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ON THE INTEGRAL TRANSFORM SHEHU FOR NONLINEAR FRACTIONAL PROBLEMS

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Abstract. The intention of this research is to establish the truncated solutions of nonlinear fractional problems, including fractional partial differential equations (FPDEs), by employing the Shehu integral transform first and the iterative method later. The numerical solutions reveal the effectiveness and accuracy of the technique.

1. Introduction

In the modeling of various processes in mathematics, biology, physics, engineering, and so on [9, 13, 15, 16, 3, 4, 10, 5], FPDEs are utilized since the results of fractional mathematical models have better results than other mathematical models. Therefore quite a few research, such as the existence, uniqueness, and regularity of solutions, on the processes of heat diffusion, modeled by FPDEs [18, 11, 17]. The contribution of fractional differential equations plays a vital role in science and technology. In the establishment of the solutions for fractional nonlinear problems, integral transform techniques are common, [14, 8]. This result leads us to establish truncated solutions of nonlinear FPDEs by utilizing the Daftardar-Jafari method (DJM) and Shehu transform together. The encountered fractional nonlinear problems modeling real-life phenomena are analyzed and solved by taking the physical background and features of the nonlinear problem into consideration.

Shehu transform, introduced by Shehu Maitama and Weidong Zhao [12], is an integral transformation, converting ordinary and partial differential equations into simpler equations. It is obtained by generalizing Laplace transformation. Moreover it is a linear transformation like Laplace and Sumudu transformations. Laplace and Yang integral transformations are obtained from Shehu transformation by taking $q = 1$ and $p = 1$ respectively. From this point of view, it could be better to use Shehu transform instead of Laplace or Yang transforms [12].

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The aim of this research is to extend Shehu Transform iterative method (STIM) to construct truncated solutions of time-space FPDEs. The STIM technique is applied to construct the solutions of various linear and nonlinear FPDEs. The truncated solutions are established in terms of Mittag-Leffler functions and fractional trigonometric functions. The advantages of this method can be listed as follows:

- CPU time is shorter,
- Robust method for nonlinear and linear FPDEs,
- Less calculation time,
- Less margin of error.

As a result, it is clear that utilizing STIM is one of the best choices for the establishment of the solutions for linear and nonlinear mathematical models including FPDEs.

2. Preliminaries

Essential knowledge such as notions and properties of fractional calculus are presented in this subsection [13, 16].
The definition of α^{th} order Riemann-Liouville time-fractional integral of a real valued function $u(x,t)$ is given as

$$I_t^{\alpha} u\left(x,t\right) = \frac{1}{\Gamma(\alpha)} \int_0^t \left(t-s\right)^{\alpha-1} u(x,s)ds.$$

The definition of the time-fractional derivative operator of order α for $u(x,t)$ in Caputo sense is given as

$$\frac{\partial^{\alpha} u(x,t)}{\partial t^{\alpha}} = I_t^{m-\alpha}\left[\frac{\partial^m u(x,t)}{\partial t^m}\right] =$$
$$\begin{cases} \frac{1}{\Gamma(m-\alpha)} \int_0^t \left(t-y\right)^{m-\alpha-1} \frac{\partial^m u(x,y)}{\partial y^m} dy, & m-1<\alpha<m, \\ \frac{\partial^m u(x,t)}{\partial t^m}, & \alpha = m. \end{cases}$$

Two parameterized Mittag-Leffler function is defined as

$$E_{\alpha,\beta}\left(z\right) = \sum_{k=0}^{\infty} \frac{z^k}{\Gamma(\alpha k+\beta)}, Re\left(\alpha\right)>0, z,\beta \in \mathbb{C}.$$

where α and β are parameters.
The definition of Shehu transformation is given by [12]

$$\mathbb{S}\left[f(t)\right] = F\left(p,q\right) = \int_0^{\infty} e^{-\frac{p}{q}t} f\left(t\right)dt.$$

