ЗА ЕДНА ХОМОГЕНА ЛИНЕАРНА ДИФЕРЕНЦИЈАЛНА РАВЕНКА ОД ЧЕТВРТ РЕД

Во оваа работа е даден одговор на прашањето*:

Да се испита дали диференцијалната равенка

(a)
$$(A_0x^4 + A_1x^3 + A_2x^2 + A_3x + A_4)\frac{d^4y}{dx^4} + (B_0x^3 + B_1x^2 + B_2x + B_3)\frac{d^3y}{dx^3}$$
$$+ (C_0x^2 + C_1x + C_2)\frac{d^2y}{dx^2} + (D_0x + D_1)\frac{dy}{dx} + y = 0$$

има партикуларни интеграли дефинирани со

(b)
$$x = a_0 + a_1t + a_2t^2,$$
$$y = b_0 + b_1t + b_2t^2,$$

кога константите

$$A_0,\ A_1,\ A_2,\ A_3,\ A_4,\ B_0,\ B_1,\ B_2,\ B_3,\ C_0,\ C_1,\ C_2,\ D_0,\ D_1,$$
$$a_0,\ a_1,\ a_2,\ b_0,\ b_1,\ b_2,$$

погодно ќе се изберат?

Бидејќи

$$\frac{dy}{dx} = \frac{\dot y}{\dot x},\quad \frac{d^2y}{dx^2} = \frac{\dot x\ddot y - \ddot x\dot y}{\dot x^3},\quad \frac{d^3y}{dx^3} = \frac{\dot x(\dot x\dddot y - \dddot x\dot y) - 3\ddot x(\dot x\ddot y - \ddot x\dot y)}{\dot x^5},$$

$$\frac{d^4y}{dx^4} = \frac{\dot x^2(\ddot x\dddot y - \dddot x\ddot y + \dot x\ddddot y - \ddddot x\dot y) - 7\dot x\ddot x(\dot x\dddot y - \dddot x\dot y) - 3(\dot x\ddot y - \ddot x\dot y)(\dot x\dddot x - 5\ddot x^2)}{\dot x^7}$$

ravenkata (a) stanuva

(c)
$$(A_0x^4 + A_1x^3 + A_2x^2 + A_3x + A_4)\,[\dot x^2(\ddot x\dddot y - \dddot x\ddot y + \dot x\ddddot y - \ddddot x\dot y) - 7\dot x\ddot x(\dot x\dddot y - \dddot x\dot y)$$
$$- 3(\dot x\ddot y - \ddot x\dot y)(\dot x\dddot x - 5\ddot x^2)] + (B_0x^3 + B_1x^2 + B_2x + B_3)\,[\dot x^3(\dot x\dddot y - \dddot x\dot y)$$
$$- 3\dot x^2\ddot x(\dot x\ddot y - \ddot x\dot y)] + (C_0x^2 + C_1x + C_2)\dot x^4(\dot x\ddot y - \ddot x\dot y) + (D_0x + D_1)\dot x^6\dot y + \dot x^7y = 0.$$

Со оглед на тоа што е

$$x = a_0 + a_1t + a_2t^2,\qquad y = b_0 + b_1t + b_2t^2,$$
$$\dot x = a_1 + 2a_2t\qquad \dot y = b_1 + 2b_2t,$$
$$\ddot x = 2a_2,\qquad \ddot y = 2b_2,$$
$$\dddot x = 0,\qquad \dddot y = 0,$$
$$\ddddot x = 0,\qquad \ddddot y = 0,$$

*) Ова прашање ни беше дадено од страна на проф. Д. С. Митриновиќ.

