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# 352. A PROCEDURE FOR OBTAINING RELATIONS BETWEEN THE ANGLES OF A TRIANGLE*

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**After accepting the paper: A family of goniometric inequalities of Bager for publication (These Publications № 339), the Editorial Committee received a paper of Ž. Madevski which deals with methods of obtaining inequalities for the elements of a triangle. Since the results of Madevski are in some connexion with those of Bager, a copy of Bager's paper was sent to Madevski with the suggestion to write for a short summary of his methods. His interesting results are to be published in more detail later. We give below the summary in question (*Editorial comment*).**

**1.** Let $\Sigma_1$ and $\Sigma_2$ be two classes of triangles and $f, g, h$ three real functions such that: if $\alpha, \beta, \gamma$ are angles of a triangle $\Delta_1 \in \Sigma_1$, then $f(\alpha)$, $g(\beta)$, $h(\gamma)$ are angles of a triangle $\Delta_2 \in \Sigma_2$. Then obviously, if $R(x, y, z)$ is a generally valid relation between the angles of the triangles of the class $\Sigma_2$, then $R(f(\alpha), g(\beta), h(\gamma))$ is a generally valid relation between the angles of the triangles of the class $\Sigma_1$. (Of course, $\Sigma_1$ or $\Sigma_2$, or both, may be the class of all triangles.)

So, if we can find convenient functions $f, g, h$, then from every known relation between the angles of a triangle, we can derive a new one.

EXAMPLES. 1° Assume that $f, g, h$ are defined by

$$(1) \qquad f(\alpha)=k\alpha+\lambda, \quad g(\beta)=k\beta+\mu, \quad h(\gamma)=k\gamma+\nu,$$

where $\lambda, \mu, \nu$ and $k$ are real numbers such that:

(i) $\lambda, \mu, k\pi+\lambda, k\pi+\mu \geqq 0$,

(ii) $\lambda+\mu, k\pi+\lambda+\mu \leqq \pi$,

(iii) $\nu=(1-k)\pi-\lambda-\mu$.

If $\alpha, \beta, \gamma$ are angles of a triangle then so are $f(\alpha)$, $g(\beta)$, $h(\gamma)$, and therefore if $R(\alpha, \beta, \gamma)$ is a generally valid relation between the angles of a tri-

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angle, then $R(k\alpha+\lambda,\ k\beta+\mu,\ k\gamma+\nu)$ is also a generally valid relation between the angles of a triangle.

As a special case for $k=-1/2$, $\lambda=\mu=\nu=\pi/2$, we obtain the transformation $\sigma$ from BAGER's paper (see these Publications № 339, pp. 5—26).

2° If we want to find a transformation of the form (1) such that the class $\Sigma_1$ of acute triangles is mapped onto the class $\Sigma_2$ of all triangles, then we get that there is only one possibility for the numbers $k$, $\lambda$, $\mu$, $\nu$, namely, $k=-2$, $\lambda=\mu=\nu=\pi$, and this is, in fact, the transformation $\tau$ from the BAGER's paper mentioned above.

**2.** Let $R(a, b, c)$ be a generally valid relation between the sides of a triangle, and let $f$, $g$, $h$ be three real functions such that if $\alpha$, $\beta$, $\gamma$ are angles of a triangle then $f(\alpha)$, $g(\beta)$, $h(\gamma)$ are sides of a triangle; then, $R(f(\alpha),\ g(\beta),\ h(\gamma))$ is also a generally valid relation between the angles. (For example, it is known that if $\alpha$, $\beta$, $\gamma$ are angles of a triangle then we can take $\cos\alpha/2$, $\cos\beta/2$, $\cos\gamma/2$ as sides of a triangle.)

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