The set of functions which has Shehu transformation is given by

$$\left\{ f(t) \middle| \exists P, \tau_1, \tau_2 > 0, \left|f\left(t\right)\right| < Pe^{\frac{|t|}{\tau_j}}, if t \in \left(-1\right)^j \times [0,\infty) \right\}.$$

A key property of it for this research is the following

$$\mathbb{S}\left[t^{\alpha}\right]=\int_{0}^{\infty}e^{-\frac{pt}{q}}t^{\alpha}dt=\Gamma\left(\alpha+1\right)\left(\frac{q}{p}\right)^{\alpha+1},Re\left(\alpha\right)>0$$

inverse Shehu transform of $\left(\frac{q}{p}\right)^{n\alpha+1}$ is the following

$$\mathbb{S}^{-1}\left[\left(\frac{q}{p}\right)^{n\alpha+1}\right]=\frac{t^{n\alpha}}{\Gamma(n\alpha+1)},Re\left(\alpha\right)>0.$$

Shehu transformation of time-fractional derivative $\frac{\partial^{\alpha}f(x,t)}{\partial t^{\alpha}}$ in Caputo sense have the following form, [1]:

$$\mathbb{S}\left[\frac{\partial^{\alpha}f(x,t)}{\partial t^{\alpha}}\right]=\left(\frac{p}{q}\right)^{\alpha}\mathbb{S}\left[f(x,t)\right]-\sum_{k=0}^{n-1}\left[\left(\frac{p}{q}\right)^{\alpha-k-1}\frac{\partial^{k}f(x,0)}{\partial t^{k}}\right],\tag{2.1}$$

$n-1<\alpha\leq n$, $n\in\mathbb{N}$.

3. The Implementation of STIM

The implementation of STIM for mathematical models, including space-time fractional differential equations, is presented in this section. Now consider the following problem

$$\frac{\partial^{\zeta}f}{\partial t^{\zeta}}=\mathbb{F}\left(x,f,\frac{\partial^{\eta}f}{\partial x^{\eta}},\ldots,\frac{\partial^{l\eta}f}{\partial x^{l\eta}}\right),j-1<\zeta\leq j,i-1<\eta\leq i,l,j,i\in\mathbb{N}\tag{3.1}$$

with the initial restrictions

$$\frac{\partial^{m}f(x,0)}{\partial t^{m}}=h_{m}(x),k=0,1,2,\ldots,j-1,\tag{3.2}$$

where $\mathbb{F}\left(x,f,\frac{\partial^{\eta}f}{\partial x^{\eta}},\ldots,\frac{\partial^{l\eta}f}{\partial x^{l\eta}}\right)$ denotes linear or nonlinear function.

After carrying out the Shehu transform to Eq. (3.1) and rearrangement, we have

$$f\left(x,t\right)=g\left(x,t\right)+G\left(x,f,\frac{\partial^{\eta}f}{\partial x^{\eta}},\ldots,\frac{\partial^{l\eta}f}{\partial x^{l\eta}}\right),\tag{3.3}$$

where the operator G and the function g is defined as:

$$g\left(x,t\right)=\mathbb{S}^{-1}\left[\sum_{m=0}^{j-1}\left[\left(\frac{q}{p}\right)^{m+1}\frac{\partial^{m}f(x,0)}{\partial t^{m}}\right]\right],$$

$$G\left(x,f,\frac{\partial^{\eta}f}{\partial x^{\eta}},\ldots,\frac{\partial^{l\eta}f}{\partial x^{l\eta}}\right)=\mathbb{S}^{-1}\left[\left(\frac{q}{p}\right)^{\zeta+1}\mathbb{S}\left[\mathbb{F}\left(x,f,\frac{\partial^{\eta}f}{\partial x^{\eta}},\ldots,\frac{\partial^{l\eta}f}{\partial x^{l\eta}}\right)\right]\right].$$

In order to construct the solution of Eq. (3.3), the DJM [7] is employed.

