Now it is easy to verify that $A = -(z+1)M - MN$. Using the previous equations we obtain

$$BAB^{-1} = -(z+1)BMB^{-1} - BMNB^{-1} = -(z+1)D - DBNB^{-1} =$$

$$= -(z+1)D - DT = -D[(z+1)I + T] =$$

$$\begin{bmatrix} 0 & 0 & 0 & \cdots & 0 & 0 \\ 1\cdot n & -1(2+z) & 0 & \cdots & 0 & 0 \\ 0 & 2\cdot(n-1) & -2(3+z) & \cdots & 0 & 0 \\ 0 & 0 & 3\cdot(n-2) & \cdots & 0 & 0 \\ \cdot & & & & & \\ \cdot & & & & & \\ \cdot & & & & & \\ 0 & 0 & 0 & \cdots & 0 & 0 \\ 0 & 0 & 0 & \cdots & -(n-1)(n+z) & 0 \\ 0 & 0 & 0 & \cdots & n\cdot 1 & -n(n+1+z) \end{bmatrix}$$

Since the last matrix is lower triangular, its eigenvalues are the diagonal elements, i.e. $\lambda_i = -i(i+1+z)$, $(0 \le i \le n)$. These eigenvalues are also eigenvalues of the matrix A because A is similar to BAB^{-1}.

In order to find the eigenvectors of the matrix A, first we find the eigenvectors of the matrix BAB^{-1}. Let $\mathbf{X}_j$ be the eigenvector corresponding to the eigenvalue $\lambda_j = -j(j+1+z)$, $(0 \le j \le n)$. Now we should solve the following system

$$BAB^{-1}\cdot \mathbf{X}_j = \lambda_j \mathbf{X}_j, \qquad (0 \le j \le n).$$

By putting $\mathbf{X}_j = \left[x_{j0}, x_{j1}, x_{j2}, \cdots, x_{jn}\right]^T$, we obtain

$$\lambda_0 x_{j0} = \lambda_j x_{j0}$$

$$u_1 x_{j0} + \lambda_1 x_{j1} = \lambda_j x_{j1}$$

$$u_2 x_{j1} + \lambda_2 x_{j2} = \lambda_j x_{j2}$$

$$u_3 x_{j2} + \lambda_3 x_{j3} = \lambda_j x_{j3}$$

$$\cdot \quad \cdot \quad \cdot$$

$$u_n x_{j,n-1} + \lambda_n x_{jn} = \lambda_j x_{jn}$$

where $u_i = (n+1-i)i$, $(1 \le i \le n)$. It follows from here that $x_{j0} = \cdots = x_{j,j-1} = 0$ and $x_{j,j}, \cdots, x_{j,n}$ satisfy the following system

$$u_{j+1} x_{j,j} + \lambda_{j+1} x_{j,j+1} = \lambda_j x_{j,j+1}$$