here, the elements of C are interpretated as constants and e as an "empty" symbol.)

If t is a $C\cup\{\cdot\}$-term such that $t=\xi^\Delta$ for an Ω-term ξ, then we say that t is a Δ-term.

Now we can give a description of semigroup representations Δ such that the classes SEM(Δ) are varieties.

6.5. Let $\Omega(0)=\emptyset$. SEM(Δ) is a variety iff the following condition is satisfied:

⊛ If t',t'' and $t_1t't_2$ are Δ-terms, then $t_1t''t_2$ is a Δ-term as well.

(In the paper [26] it is shown that ⊛ is a sufficient condition, and the necessarity of this condition is shown in [21].)

Let Δ be a semigroup representation of Ω, and $\underline{A}=(A,\Omega)$ an Ω-algebra. Then the semigroup $\underline{A}^\Delta$ with the presentation:

$$\langle A\cup C;\{a=\omega_o a_1\dots\omega_{n-1}a_n\omega_n \mid a=\omega(a_1,\dots,a_n) \text{ in } \underline{A}\}\rangle \qquad (6.1)$$

is called a universal Δ-envelopping semigroup of $\underline{A}$. If $\mathcal{V}$ is a variety of semigroups, then by $\underline{A}^\Delta_{\mathcal{V}}$ is denoted the semigroup with the presentation (6.1) in the variety $\mathcal{V}$.

Now we can give the following characterization of $\mathcal{V}(\Delta)$.

6.6. $\underline{A}\in\mathcal{V}(\Delta)$ iff:

$$a,b\in A \Rightarrow (a=b \text{ in } \underline{A}^\Delta_{\mathcal{V}} \Rightarrow a=b \text{ in } \underline{A}) \qquad (6.2)$$

We notice that the statement that $\mathcal{V}(\Delta)$ is a variety one proves by showing that if an Ω-algebra $\underline{A}$ satisfies all the identities of $\mathcal{V}(\Delta)$, then it satisfies (6.2), as well.

*) $S^1=S$ if $\underline{S}$ has a unity, and if this is not the case then $1\notin S$ and 1 is the unity of $\underline{S}^1$.

6.7. If $\langle B;\Lambda\rangle$ is a presentation of $\underline{A}$ in the class of Ω-algebras, then $\underline{A}^{\Delta}_{V}$ has a presentation $\langle B\cup C;\Lambda^{\Delta}\rangle$ in V. Moreover, if Δ is a semigroup representation of the kind given in 6.1, 6.3 or 6.4, then $\langle B;\Lambda\rangle$ is solvable iff $\langle B\cup C;\Lambda^{\Delta}\rangle$ is solvable ([10]).

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СМЕСТУВАЊЕ НА УНИВЕРЗАЛНИ АЛГЕБРИ ВО ПОЛУГРУПИ

Р е з и м е

Во оваа работа се дава приказ на резултати што се однесуваат за сместување на универзални алгебри во полугрупи.