EQUIVALENCE OF INTRINSIC SHAPE, BASED ON $\mathcal{V}$-CONTINUOUS FUNCTIONS, AND SHAPE

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Abstract. In this paper is given a direct proof that the intrinsic shape category $\text{InSh}$ constructed with continuous functions over coverings, is equivalent to original shape category $\text{Sh}$ of Borsuk obtained by embedding compact metric spaces in Hilbert cube $Q$. The functor $\text{Sh} \to \text{InSh}$ is established taking a fundamental sequence $(\tilde{f}_n)$ from $X$ to $Y$ in the sense of Borsuk, and by associating to the continuous function $\tilde{f}_n : Q \to Q$ mapping some neighborhood of $X$ into a union of the members of a covering $\mathcal{V}$ of $Y$, a $\mathcal{V}$-continuous function $f_n : X \to Y$, and forming the proximate sequence $(f_n)$ in the sense of N. Shekutkovski, Top. Proc. 39 (2012).

1. Introduction

For compact metric spaces, shape theory was introduced by K. Borsuk. His original approach was by embedding a compact metric space in Hilbert cube. Further on we will denote his shape category of compact metric spaces by $\text{Sh}$.

From the early beginning of the theory arised the question of intrinsic definition of shape i.e., a definition without external spaces like Hilbert cube. A shape category $\text{HN}$ is obtained by intrinsic definition by Sanjurjo in [6], and it is shown that $\text{HN}$ and $\text{Sh}$ are equivalent constructing a functor $\text{HN} \to \text{Sh}$. A shape category, also by intrinsic approach is obtained by Shekutkovski in [7], which we will further on denote by $\text{InSh}$. Using the result [3] and [4] about equivalence of categories $\text{InSh}$ and $\text{HN}$, and the isomorphic functor $\text{Sh} \to \text{HN}$ the equivalence categories $\text{Sh}$ and $\text{InSh}$ is indirectly proven in [5], using.
In this paper we will give a direct proof of equivalence of categories $\text{Sh}$ and $\text{InSh}$ constructing a functor $\text{Sh} \rightarrow \text{InSh}$, i.e. compared with the functor from [6], it is in the opposite direction. The construction requires some new ideas, among them the introduced notions of depth and regular covering.

Let $X$ and $Y$ be compact metric spaces. By a covering we understand a covering consisting of open sets. We repeat the intrinsic approach to shape from [7] (also [9]):

**Definition 1.** Suppose $V$ is a finite covering of $Y$. A function $f : X \rightarrow Y$ is $V$-continuous at point $x \in X$, if there exists a neighborhood $U_x$ of $x$, and $V \in V$, such that $f(U_x) \subseteq V$.

A function $f : X \rightarrow Y$ is $V$-continuous, if it is $V$-continuous at every point $x \in X$.

In this case, the family of all neighborhoods $U_x$, form a covering of $X$. By this, $f : X \rightarrow Y$ is $V$-continuous if there exists a finite covering $U$ of $X$, such that for any $U \in U$, there exists $V \in V$ such that $f(U) \subseteq V$. We denote shortly: there exists $V$, such that $f(U) \prec V$.

If $f : X \rightarrow Y$ is $V$-continuous, then $f : X \rightarrow Y$ is $W$-continuous for any $W$, such that $V \prec W$.

If $V$ is a finite covering of $Y$, and $V \in V$, than star of $V$ is the open set $st(V) = \{ W | W \in V, W \cap V \neq \emptyset \}$. We form a new covering $st(V) = \{ st(V) | V \in V \}$.

**Definition 2.** The functions $f, g : X \rightarrow Y$ are $V$-homotopic, if there exists a function $F : X \times I \rightarrow Y$ such that:

1) $F$ is $st(V)$-continuous,
2) $F$ is $V$-continuous at all points of $X \times \partial I$, and
3) $F(x, 0) = f(x), F(x, 1) = g(x)$.

The relation of $V$-homotopy is denoted by $f \sim_V g$. This is an equivalence relation.

Usually, the condition 2) of the previous statement is formulated as:

2) there exists an neighbourhood $N$ of $\partial I = \{0, 1\}$ such that $F|_{X \times N}$ is $V$-continuous.
Definition 3. A cofinal sequence of finite coverings $\mathcal{V}_1 \succ \mathcal{V}_2 \succ \ldots \mathcal{V}_n \succ \ldots$ is a sequence of finite coverings of spaces, such that for any covering $\mathcal{V}$, there exists $n$, such that $\mathcal{V}_n \prec \mathcal{V}$.