$$f=\sum_{n=0}^{\infty}f_{n},\tag{3.4}$$

is the series form of the solution in terms of f_n, established recursively.
After decomposition of the operator G, we have

$$G\left(x,\sum_{n=0}^{\infty}f_n,\frac{\partial^{\eta}\left(\sum_{n=0}^{\infty}f_n\right)}{\partial x^{\eta}},\ldots,\frac{\partial^{l\eta}\left(\sum_{n=0}^{\infty}f_n\right)}{\partial x^{l\eta}}\right)=G\left(x,f_0,\tfrac{\partial^{\eta}f_0}{\partial x^{\eta}},\ldots,\tfrac{\partial^{l\eta}f_0}{\partial x^{l\eta}}\right)+$$
$$+\sum_{c=1}^{\infty}\left(G\left(x,\sum_{n=0}^{c}f_n,\frac{\partial^{\eta}\left(\sum_{n=0}^{c}f_n\right)}{\partial x^{\eta}},\ldots,\frac{\partial^{l\eta}\left(\sum_{n=0}^{c}f_n\right)}{\partial x^{l\eta}}\right)\right)$$
$$-\sum_{c=1}^{\infty}\left(G\left(x,\sum_{n=0}^{c-1}f_n,\frac{\partial^{\eta}\left(\sum_{n=0}^{c-1}f_n\right)}{\partial x^{\eta}},\ldots,\frac{\partial^{l\eta}\left(\sum_{n=0}^{c-1}f_n\right)}{\partial x^{l\eta}}\right)\right).$$

$$\mathbb{S}^{-1}\left[\left(\tfrac{q}{p}\right)^{\zeta+1}\mathbb{S}\left[\mathbb{F}\left(x,\textstyle\sum_{n=0}^{\infty}f_n,\frac{\partial^{\eta}\left(\sum_{n=0}^{\infty}f_n\right)}{\partial x^{\eta}},\ldots,\frac{\partial^{l\eta}\left(\sum_{n=0}^{\infty}f_n\right)}{\partial x^{l\eta}}\right)\right]\right]=$$
$$=\mathbb{S}^{-1}\left[\left(\tfrac{q}{p}\right)^{\zeta+1}\mathbb{S}\left[\mathbb{F}\left(x,f_0,\tfrac{\partial^{\eta}f_0}{\partial x^{\eta}},\ldots,\tfrac{\partial^{l\eta}f_0}{\partial x^{l\eta}}\right)\right]\right]+$$
$$+\sum_{c=1}^{\infty}\mathbb{S}^{-1}\left[\left(\frac{q}{p}\right)^{\zeta+1}\mathbb{S}\left[\mathbb{F}\left(x,\sum_{n=0}^{c}f_n,\frac{\partial^{\eta}\left(\sum_{n=0}^{c}f_n\right)}{\partial x^{\eta}},\ldots,\frac{\partial^{l\eta}\left(\sum_{n=0}^{c}f_n\right)}{\partial x^{l\eta}}\right)\right]\right]-$$
$$-\sum_{c=1}^{\infty}\mathbb{S}^{-1}\left[\left(\frac{q}{p}\right)^{\zeta+1}\mathbb{S}\left[\mathbb{F}\left(x,\sum_{n=0}^{c-1}f_n,\frac{\partial^{\eta}\left(\sum_{n=0}^{c-1}f_n\right)}{\partial x^{\eta}},\ldots,\frac{\partial^{l\eta}\left(\sum_{n=0}^{c-1}f_n\right)}{\partial x^{l\eta}}\right)\right]\right]. \tag{3.5}$$

