

MATEMATICHKI BILTEN

BULLETIN MATHÉMATIQUE DE LA SOCIÉTÉ DES MATHÉMATIENS DE LA RÉPUBLIQUE MACÉDOINE

[Mat. Bilten Vol. 45 \(LXXI\), No. 1, 2021](#), 35–47

<https://doi.org/10.37560/matbil21451035s>

On numerical solutions of linear fractional differential equations

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Abstract: Fractional differential equations have excited considerable interest recently, both in pure and applied mathematics. In this paper, we apply Fractional Adams-Bashforth Method (FAB), Fractional Adams-Bashforth-Moulton Method (FABM) and Fractional Multistep Differential Transform Method (FMDTM), for obtaining the numerical solutions of two distinct linear systems of fractional differential equations with fractional derivatives described in the Caputo sense. The numerical results for the three methods are compared with the exact solution for each linear system by using the relative difference between the exact and the approximate solution at each integration point. The results are given both graphically and tabularly, concluding that, aside from occasional non-monotonicity for small time values, all three numerical methods gradually diverge from the exact solution with increasing integration time, and the superiority of each numerical method over the others depends on the particular system under investigation.

Key words and phrases: Fractional differential equations, Fractional Adams-Bashforth Method, Fractional Multistep Differential Transform Method, Fractional Adams-Bashforth-Moulton Method.

AMS Mathematics Subject Classification (2010): 34C28, 34A08, 74H15.

Document type: article.

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