In a compact metric space there exists such a sequence. This fact allows working with proximate sequences instead with proximate nets.

Definition 4. The sequence $(f_n)$ of functions $f_n : X \to Y$ is a proximate sequence from $X$ to $Y$, if there exists a cofinal sequence of finite coverings of $Y$, $\mathcal{V}_1 \succ \mathcal{V}_2 \succ \ldots \mathcal{V}_n \succ \ldots$, and for all indexes $f_n$ and $f_{n+1}$ are $\mathcal{V}_n$-homotopic.

In this case we say that $(f_n)$ is a proximate sequence over $(\mathcal{V}_n)$.

If $(f_n)$ and $(f'_n)$ are proximate sequences from $X$ to $Y$, than there exists a cofinal sequence of finite coverings $\mathcal{V}_1 \succ \mathcal{V}_2 \succ \ldots \mathcal{V}_n \succ \ldots$ such that $(f_n)$ and $(f'_n)$ are proximate sequences over $(\mathcal{V}_n)$.

Definition 5. Two proximate sequences $(f_n)$ and $(f'_n)$ are homotopic if there exists a cofinal sequence of finite coverings $\mathcal{V}_1 \succ \mathcal{V}_2 \succ \ldots \mathcal{V}_n \succ \ldots$ of $Y$, such that $(f_n)$ and $(f'_n)$ are $\mathcal{V}_n$-homotopic for all integers $n$.

We say that $(f_n)$ and $(f'_n)$ are homotopic over $(\mathcal{V}_n)$.

Let $(f_n) : X \to Y$ be a proximate sequence over $(\mathcal{V}_n)$ and $(g_k) : Y \to Z$ be a proximate sequence over $(\mathcal{W}_k)$. For a covering $\mathcal{W}_k$ of $Z$, there exists a covering $\mathcal{V}_{n_k}$ of $Y$ such that $g(\mathcal{V}_{n_k}) \prec \mathcal{W}_k$. Then, the composition is the proximate sequence $(h_k) = (g_k f_{n_k}) : X \to Z$. In [7] is proven that compact metric spaces and homotopy classes of proximate sequences $[(f_n)]$ form the shape category $InSh$ i.e. isomorphic spaces in this category has the same shape.

We repeat the original definition of Bosuk of shape category $Sh$. Let $X$, $Y$ and $Z$, be compact metric spaces, embedded in the Hilbert space $Q$.

A sequence of maps $f_k : Q \to Q$, $k = 1, 2, 3, \ldots$, is fundamental sequence from $X$ to $Y$, if for every neighborhood $V$ of $Y$, there exist a neighborhood $U$ of $X$ and there exists $k_0 \in \mathbb{N}$, such that $f_k|_U \simeq f_{k+1}|_U$ in $V$ for all $k \geq k_0$. A fundamental sequence is denoted with $(f_k : X \to Y)_{Q, Q}$.

Proposition 1. If $(f_k : X \to Y)_{Q, Q}$ is a fundamental sequence, then there exists a decreasing sequence of neighborhoods of $Y$, $V_1 \supseteq V_2 \supseteq \ldots$ such that $\cap V_n = Y$, and there exists a decreasing sequence of neighborhoods of
Two fundamental sequences \((f_k : X \to Y)_{Q,Q}\) and \((f'_k : X \to Y)_{Q,Q}\) are homotopic, if for every neighborhood \(V\) of \(Y\) in \(Q\), there exist neighborhood \(U\) of \(X\) in \(Q\) and \(k_0 \in \mathbb{N}\), such that \(f_k\) is homotopic to \(f'_k\) in \(V\), for all \(k \geq k_0\).

The relation of homotopy \((f_k : X \to Y) \simeq (f'_k : X \to Y)\) of fundamental sequences is an equivalence relation. We use symbol \([\_]\) to denote homotopy classes.

The composition of fundamental sequences \((f_k : X \to Y)\) and \((g_k : Y \to Z)\) is the fundamental sequence \((g_k f_k : X \to Z)\). The composition of classes \([f_k : X \to Y]\) and \([g_k : Y \to Z]\) is the class \([g_k f_k : X \to Z]\).