$$A_0 x^4 + A_1 x^3 + A_2 x^2 + A_3 x + A_4 = a_0^4 A_0 + a_0^3 A_1 + a_0^2 A_2 + a_0 A_3 + A_4$$
$$+ (4a_0^3 a_1 A_0 + 3a_0^2 a_1 A_1 + 2a_0 a_1 A_2 + a_1 A_3)\, t + (6a_0^2 a_1^2 A_0 + 4a_0^3 a_2 A_0 + 3a_0^2 a_2 A_1$$
$$+ 3a_0 a_1^2 A_1 + a_1^2 A_2 + 2a_0 a_2 A_2 + a_2 A_3)\, t^2 + (12a_0^2 a_1 a_2 A_0 + 4a_0 a_1^3 A_0 + a_1^3 A_1$$
$$+ 6a_0 a_1 a_2 A_1 + 2a_1 a_2 A_2)\, t^3 + (6a_0^2 a_2^2 A_0 + 12a_0 a_1^2 a_2 A_0 + a_1^4 A_0 + 3a_0 a_2^2 A_1$$
$$+ 3a_1^2 a_2 A_1 + a_2^2 A_2)\, t^4 + (12a_0 a_1 a_2^2 A_0 + 4a_1^3 a_2 A_0 + 3a_1 a_2^2 A_1)\, t^5 + (6a_1^2 a_2^2 A_0$$
$$+ 4a_0 a_2^3 A_0 + a_2^3 A_1)\, t^6 + 4a_1 a_2^3 A_0 t^7 + a_2^4 A_0 t^8;$$
$$B_0 x^3 + B_1 x^2 + B_2 x + B_3 = a_0^3 B_0 + a_0^2 B_1 + a_0 B_2 + B_3 + (3a_0^2 a_1 B_0 + 2a_0 a_1 B_1 + a_1 B_2)\, t$$
$$+ (3a_0 a_1^2 B_0 + 3a_0^2 a_2 B_0 + a_1^2 B_1 + 2a_0 a_2 B_1 + a_2 B_2)\, t^2 + (a_1^3 B_0 + 6a_0 a_1 a_2 B_0$$
$$+ 2a_1 a_2 B_1)\, t^3 + (3a_0 a_2^2 B_0 + 3a_1^2 a_2 B_0 + a_2^2 B_1)\, t^4 + 3a_1 a_2^2 B_0 t^5 + a_2^3 B_0 t^6;$$
$$C_0 x^2 + C_1 x + C_2 = a_0^2 C_0 + a_0 C_1 + C_2 + (2a_0 a_1 C_0 + a_1 C_1)\, t + (a_1^2 C_0 + 2a_0 a_2 C_0$$
$$+ a_2 C_1)\, t^2 + 2a_1 a_2 C_0 t^3 + a_2^4 C_0 t^4;$$
$$D_0 x + D_1 = a_0 D_0 + D_1 + a_1 D_0 t + a_2 D_0 t^2;$$

