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On generalized fractional integral inequalities for functions of bounded variation with two variables

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Abstract: In this paper, authors establish some identities for generalized fractional integrals. Utilizing these identities, some Ostrowski and Midpoint type inequalities for generalized fractional integrals for functions of bounded variation with two variables are obtained. Moreover, some new inequalities involving Riemann–Liouville fractional integrals are presented as special cases of our main results.

Key words and phrases: Function of bounded variation, Ostrowski inequality, generalized fractional integral.

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