Plugging Eqs. (3.4), (3.5) into Eq. (3.3) leads to

$$\sum_{n=0}^{\infty}f_n=$$
$$\mathbb{S}^{-1}\left[\sum_{m=0}^{j-1}\left[\left(\frac{q}{p}\right)^{m+1}\frac{\partial^m f(x,0)}{\partial t^m}\right]\right]+\mathbb{S}^{-1}\left[\left(\tfrac{q}{p}\right)^{\zeta+1}\mathbb{S}\left[\mathbb{F}\left(x,f_0,\tfrac{\partial^{\eta}f_0}{\partial x^{\eta}},\ldots,\tfrac{\partial^{l\eta}f_0}{\partial x^{l\eta}}\right)\right]\right]$$

$$+\sum_{c=1}^{\infty}\left(\mathbb{S}^{-1}\left[\left(\frac{q}{p}\right)^{\zeta+1}\mathbb{S}\left[\mathbb{F}\left(x,\sum_{n=0}^{c}f_n,\frac{\partial^{\eta}\left(\sum_{n=0}^{c}f_n\right)}{\partial x^{\eta}},\ldots,\frac{\partial^{l\eta}\left(\sum_{n=0}^{c}f_n\right)}{\partial x^{l\eta}}\right)\right]\right]\right)$$
$$-\sum_{c=1}^{\infty}\left(\mathbb{S}^{-1}\left[\left(\frac{q}{p}\right)^{\zeta+1}\mathbb{S}\left[\mathbb{F}\left(x,\sum_{n=0}^{c-1}f_n,\frac{\partial^{\eta}\left(\sum_{n=0}^{c-1}f_n\right)}{\partial x^{\eta}},\ldots,\frac{\partial^{l\eta}\left(\sum_{n=0}^{c-1}f_n\right)}{\partial x^{l\eta}}\right)\right]\right]\right)$$

The functions f_n are recursively computed as follows:

$$\left.\begin{aligned}
f_0 &= \mathbb{S}^{-1}\left[\sum_{m=0}^{j-1}\left[\left(\frac{q}{p}\right)^{m+1}\frac{\partial^m f(x,0)}{\partial t^m}\right]\right],\\
f_1 &= \mathbb{S}^{-1}\left[\left(\frac{q}{p}\right)^{\zeta+1}\mathbb{S}\left[\mathbb{F}\left(x,f_0,\frac{\partial^{\eta}f_0}{\partial x^{\eta}},\ldots,\frac{\partial^{l\eta}f_0}{\partial x^{l\eta}}\right)\right]\right],\\
f_{r+1} &= \mathbb{S}^{-1}\left[\left(\frac{q}{p}\right)^{\zeta+1}\mathbb{S}\left[\mathbb{F}\left(x,\sum_{n=0}^{c}f_n,\frac{\partial^{\eta}\left(\sum_{n=0}^{c}f_n\right)}{\partial x^{\eta}},\ldots,\frac{\partial^{l\eta}\left(\sum_{n=0}^{c}f_n\right)}{\partial x^{l\eta}}\right)\right]\right]\\
&\quad-\mathbb{S}^{-1}\left[\left(\frac{q}{p}\right)^{\zeta+1}\mathbb{S}\left[\mathbb{F}\left(x,\sum_{n=0}^{c-1}f_n,\frac{\partial^{\eta}\left(\sum_{n=0}^{c-1}f_n\right)}{\partial x^{\eta}},\ldots,\frac{\partial^{l\eta}\left(\sum_{n=0}^{c-1}f_n\right)}{\partial x^{l\eta}}\right)\right]\right].
\end{aligned}\right\}\tag{3.6}$$

The truncated solutions of r-terms of Eqs. (3.1), (3.2) is established as $f \approx f_0 + f_1 + \ldots + f_{r-1}$. For the convergence analysis of DJM, refer to [2].