$$\dot{x}^2 (\ddot{x}\ddddot{y} - \ddddot{x}\ddot{y} + \dot{x}\dddot{y}\,'' - \dddot{x}\,''\dot{y}) - 7\dot{x}\ddot{x}(\dot{x}\ddddot{y} - \ddddot{x}\dot{y}) - 3(\dot{x}\dddot{y} - \dddot{x}\dot{y})(\dot{x}\dddot{x} - 5\ddot{x}^2) = 120\, a_2^2 \alpha_{12};$$
$$\dot{x}^3 (\dot{x}\dddot{y} - \dddot{x}\dot{y}) - 3\dot{x}^2 \ddot{x} (\dot{x}\ddot{y} - \ddot{x}\dot{y}) = -12\, a_2 \alpha_{12};$$
$$\dot{x}^4 (\dot{x}\ddot{y} - \ddot{x}\dot{y}) = 2\alpha_{12} (a_1^4 + 8 a_1^3 a_2 t + 24 a_1^2 a_2^2 t^2 + 32 a_1 a_2^3 t^3 + 16 a_2^4 t^4);$$
$$\dot{x}^6 \dot{y} = a_1^6 b_1 + (12 a_1^5 a_2 b_1 + 2 a_1^6 b_2)\, t + (60 a_1^4 a_2^2 b_1 + 24 a_1^5 a_2 b_2)\, t^2$$
$$+ (160 a_1^3 a_2^3 b_1 + 120 a_1^4 a_2^2 b_2)\, t^3 + (240 a_1^2 a_2^4 b_1 + 320 a_1^3 a_2^3 b_2)\, t^4$$
$$+ (192 a_1 a_2^5 b_1 + 480 a_1^2 a_2^4 b_2)\, t^5 + (64 a_2^6 b_1 + 384 a_1 a_2^5 b_2)\, t^6 + 128 a_2^6 b_2 t^7;$$
$$x^7 y = a_1^7 b_0 + (14 a_1^6 a_2 b_0 + a_1^7 b_1)\, t + (84 a_1^5 a_2^2 b_0 + 14 a_1^6 a_2 b_1 + a_1^7 b_2)\, t^2$$
$$+ (280 a_1^4 a_2^3 b_0 + 84 a_1^5 a_2^2 b_1 + 14 a_1^6 a_2 b_2)\, t^3 + (560 a_1^3 a_2^4 b_0 + 280 a_1^4 a_2^3 b_1 + 84 a_1^5 a_2^2 b_2)\, t^4$$
$$+ (672 a_1^2 a_2^5 b_0 + 560 a_1^3 a_2^4 b_1 + 280 a_1^4 a_2^3 b_2)\, t^5 + (448 a_1 a_2^6 b_0 + 672 a_1^2 a_2^5 b_1$$
$$+ 560 a_1^3 a_2^4 b_2)\, t^6 + (128 a_2^7 b_0 + 448 a_1 a_2^6 b_1 + 672 a_1^2 a_2^5 b_2)\, t^7 + (128 a_2^7 b_1$$
$$+ 448 a_1 a_2^6 b_2)\, t^8 + 128 a_2^7 b_2 t^9,$$
$$\alpha_{ik} = \begin{vmatrix} a_i & a_k \\ b_i & b_k \end{vmatrix} \quad (i, k = 0, 1, 2),$$

од (c), во однос на t, го добиваме равенството

(d)
$$(128 a_2^7 b_2 D_0 + 128 a_2^7 b_2)\, t^9$$
$$+ (120 a_2^6 \alpha_{12} A_0 - 48 a_2^6 \alpha_{12} B_0 + 32 a_2^6 \alpha_{12} C_0 + 64 a_2^7 b_1 D_0 + 512 a_1 a_2^6 b_2 D_0$$
$$+ 128 a_2^7 b_1 + 448 a_1 a_2^6 b_2)\, t^8$$
$$+ (480 a_1 a_2^5 \alpha_{12} A_0 - 192 a_1 a_2^5 \alpha_{12} B_0 + 128 a_1 a_2^5 \alpha_{12} C_0 + 128 a_0 a_2^6 b_2 D_0 + 128 a_2^6 b_2 D_1$$
$$+ 256 a_1 a_2^6 b_1 D_0 + 864 a_1^2 a_2^5 b_2 D_0 + 128 a_2^7 b_0 + 448 a_1 a_2^6 b_1 + 672 a_1^2 a_2^5 b_2)\, t^7$$
$$+ (720 a_1^2 a_2^4 \alpha_{12} A_0 + 480 a_0 a_2^5 \alpha_{12} A_0 + 120 a_2^5 \alpha_{12} A_1 - 300 a_1^2 a_2^4 \alpha_{12} B_0 - 144 a_0 a_2^5 \alpha_{12} B_0$$
$$- 48 a_2^5 \alpha_{12} B_1 + 208 a_1^2 a_2^4 \alpha_{12} C_0 + 64 a_0 a_2^5 \alpha_{12} C_0 + 32 a_2^5 \alpha_{12} C_1 + 64 a_0 a_2^6 b_1 D_0$$
$$+ 384 a_0 a_1 a_2^5 b_2 D_0 + 64 a_2^6 b_1 D_1 + 384 a_1 a_2^5 b_2 D_1 + 432 a_1^2 a_2^5 b_1 D_0 + 800 a_1^3 a_2^4 b_2 D_0$$
$$+ 448 a_1 a_2^6 b_0 + 672 a_1^2 a_2^5 b_1 + 560 a_1^3 a_2^4 b_2)\, t^6$$
$$+ (1440 a_0 a_1 a_2^4 \alpha_{12} A_0 + 480 a_1^3 a_2^3 \alpha_{12} A_0 + 360 a_1 a_2^4 \alpha_{12} A_1 - 204 a_1^3 a_2^3 \alpha_{12} B_0$$
$$- 432 a_0 a_1 a_2^4 \alpha_{12} B_0 - 144 a_1 a_2^4 \alpha_{12} B_1 + 176 a_1^3 a_2^3 \alpha_{12} C_0 + 192 a_0 a_1 a_2^4 \alpha_{12} C_0$$
$$+ 96 a_1 a_2^4 \alpha_{12} C_1 + 192 a_0 a_1 a_2^5 b_1 D_0 + 480 a_0 a_1^2 a_2^4 b_2 D_0 + 192 a_1 a_2^5 b_1 D_1 + 480 a_1^2 a_2^4 b_2 D_1$$
$$+ 400 a_1^3 a_2^4 b_1 D_0 + 440 a_1^4 a_2^3 b_2 D_0 + 672 a_1^2 a_2^5 b_0 + 560 a_1^3 a_2^4 b_1 + 280 a_1^4 a_2^3 b_2)\, t^5$$
$$+ (720 a_0^2 a_2^4 \alpha_{12} A_0 + 1440 a_0 a_1^2 a_2^3 \alpha_{12} A_0 + 120 a_1^4 a_2^2 \alpha_{12} A_0 + 360 a_0 a_2^4 \alpha_{12} A_1$$