2. Equivalence of categories

Let \(X\) be a set and \(\mathcal{V} = \{V_i | i = 1, 2, ..., n\}\) be a finite set of subsets of \(X\). If \(V \in \mathcal{V}\), we define depth of \(V\) in \(\mathcal{V}\), to be the biggest number \(k \in \mathbb{N}\) such that there exist sequence of elements of \(\mathcal{V}\) such that \(V \subset V_2 \subset V_3 \subset ... \subset V_k\). (if \(V\) is not a proper subset of any element in \(\mathcal{V}\) then depth of \(V\) is 1). The depth of \(V\) we denote with \(\text{depth}(V)\).

A covering \(\mathcal{V}\) of \(Y\) in \(X\) is regular if it satisfies the following conditions:

1) If \(V \in \mathcal{V}\) than \(V \cap Y \neq \emptyset\).

2) If \(U, V \in \mathcal{V}\) and \(U \cap V \neq \emptyset\), than \(U \cap V \in \mathcal{V}\).

About the condition 1) see definition of proper covering, ([10], Definition 8.1., p. 249), while the condition 2) together with 1) shows that \(\mathcal{V}\) is a regular family relative to \(Y\) in the sense of [10] (Definition 3.5. p. 262).

For a covering \(\mathcal{V}\) we introduce the notation \(|\mathcal{V}| = \cup \{V | V \in \mathcal{V}\}\).

We define a function \(r_{\mathcal{V}} : |\mathcal{V}| \to Y\) in the following way:

Suppose \(n\) is the biggest depth of the elements of \(\mathcal{V}\). A function \(r_{\mathcal{V}} : |\mathcal{V}| \to Y\) will be defined by induction.

For points \(y\) belonging to \(V \in \mathcal{V}\), such that \(\text{depth}(V) = n\), we choose a fixed point \([V] \in V \cap Y\) and put \(r_{\mathcal{V}}(y) = [V]\).

Suppose the function is defined for all \(y\) belonging to some \(V \in \mathcal{V}\), such that \(\text{depth}(V) > n - k\) for some natural number \(k\).
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If \( y \) belongs to some \( V \in \mathcal{V} \) with \( \text{depth}(V) = n - k \), and \( r_V(y) \) is not defined yet, i.e. \( y \in V \setminus \bigcup \{ V \mid V \in \mathcal{V}, \text{depth}(V) > n - k \} \), we choose a fixed point \([V], [V] \in (V \setminus \bigcup \{ V \mid V \in \mathcal{V}, \text{depth}(V) > n - k \}) \cap Y \) and put \( r_V(y) = [V] \).

The function \( r_V \) is well defined and is \( V \)-continuous.

In fact, \( r_V(y) = [V] \) if and only if \( V \) is the smallest set in \( \mathcal{V} \) which contains \( y \), i.e. \( V = \bigcap_{U \in \mathcal{V}} U \).

Now, if \( \mathcal{V} \) is a regular covering of \( Y \) and \( \bar{f} : X \to |\mathcal{V}| \) is a continuous function, we define function \( f : X \to Y \) with \( f(x) = r_V \bar{f}(x) \) for all \( x \in X \).

The function \( f \) is well defined and since \( \bar{f} \) is continuous, the function \( f \) is \( \mathcal{V} \)-continuous.

We will say that the function is \( f \) is obtained from a continuous function \( \bar{f} \) and covering \( \mathcal{V} \).

**Theorem 1.** If \( Y \) is compact metric space embedded in Hilbert cube \( Q \), \( \mathcal{V} \) and \( \mathcal{W} \) are regular coverings of \( Y \) in \( Q \) such that \( \mathcal{W} \prec \mathcal{V} \), then \( r_V : |\mathcal{W}| \to Y \) (the restriction of \( r_V \) to \( |\mathcal{W}| \)) and \( r_W \) are \( \mathcal{V} \)-homotopic.

**Proof.** We consider the function \( R : |\mathcal{W}| \times I \to Y \) defined by

\[
R(x, t) = \begin{cases} r_V(x), & (x, t) \in |\mathcal{W}| \times [0, 1) \\ r_W(x), & (x, 1) \in |\mathcal{W}| \times \{1\} \end{cases}
\]

If \((x, t) \in |\mathcal{W}| \times [0, 1)\), then \( R(x, t) = r_V(x) \), and \( R \) is \( \mathcal{V} \)-continuous in \((x, t)\).

If \((x, 1) \in |\mathcal{W}| \times \{1\}\), then \( R(x, 1) = r_W(x) = [W] \), where \( W \) is the smallest set in \( \mathcal{W} \) that contains \( x \).