4. Elucidative Example

Take the following mathematical model

$$\frac{\partial^{\zeta}f}{\partial t^{\zeta}} = \left(\frac{\partial^{\eta}f}{\partial x^{\eta}}\right)^2 - f\left(\frac{\partial^{\eta}f}{\partial x^{\eta}}\right), t > 0, \zeta, \eta \in (0,1], \tag{4.1}$$

subject to the initial datum

$$u(x,0) = 3 + \frac{5}{2}E_{\eta}(x^{\eta}), \tag{4.2}$$

into account. Applying the Shehu transform to Eq. (4.1) leads to

$$\mathbb{S}\left[\frac{\partial^{\zeta}f}{\partial t^{\zeta}}\right] = \mathbb{S}\left[\left(\frac{\partial^{\eta}f}{\partial x^{\eta}}\right)^2 - f\left(\frac{\partial^{\eta}f}{\partial x^{\eta}}\right)\right].$$

The property (2.1) allow us to have the following

TABLE 1. The values of the analytic solution and truncated solutions of 10-term by STIM for example.

	STIM	Exact	STIM	Exact	STIM	Exact
t	ζ,η=0.9	ζ,η=0.9	ζ,η=0.95	ζ,η=0.95	ζ,η=1	ζ,η=1
	time= 0.1139s		time= 0.1173s		time= 0.1125s	
0	9.9292	9.9292	9.8664	9.8664	9.7957	9.7957
0.1	7.7178	8.0988	7.8855	8.0681	8.0344	8.0344
0.2	6.4308	6.7935	6.5797	6.7610	6.7296	6.7296
0.3	5.5702	5.8563	5.6615	5.8082	5.7629	5.7629
0.4	4.9704	4.9495	5.0029	4.3881	5.0468	5.0468
0.5	4.5413	4.5055	4.5245	4.3661	4.5163	4.5163
0.6	4.2280	4.2772	4.1736	3.9080	4.1233	4.1233
0.7	3.9958	4.0783	3.9139	3.9112	3.8322	3.8322
0.8	3.8225	3.8596	3.7204	3.9075	3.6166	3.6165
0.9	3.6965	3.6961	3.5763	3.5912	3.4571	3.4567
1	3.6187	3.5796	3.4712	3.4481	3.3398	3.3383

$$\mathbb{S}\left[f(x,t)\right]=\left(\frac{q}{p}\right)f\left(x,0\right)+\left(\frac{q}{p}\right)^{\zeta+1}\left(\mathbb{S}\left[\left(\frac{\partial^{\eta}f}{\partial x^{\eta}}\right)^{2}-f\left(\frac{\partial^{\eta}f}{\partial x^{\eta}}\right)\right]\right).$$

Utilizing the inverse transformation of Shehu leads to

$$f\left(x,t\right)=\mathbb{S}^{-1}\left[\left(\frac{q}{p}\right)f\left(x,0\right)\right]+\mathbb{S}^{-1}\left[\left(\frac{q}{p}\right)^{\zeta+1}\left(\mathbb{S}\left[\left(\frac{\partial^{\eta}f}{\partial x^{\eta}}\right)^{2}-f\left(\frac{\partial^{\eta}f}{\partial x^{\eta}}\right)\right]\right)\right].$$

Utilizing the recurrence relation (3.6), we have

$$\left.\begin{aligned}
f_0&=\mathbb{S}^{-1}\left[\left(\frac{q}{p}\right)f\left(x,0\right)\right]=3+\frac{5}{2}E_{\eta}\left(x^{\eta}\right),\\
f_1&=\mathbb{S}^{-1}\left[\left(\frac{q}{p}\right)^{\zeta+1}\left(\mathbb{S}\left[\left(\frac{\partial^{\eta}f_0}{\partial x^{\eta}}\right)^{2}-f\left(\frac{\partial^{\eta}f_0}{\partial x^{\eta}}\right)\right]\right)\right]=-\frac{15t^{\zeta}E_{\eta}\left(x^{\eta}\right)}{2\Gamma\left(\zeta+1\right)},\\
f_2&=\mathbb{S}^{-1}\left[\left(\frac{q}{p}\right)^{\zeta+1}\left(\mathbb{S}\left[\left(\frac{\partial^{\eta}\left(f_0+f_1\right)}{\partial x^{\eta}}\right)^{2}-f\left(\frac{\partial^{\eta}\left(f_0+f_1\right)}{\partial x^{\eta}}\right)\right]\right)\right]\\
&\quad-\mathbb{S}^{-1}\left[\left(\frac{q}{p}\right)^{\zeta+1}\left(\mathbb{S}\left[\left(\frac{\partial^{\eta}f_0}{\partial x^{\eta}}\right)^{2}-f\left(\frac{\partial^{\eta}f_0}{\partial x^{\eta}}\right)\right]\right)\right]=\frac{45t^{2\zeta}E_{\eta}\left(x^{\eta}\right)}{2\Gamma\left(2\zeta+1\right)},\\
f_3&=-\frac{135t^{3\zeta}E_{\eta}\left(x^{\eta}\right)}{2\Gamma\left(3\zeta+1\right)},\\
f_4&=\frac{405t^{4\zeta}E_{\eta}\left(x^{\eta}\right)}{2\Gamma\left(4\zeta+1\right)}.
\end{aligned}\right\}$$