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\begin{aligned}
&+360a_1^2a_2^3\,\alpha_{12}A_1+120a_2^4\alpha_{12}A_2-468a_0\,a_1^2a_2^3\alpha_{12}B_0-84a_1^2a_2^2\alpha_{12}B_0-144a_0^2a_2^4\alpha_{12}B_0\\
&-156a_1^2a_2^3\alpha_{12}B_1-96a_0\,a_2^4\alpha_{12}B_1-48a_2^4\alpha_{12}B_2+82a_1^4a_2^2\alpha_{12}C_0+224a_0\,a_1^2a_2^3\alpha_{12}C_0\\
&+32a_0^2a_2^4\,\alpha_{12}C_0+112a_1^2\alpha_2^3\alpha_{12}C_1+32a_0\,a_2^4\,\alpha_{12}C_1+32\alpha_2^4\alpha_{12}C_2+240a_0\,a_1^2a_2^4b_1D_0\\
&+320a_0\,a_1^3a_2^3b_2\,D_0+240\alpha_1^2a_2^4b_1\,D_1+320a_1^3a_2^3b_2\,D_1+220a_1^4a_2^3b_1\,D_0+144a_1^5a_2^2b_2\,D_0\\
&+560a_1^3a_2^4b_0+280a_1^4a_2^3b_1+84a_1^5a_2^2b_2)\,t^4\\
&+(1440a_0^2\,a_1a_2^3\alpha_{12}A_0+480a_0\,a_1^3a_2^2\,\alpha_{12}A_0+120a_1^3a_2^2\alpha_{12}A_1+720a_0\,a_1\,a_2^3\alpha_{12}A_1\\
&+240a_1a_2^3\,\alpha_{12}A_2-12a_1^5\,a_2\,\alpha_{12}B_0-216a_0\,a_1^3a_2^2\alpha_{12}B_0-288a_0^2\,a_1a_2^3\alpha_{12}B_0\\
&-72a_1^3a_2^2\alpha_{12}B_1-192a_0\,a_1\,a_2^3\,\alpha_{12}B_1-96a_1\,a_2^3\,\alpha_{12}B_2+20a_1^5\,a_2\,\alpha_{12}C_0\\
&+128a_0\,a_1^3a_2^2\,\alpha_{12}C_0+64a_0^2\,a_1a_2^3\,\alpha_{12}C_0+64a_1^3a_2^2\alpha_{12}C_1\\
&+64a_0a_1\,a_2^3\alpha_{12}C_1+64a_1\,a_2^3\alpha_{12}C_2+160a_0\,a_1^3a_2^3\,b_1D_0+120a_0\,a_1^4a_2^2b_2\,D_0\\
&+160a_1^3a_2^3b_1\,D_1+120a_1^4a_2^2b_2\,D_1+72a_1^5a_2^2b_1\,D_0+26a_1^6\,a_2\,b_2\,D_0+280a_1^4a_2^3b_0\\
&+84a_1^5a_2^2b_1+14a_1^6\,a_2\,b_2)\,t^3\\
&+(720a_0^2a_1^2a_2^2\alpha_{12}A_0+480a_0^3a_2^3\alpha_{12}A_0+360a_0\,a_1^2a_2^2\alpha_{12}A_1+360a_0^2a_2^3\alpha_{12}A_1\\
&+120a_1^2a_2^2\alpha_{12}A_2+240a_0\,a_2^3\,\alpha_{12}A_2+120a_2^3\alpha_{12}A_3-36a_0\,a_1^4\,a_2\,\alpha_{12}B_0\\
