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ON INEQUALITY (GI; 5.47)

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Abstract. In the proof of the inequality (GI; 5.47) is assumed that

$$a = s-a, \quad b = s-b, \quad c = s-c$$

(aren't true) becouse 2s=a+b+c.

So, the inequality in view may be which is not correct as

$$\sqrt{\frac{a}{r_a}} + \sqrt{\frac{b}{r_b}} + \sqrt{\frac{c}{r_c}} < \frac{3}{2}\sqrt{\frac{s}{r}}, \tag{1}$$

which is not of interest.

Using the Cauchy's inequality

$$(\Sigma a_i b_i)^2 \le \Sigma a_i^2 \cdot \Sigma b_i^2, \tag{2}$$

we can derive some inequalities of type (1).

Let $a,b,c,h_a,h_b,h_c,r_a,r_b,r_c$ be the sides, altitudes and radii of excircles of a triangle, respectively.

$\underline{1}^o$ If $a_1=\sqrt{a}$, $a_2=\sqrt{b}$, $a_3=\sqrt{c}$ and $b_1 = \frac{1}{\sqrt{r_a}}$, $b_2 = \frac{1}{\sqrt{r_b}}$, $b_3 = \frac{1}{\sqrt{r_c}}$, then from (2) we get

$$\sqrt{\frac{a}{r_a}} + \sqrt{\frac{b}{r_b}} + \sqrt{\frac{c}{r_c}} \le \sqrt{2}\sqrt{\frac{s}{r}};$$

this inequality is stronger than (1).

$\underline{2}^o$ If $a_1=\sqrt{r_a}$, $a_2=\sqrt{r_b}$, $a_3=\sqrt{r_c}$ and $b_1 = \frac{1}{\sqrt{a}}$, $b_2 = \frac{1}{\sqrt{b}}$, $b_3 = \frac{1}{\sqrt{c}}$, then from (2), by virtue (GI; 5.22), (GI; 5.1) and (GI; 5.12), we obtain

$$\sqrt{\frac{r_a}{a}} + \sqrt{\frac{r_b}{b}} + \sqrt{\frac{r_c}{c}} \le \sqrt{\frac{\sqrt{3}}{2}\cdot\frac{\sqrt{4R+r}}{\sqrt{r}}} \le \frac{3\sqrt[4]{3}}{2}\sqrt{\frac{R}{r}} \le \frac{1}{\sqrt{2\sqrt{3}}}\cdot\frac{s}{r}.$$

$\underline{3}^o$ If $a_1=\sqrt{a}$, $a_2=\sqrt{b}$, $a_3=\sqrt{c}$ and $b_1 = \frac{1}{\sqrt{h_a}}$, $b_2 = \frac{1}{\sqrt{h_b}}$, $b_3 = \frac{1}{\sqrt{h_c}}$, then (2) provides

$$\sqrt{\frac{a}{h_a}} + \sqrt{\frac{b}{h_a}} + \sqrt{\frac{c}{h_c}} \le \sqrt{2}\sqrt{\frac{s}{r}}.$$

$\underline{4}^{\circ}$ If $a_1=\sqrt{h_a}$, $a_2=\sqrt{h_b}$, $a_3=\sqrt{h_c}$ and $b_1 = \frac{1}{\sqrt{a}}$, $b_2 = \frac{1}{\sqrt{b}}$, $b_3 = \frac{1}{\sqrt{c}}$, then from (2), by virtue (GI; 5.22) and (GI; 6.1), we obtain

$$\sqrt{\frac{h_a}{a}} + \sqrt{\frac{h_b}{b}} + \sqrt{\frac{h_c}{c}} \le \sqrt{\frac{3}{2}}\sqrt{\frac{s}{r}}.$$

$\underline{5}^{\circ}$ If $a_1=\sqrt{r_a}$, $a_2=\sqrt{r_b}$, $a_3=\sqrt{r_c}$ and $b_1 = \frac{1}{\sqrt{h_a}}$, $b_2 = \frac{1}{\sqrt{h_b}}$, $b_3 = \frac{1}{\sqrt{h_c}}$, then from (2) we get

$$\sqrt{\frac{r_a}{h_a}} + \sqrt{\frac{r_b}{h_b}} + \sqrt{\frac{r_c}{h_c}} \le \frac{\sqrt{4R+r}}{\sqrt{r}} \le \frac{\sqrt{3}}{3}\sqrt{\frac{s}{r}}.$$

$\underline{6}^{\circ}$ If $a_1=\sqrt{h_a}$, $a_2=\sqrt{h_b}$, $a_3=\sqrt{h_c}$ and $b_1 = \frac{1}{\sqrt{r_a}}$, $b_2 = \frac{1}{\sqrt{r_b}}$, $b_3 = \frac{1}{\sqrt{r_c}}$, then from (2) we get

$$\sqrt{\frac{h_a}{r_a}} + \sqrt{\frac{h_b}{r_b}} + \sqrt{\frac{h_c}{r_c}} \le \sqrt[4]{3}\sqrt{\frac{s}{r}}.$$

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* Equalities in $\underline{1}^{\circ}$-$\underline{6}^{\circ}$ hold if and only if the triangle is equilateral.

R E F E R E N C E S

[GI] Bottema D., Djordjević R.Ž., Janić R.R., Mitrinović D.S., Vasić P.M.: GEOMETRIC INEQUALITIES, Groningen, 1969

ЗА НЕРАВЕНСТВОТО (GI; 5.47)

Ж. Мадевски

Р е з и м е

Во доказот на неравенството (GI; 5.47) се зема дека a=s-a, b=s-b, c=s-c што претставува превид, бидејќи 2s=a+b+c, па поради тоа неравенството може да се запише само во обликот (1).

Користејќи го неравенството на Коши, може да се подобри (1), и да се добијат и други неравенства.