Finally, the series solution allows us to establish the analytic solution of the problem (4.1)-(4.2):

$$f(x,t) = f_0 + f_1 + f_2 + f_3 + \ldots = 3 + \left[\frac{5}{2} E_{\zeta}\left(-3t^{\zeta}\right)\right] E_{\eta}(x^{\eta}),$$

which is the same exact solution obtained in [6].

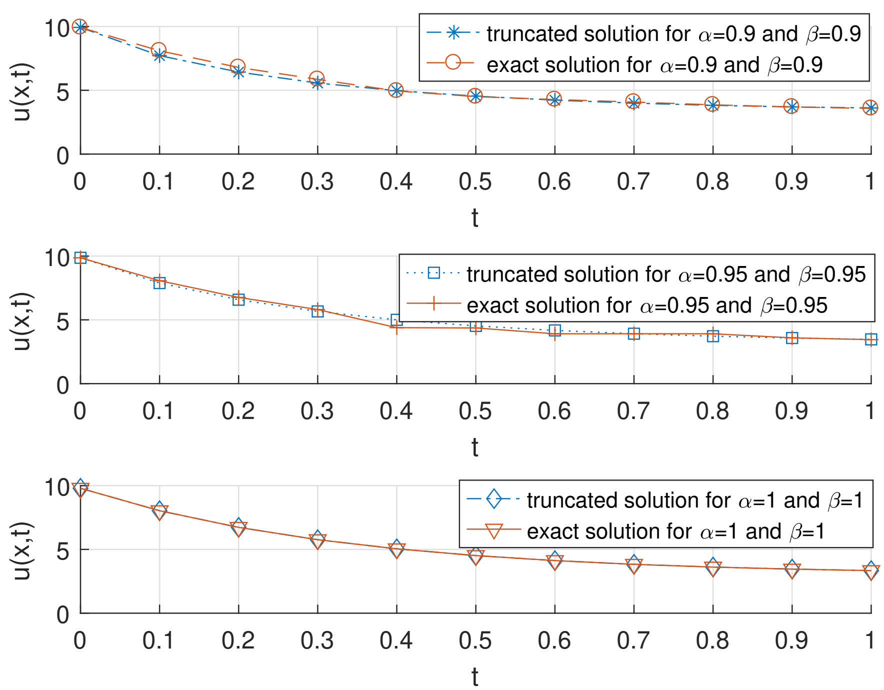

FIGURE 1. Figures of 10-term truncated solution and exact solution for different ζ and η at $x = 1$.

It is clear from Table 1 that the values of the solution for $\zeta = \eta = 1$ and exact solution are very close each other which indicates that the method utilized in this research is effective and accurate one for the solution of space-time FPDEs. The programming language is MATLAB 2016b. The computer used has an Intel (R) Core (TM) i3 CPU M 370.