&-180a_0^2a_1^2a_2^2\alpha_{12}B_0-48a_0^3a_2^3\alpha_{12}B_0-12a_1^4\,a_2\,\alpha_{12}B_1-120a_0\,a_1^2a_2^2\alpha_{12}C_0\\
&-48a_0^2a_2^3\alpha_{12}B_1-60a_1^2a_2^2\alpha_{12}B_2-48a_0\,a_2^3\alpha_{12}B_2-48a_2^3\alpha_{12}B_3+2a_1^6\alpha_{12}C_0\\
&+36a_0\,a_1^4\,a_2\,\alpha_{12}C_0+48a_0^2a_1^2a_2^2\,\alpha_{12}C_0+18a_1^4\,a_2\,\alpha_{12}C_1+48a_0\,a_1^2a_2^2\alpha_{12}C_1\\
&+48a_1^2a_2^2\alpha_{12}C_2+60a_0\,a_1^4a_2^2b_1\,D_0+24a_0\,a_1^5\,a_2\,b_2\,D_0+60a_1^4a_2^2b_1\,D_1+24a_1^5\,a_2\,b_2\,D_1\\
&+13a_1^6\,a_2b_1D_0+2a_1^7b_2\,D_0+84a_1^5a_2^2b_0+14a_1^6\,a_2\,b_1+a_1^7\,b_2)\,t^2\\
&+(480a_0^3\,a_1a_2^2\alpha_{12}A_0+360a_0^2\,a_1a_2^2\alpha_{12}A_1+240a_0\,a_1\,a_2^2\alpha_{12}A_2+120a_1\,a_2^2\alpha_{12}A_3\\
&-48a_0^3\,a_1a_2^2\,\alpha_{12}B_0-36a_0^2a_1^3\,a_2\,\alpha_{12}B_0-48a_0^2\,a_1a_2^2\alpha_{12}B_1-24a_0\,a_1^3\,a_2\,\alpha_{12}B_1\\
&-48a_0\,a_1\,a_2^2\alpha_{12}B_2-12a_1^3\,a_2\,\alpha_{12}B_2-48a_1\,a_2^2\alpha_{12}B_3+4a_0\,a_1^5\alpha_{12}C_0+16a_0^2a_1^3\,a_2\,\alpha_{12}C_0\\
&+2a_1^5\alpha_{12}C_1+16a_0\,a_1^3\,a_2\,\alpha_{12}C_1+16a_1^3\,a_2\,\alpha_{12}C_2+12a_0\,a_1^5\,a_2\,b_1\,D_0+2a_0\,a_1^6\,b_2\,D_0\\
&+12a_1^5\,a_2\,b_1\,D_1+2a_1^6\,b_2\,D_1+a_1^7\,b_1\,D_0+14a_1^6\,a_2\,b_0+a_1^7\,b_1)\,t\\
&+120a_0^4a_2^2\alpha_{12}A_0+120a_0^3a_2^2\alpha_{12}A_1+120a_0^2a_2^2\alpha_{12}A_2+120a_0\,a_2^2\,\alpha_{12}A_3+120a_2^2\alpha_{12}A_4\\
&-12a_0^3a_1^2\,a_2\,\alpha_{12}B_0-12a_0^2a_1^2\,a_2\,\alpha_{12}B_1-12a_0\,a_1^2\,a_2\,\alpha_{12}B_2-12a_1^2\,a_2\,\alpha_{12}B_3+2a_0^2a_1^4\alpha_{12}C_0\\
&+2a_0a_1^4\,\alpha_{12}C_1+2a_1^4\alpha_{12}C_2+a_0\,a_1^6\,b_1\,D_0+a_1^6\,b_1\,D_1+a_1^7\,b_0=0.
\end{aligned}
$$