From \( \mathcal{W} \prec \mathcal{V} \), it follows that \( W \subseteq V \in \mathcal{V} \), and we can choose \( V \) to be the smallest set in \( \mathcal{V} \), with the property \( W \subseteq V \). Then \( r_V(V) \in V \cap Y \) and

\[
R(W \times 1) = r_W(W) \in W \cap Y \subseteq V \cap Y.
\]

We take the neighborhood \( W \times [0, 1] \) of \((x, 1)\) and \((w, t) \in W \times [0, 1)\).

There is a smallest set \( V_w \) in \( \mathcal{V} \) such that \( w \in V_w \). We obtain

\[
R(w, t) = r_V(w) = [V_w] \in V_w \cap Y, \text{ for all } t \in [0, 1).
\]

From the construction \( V_w \subseteq V \) for all \( w \in W \). Finally from \((*)\) and \((***)\),

\[
R(W \times [0, 1]) = R(W \times [0, 1]) \cup R(W \times 1) \subseteq V \cap Y.
\]
It follows that \( R \) is \( \mathcal{V} \)-continuous at \((x, 1) \in |\mathcal{W}| \times \{1\}\), and \( R(x, 0) = r_{\mathcal{V}}(x), R(x, 1) = r_{\mathcal{W}}(x)\).

By Proposition 1, if \((\tilde{f}_n)\) is fundamental sequence from \(X\) to \(Y\), there exists \(\mathcal{V}_1 \succ \mathcal{V}_2 \succ \ldots\), a cofinal sequence of finite regular coverings of \(Y\) in \(Q\), and there exists a cofinal sequence of finite regular coverings of \(X\) in \(Q\), \(\mathcal{U}_1 \succ \mathcal{U}_2 \succ \ldots\) such that \(\tilde{f}_n(\mathcal{U}_n) \subset |\mathcal{V}_n|\), and continuous functions \(f_n\) and \(\tilde{f}_{n+1}\), are homotopic in \(|\mathcal{V}_n|\). \(\square\)

We define a function \(f_n : X \to Y\), \(n \in \mathbb{N}\) by \(f_n(x) = r_{\mathcal{V}_n} \tilde{f}_n(x)\) for \(x \in X\).

**Theorem 2.** 1) If \((\tilde{f}_n)\) is fundamental sequence then \((f_n)\) is proximate sequence.

2) If two fundamental sequences \((f_n)\) and \((f'_n)\) are homotopic, then the obtained from them proximate sequences \((f_n)\) and \((f'_n)\) are homotopic.

**Proof.** 1) Suppose \(\overline{f}_{n,n+1}\), is the homotopy connecting \(\overline{f}_n\) and \(\overline{f}_{n+1}\). We define \(f_{n,n+1} : X \times I \to Y\) by

\[
f_{n,n+1}(x, t) = r_{\mathcal{V}_n} \overline{f}_{n,n+1}(x, t).
\]

Then \(f_{n,n+1}\) is \(\mathcal{V}_n\)-continuous and

\[
f_{n,n+1}(x, 0) = r_{\mathcal{V}_n} \overline{f}_n(x), \quad f_{n,n+1}(x, 1) = r_{\mathcal{V}_n} \overline{f}_{n+1}(x).
\]

By the previous theorem \(r_{\mathcal{V}_n} |_{|\mathcal{V}_{n+1}|}\) (the restriction of \(r_{\mathcal{V}_n}\) to \(|\mathcal{V}_n|\)) and \(r_{\mathcal{V}_{n+1}}\) are \(\mathcal{V}_n\)-homotopic, by a homotopy \(R : |\mathcal{V}_{n+1}| \times I \to Y\) i.e.

\[
R(x, 0) = r_{\mathcal{V}_n}(x), \quad R(x, 1) = r_{\mathcal{V}_{n+1}}(x).
\]

Then the \(\mathcal{V}_n\)-homotopy \(R\overline{f}_{n+1} : X \times I \to Y\) satisfies

\[
R\overline{f}_{n+1}(x, 0) = r_{\mathcal{V}_n} \overline{f}_{n+1}(x), \quad R\overline{f}_{n+1}(x, 1) = r_{\mathcal{V}_{n+1}} \overline{f}_{n+1}(x) = f_{n+1}(x).
\]

Since \(\mathcal{V}_n\)-homotopy is an equivalence relation by (*) and (**) it follows that \(r_{\mathcal{V}_n} \overline{f}_n(x) = f_n(x)\) and \(r_{\mathcal{V}_{n+1}} \overline{f}_{n+1}(x) = f_{n+1}(x)\) are \(\mathcal{V}_n\)-homotopic.