5. CONCLUSION

In this study, we build numerical or analytical solutions of mathematical models, including nonlinear space time FPDEs by means of STIM. This approach is proved to be more convenient and effective for nonlinear FPDEs than the methods obtained by taking the various combinations of different integral transformations. The illustrative example verifies this result.

References

[1] R. Belgacem, D. Baleanu and A. Bokharı, *Shehu Transform and applications to Caputo-fractional differential equations*, International Journal of Analysis and Applications, 17(6), (2019) 917–927.

[2] S. Bhalekar and V. Daftardar-Gejji, *Convergence of the new iterative method*, International Journal of Differential Equations, 2011, (2011), Article ID 989065.

[3] S. Cetinkaya, A. Demir and H. Kodal Sevindir, *Solution of Space-Time-Fractional Problem by Shehu Variational Iteration Method*, Advances in Mathematical Physics, 2021, (2021), Article ID 5528928.

[4] S. Cetinkaya and A. Demir, *On the Solution of Bratu's Initial Value Problem in the Liouville-Caputo Sense by ARA Transform and Decomposition Method*, Comptes rendus de l'Academie bulgare des Sciences, 74(12), (2021), 1729–1738.

[5] S. Cetinkaya and A. Demir, *The Analytic Solution of Time-Space Fractional Diffusion Equation via New Inner Product with Weighted Function*, Communications in Mathematics and Applications, 10(4) (2019), 865–873.

[6] S. Choudhary and V. Daftardar-Gejji, *Invariant subspace method: a tool for solving fractional partial differential equations*, Fract. Calc. Appl. Anal., 20(2), (2017), 477-–493.

[7] V. Daftardar-Gejji and H. Jafari, *An iterative method for solving nonlinear functional equations*, J. Math. Anal. Appl., 316(2), (2006), 753-–763.

[8] L. Debnath and D. D. Bhatta, *Solutions to few linear fractional inhomogeneous partial differential equations in fluid mechanics* , Fract. Calc. Appl. Anal., 7(1), (2018), 21-–36.

[9] A. Kilbas, H. M. Srivastava and J. J. Trujillo, *Theory and Applications of Fractional Differential Equations*, North-Holland Mathematics Studies, Elsevier Science B.V., Amsterdam, 2006.

[10] H. Kodal Sevindir, S. Cetinkaya and A. Demir, *On Effects of a New Method for Fractional Initial Value Problems*, Advances in Mathematical Physics, 2021, (2021), Article ID 7606442.

[11] L. Mahto, S. Abbas, M. Hafayed and H. M. Srivastava, *Approximate Controllability of Sub-Diffusion Equation with Impulsive Condition*, Mathematics, 7(190), (2019), 1–16.

[12] S. Maitama and W. Zhao, *New Integral Transform: Shehu Transform a Generalization of Sumudu and Laplace Transform for Solving Differential Equations*, Int. J. Anal. Appl., 17(2), (2019), 167–190.

[13] I. Podlubny, *Fractional differential equation*, Academic Press, San Diego, CA, 1999.

[14] I. Podlubny, *Fractional Differential Equations: An Introduction to Fractional Derivatives, Fractional Differential Equations, to Methods of Their Solution and Some of Their Applications*, Academic Press, New York, 1998.

[15] J. Sabatier, O. P. Agarwal and J. A. T. Machado, *Advances in fractional calculus: theoretical developments and applications in physics and engineering*, Springer, Dordrecht, 2007.

[16] S. G. Samko, A. A. Kilbas and O. I. Marichev, *Fractional integrals and derivatives theory and applications*, Gordon and Breach, Amsterdam, 1993.

[17] X. J. Yang, H. M. Srivastava, D. F. M.Torres and A. Debbouche, *General Fractional-order Anomalous Diffusion with Non-singuler Power-Law Kernel*, Thermal Science, 21(1), (2017), 1–9.

[18] K. V. Zhukovsky and H. M. Srivastava, *Analytical solutions for heat diffusion beyond Fourier law*, Applied Mathematics and Computation, 293, (2017), 423–437.

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