Бидејќи пак равенството (d) треба да биде задоволено за секое t, за оцеделувањето на врските помеѓу константите A_0, A_1, A_2, A_3, A_4, B_0, B_1, B_2, B_3, C_0, C_1, C_2, D_0, D_1, a_0, a_1, a_2, b_0, b_1, b_2 го добиваме системот од равенки:

(1) $$128a_2^7b_2D_0+128a_2^7b_2=0,$$

(2) $$120a_2^6\,\alpha_{12}A_0-48a_2^6\alpha_{12}B_0+32a_2^6\,\alpha_{12}C_0+64a_2^7b_1\,D_0+512a_1\,a_2^6b_2\,D_0+128a_2^7\,b_1+448a_1\,a_2^6b_2=0,$$

(3) $$480a_1\,a_2^5\alpha_{12}A_0-192a_1\,a_2^5\alpha_{12}B_0+128a_1\,a_2^5\alpha_{12}C_0+128a_0a_2^6b_2\,D_0+128a_2^6b_2\,D_1+256a_1\,a_2^6b_1\,D_0+864a_1^2a_2^5b_2\,D_0+128a_2^7\,b_0+448a_1\,a_2^6b_1+672a_1^2a_2^5b_2=0,$$

(4) $$720a_1^2a_2^4\alpha_{12}A_0+480a_0\,a_2^5\alpha_{12}A_0+120a_2^5\alpha_{12}A_1-300a_1^2a_2^4\alpha_{12}B_0-144a_0\,a_2^3\alpha_{12}B_0-48a_2^5\alpha_{12}B_1+208a_1^2a_2^4\alpha_{12}C_0+64a_0\,a_2^5\alpha_{12}C_0+32a_2^5\alpha_{12}C_1+64a_0\,a_2^6b_1\,D_0+384a_0\,a_1\,a_2^5b_2\,D_0+64a_2^6\,b_1\,D_1+384a_1\,a_2^5b_2\,D_1+432a_1^2a_2^5b_1\,D_0+800a_1^3a_2^4b_2\,D_0+448a_1\,a_2^6b_0+672a_1^2a_2^5b_1+560\,a_1^3\,a_2^4b_2=0$$

(5) $$1440 a_0 a_1 a_2^4 \alpha_{12} A_0 + 480 a_1^3 a_2^3 \alpha_{12} A_0 + 360 a_1 a_2^4 \alpha_{12} A_1 - 204 a_1^3 a_2^3 \alpha_{12} B_0$$
$$-432 a_0 a_1 a_2^4 \alpha_{12} B_0 - 144 a_1 a_2^4 \alpha_{12} B_1 + 176 a_1^3 a_2^3 \alpha_{12} C_0 + 192 a_0 a_1 a_2^4 \alpha_{12} C_0$$
$$-96 a_1 a_2^4 \alpha_{12} C_1 + 192 a_0 a_1 a_2^5 b_1 D_0 + 480 a_0 a_1^2 a_2^4 b_2 D_0 + 192 a_1 a_2^5 b_1 D_1 + 480 a_1^2 a_2^4 b_2 D_1$$
$$+400 a_1^3 a_2^4 b_1 D_0 + 440 a_1^4 a_2^3 b_2 D_0 + 672 a_1^2 a_2^5 b_0 + 560 a_1^3 a_2^4 b_1 + 280 a_1^4 a_2^3 b_2 = 0,$$