2) Suppose \(\overline{f}_n\), is the homotopy connecting \(\overline{f}_n\) and \(\overline{f}_n\). We define \(F_n : X \times I \to Y\) by \(F_n(x, t) = r_{\mathcal{V}_n} \overline{f}_n(x, t)\).

Then \(F_n\) is \(\mathcal{V}_n\)-continuous and \(st(\mathcal{V}_n)\) continuous at all points of \(X \times \partial I\), and \(F_n(x, 0) = f_n(x), F_n(x, 1) = f'_n(x)\). \(\square\)

We will describe a functor \(\Phi : Sh \to InSh\).

1) On compact metric spaces is defined by \(\Phi(X) = X\), for every compact metric space \(X\).
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2) and is defined with $\Phi \left( \left[ (\tilde{f}^n) \right] \right) = \left[ (f^n) \right]$ for every class of fundamental sequences $\left[ (f^n) \right]$ from $X$ to $Y$.

**Theorem 3.** $\Phi : Sh \to InSh$ is a functor which is isomorphism of categories.

**Proof.** First we will prove that for two fundamental sequences $(f^n) : X \to Y$ and $(g^n) : Y \to X$ holds

$$\Phi \left( \left[ (\tilde{g}^n) (\tilde{f}^n) \right] \right) = \Phi \left( \left[ (\tilde{g}^n) \right] \right) \Phi \left( \left[ (\tilde{f}^n) \right] \right).$$

As in the beginning of this section there exists a cofinal sequence of finite regular coverings $W_1 \triangleright W_2 \triangleright \ldots$ of $Z$ in $Q$, there exists a cofinal sequence of finite regular coverings $V_1 \triangleright V_2 \triangleright \ldots$ of $Y$ in $Q$, and there exists a cofinal sequence of finite regular coverings $U_1 \triangleright U_2 \triangleright \ldots$ of $X$ in $Q$, such that $\tilde{f}^n(|U_n|) \subseteq |V_n|$ and $\tilde{f}^n(|V_n|) \subseteq |W_n|$, and such that continuous functions $\tilde{f}^n, \tilde{f}^{n+1}$, are homotopic in $|V_n|$ and $\tilde{g}^n, \tilde{g}^{n+1}$, are homotopic in $|W_n|$ for all $n$.

Suppose a proximate sequence $(g^n)$ from $Y$ to $Z$ is obtained from fundamental sequence $(\tilde{g}^n)$, taking a cofinal sequence of finite regular coverings $(W_k)$ of $Z$ in $Q$.

Suppose $(f_{nk})$ from $X$ to $Y$ is a proximate subsequence of the proximate sequence $(f^n)$ obtained from fundamental sequence $(\tilde{f}^n)$. The subsequence of natural numbers is chosen such that $f_{nk}(V_{nk}) \triangleright W_k$.

The fundamental sequences $(\tilde{f}^n)$ and $(\tilde{f}_{nk})$ are in the same class. By theorem from [7], $(f_{nk})$ and $(f^n)$ are in the same class and if we put $\tilde{g}_k \tilde{f}_{nk} = \tilde{h}_k$.

In fact we have to prove

$$[r_{W_k} \tilde{h}_k] = [(r_{W_k} \tilde{g}_k)(r_{V_{nk}} \tilde{f}_{nk})]$$

or that $r_{W_k} \tilde{h}_k$ and $(r_{W_k} \tilde{g}_k)(r_{V_{nk}} \tilde{f}_{nk})$ are homotopic.

Take a point $x$ in $X$. By definition

$$r_{W_k} \tilde{h}_k(x) = [W] \quad (*)$$

where $W$ is the smallest set $W$ in $W_k$ such that $\tilde{g}_k \tilde{f}_{nk} (x) = \tilde{h}_k (x) \in W$.

Since $\tilde{g}_k (V_{nk}) \triangleright W_k$ there exist $V'$ in $V$ such that $\tilde{g}_k (V') \subseteq W$.

On the other hand, by definition $r_{V_{nk}} \tilde{f}_{nk} (x) = [V]$ where $V$ is the smallest set $V$ in $V_{nk}$ such that $\tilde{f}_{nk} (x) \in V$. 


Since $V$ is the smallest, then $V \subseteq V'$ and it follows $\bar{g}_k(V) \subseteq W$. Then

$$(r_{\mathcal{W}_k}\bar{g}_k)(r_{\mathcal{V}_{nk}}\bar{f}_{nk})(x) = r_{\mathcal{W}_k}(\bar{g}_k[V]) \in W \quad (**)$$

Then, by (**) and (*) $r_{\mathcal{W}_k}\bar{h}_k = h_k$ and $(r_{\mathcal{W}_k}\bar{g}_k)(r_{\mathcal{V}_{nk}}\bar{f}_{nk}) = g_kf_{nk}$ are $\mathcal{W}_k$-near, and since $h_k$ is $\mathcal{W}_k$-continuous, by Lemma 1.1 from [14] we have that $h_k$ and $g_kf_{nk}$ are $\mathcal{W}_k$-homotopic.

Now, we will prove that

$$\Phi([[1_X]]) = 1_{\Phi(X)}.$$  

One represent of the identical morphism in $Sh$ is the class of fundamental sequences from $X$ to $X$ is $(\bar{T}_n)$, where $\bar{T}_n : Q \to Q$, $n \in \mathbb{N}$ are copies of identical map defined by $\bar{T}_n(x) = x$, $x \in X$.

Then $\Phi([[\bar{T}_n]]) = [(1_n)]$, where $1_n : X \to X$, $n \in \mathbb{N}$ are copies of identical map.

$[(1_n)]$ is the identical morphism in $InSh$ since for proximate sequences $(f_n) : X \to Y$ and $(g_n) : Y \to X$ holds $(f_n)(1_n) = (f_n)$ and $(g_n)(1_n) = (g_n)$.

It follows $\Phi([[1_X]]) = 1_{\Phi(X)}$.

To prove that $\Phi : Sh \to InSh$ is a functor which is an isomorphism of categories we use the following reformulation of theorem 1 of [5]: For every proximate sequence $(g_n) : X \to Y$ there exists a fundamental sequence $(\bar{f}_n)$ from $X$ to $Y$ and a cofinal sequence of coverings $\mathcal{V}_1 \succ \mathcal{V}_2 \succ \ldots$, such that for such that $\bar{f}_n|_X$ and $g_n$ are $\mathcal{V}_n$-close for all integers. Also, all fundamental sequences obtained from $(f_n)$ in this way are homotopic.

One proximate sequence $(f_n)$ obtained from fundamental sequence $(\bar{f}_n)$, it consists of $\mathcal{V}_n$-close functions $f_n$ and $\bar{f}_n$. Therefore $(f_n)$ and $(g_n)$ are homotopic and it follows that the functor is surjective.

To prove that the functor is injective, suppose the proximate sequences $(f_n)$ and $(f'_n)$ from $X$ to $Y$ are obtained from fundamental sequences $(\bar{f}_n)$ and $(\bar{f'}_n)$ from $X$ to $Y$, respectively. Suppose $(f_n)$ and $(f'_n)$ are homotopic, i.e. $f_n$ and $f'_n$ are connected by homotopy $F_n$ for all positive integers. By Ho’s theorem, in fact from its form Lemma 1 from [6], it follows that there exists a continuous homotopy $\bar{F}_n$ connecting $\bar{f}_n$ and $\bar{f'}_n$ for all natural numbers $n$. □
References


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ЕКВИВАЛЕНЦИЈА НА ВНАТРЕШЕН ОБЛИК, БАЗИРАННА НА $\mathcal{V}$—НЕПРЕКИНИТА ФУНКЦИИ, И ОБЛИК

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Р е з и м е

Во овој труд даден е директен доказ дека категоријата на внатрешен облик $\text{InSh}$ конструирана со непрекинати функции над покривајќи е еквијвалентна со оригиналната категорија на облик $\text{Sh}$ на Борсукин добиена со вложување на компактни метрички простори во Хилбертовиот куб $Q$. Функторот $\text{Sh} \to \text{InSh}$ е добиен земајќи фундаментална низа $(\bar{f}_n)$ од $X$ во $Y$ во смисла на Борсуки и на непрекинатата функција $\bar{f}_n : Q \to Q$ која пресликува некоја околина на $X$ во унија на членови на покривање $\mathcal{V}$ на $Y$, и се придружува $\mathcal{V}$-непрекината функција $f_n : X \to Y$, и формирајќи проксимативната низа $(f_n)$ во смисла на N. Shekutkovski, Top. Proc. 39 (2012).

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