(6) $$720 a_0^2 a_2^4 \alpha_{12} A_0 + 1440 a_0 a_1^2 a_2^3 \alpha_{12} A_0 + 120 a_1^4 a_2^2 \alpha_{12} A_0 + 360 a_0 a_2^4 \alpha_{12} A_1$$
$$+ 360 a_1^2 a_2^3 \alpha_{12} A_1 + 120 a_2^4 \alpha_{12} A_2 - 468 a_0 a_1^2 a_2^3 \alpha_{12} B_0 - 84 a_1^4 a_2^2 \alpha_{12} B_0 - 144 a_0^2 a_2^4 \alpha_{12} B_0$$
$$- 156 a_1^2 a_2^3 \alpha_{12} B_1 - 96 a_0 a_2^4 \alpha_{12} B_1 - 48 a_2^4 \alpha_{12} B_2 + 82 a_1^4 a_2^2 \alpha_{12} C_0 + 224 a_0 a_1^2 a_2^3 \alpha_{12} C_0$$
$$+ 32 a_0^2 a_2^4 \alpha_{12} C_0 + 112 a_1^2 a_2^3 \alpha_{12} C_1 + 32 a_0 a_2^4 \alpha_{12} C_1 + 32 a_2^4 \alpha_{12} C_2 + 240 a_0 a_1^2 a_2^4 b_1 D_0$$
$$+ 320 a_0 a_1^3 a_2^3 b_2 D_0 + 240 a_1^2 a_2^4 b_1 D_1 + 320 a_1^3 a_2^3 b_2 D_1 + 220 a_1^4 a_2^3 b_1 D_0 + 144 a_1^5 a_2^2 b_2 D_0$$
$$+ 560 a_1^3 a_2^4 b_0 + 280 a_1^4 a_2^3 b_1 + 84 a_1^5 a_2^2 b_2 = 0,$$

(7) $$1440 a_0^2 a_1 a_2^3 \alpha_{12} A_0 + 480 a_0 a_1^3 a_2^2 \alpha_{12} A_0 + 120 a_1^3 a_2^2 \alpha_{12} A_1 + 720 a_0 a_1 a_2^3 \alpha_{12} A_1$$
$$+ 240 a_1 a_2^3 \alpha_{12} A_2 - 12 a_1^5 a_2 \alpha_{12} B_0 - 216 a_0 a_1^3 a_2^2 \alpha_{12} B_0 - 288 a_0^2 a_1 a_2^3 \alpha_{12} B_0$$
$$- 72 a_1^3 a_2^2 \alpha_{12} B_1 - 192 a_0 a_1 a_2^3 \alpha_{12} B_1 - 96 a_1 a_2^3 \alpha_{12} B_2 + 20 a_1^5 a_2 \alpha_{12} C_0$$
$$+ 128 a_0 a_1^3 a_2^2 \alpha_{12} C_0 + 64 a_0^2 a_1 a_2^3 \alpha_{12} C_0 + 64 a_1^3 a_2^2 \alpha_{12} C_1 + 64 a_0 a_1 a_2^3 \alpha_{12} C_1$$
$$+ 64 a_1 a_2^3 \alpha_{12} C_2 + 160 a_0 a_1^3 a_2^3 b_1 D_0 + 120 a_0 a_1^4 a_2^2 b_2 D_0 + 160 a_1^3 a_2^3 b_1 D_1$$
$$+ 120 a_1^4 a_2^2 b_2 D_1 + 72 a_1^5 a_2^2 b_1 D_0 + 26 a_1^6 a_2 b_2 D_0 + 280 a_1^4 a_2^3 b_0 + 84 a_1^5 a_2^2 b_1 + 14 a_1^6 a_2 b_2 = 0,$$

(8) $$720 a_0^2 a_1^2 a_2^2 \alpha_{12} A_0 + 480 a_0^3 a_2^3 \alpha_{12} A_0 + 360 a_0 a_1^2 a_2^2 \alpha_{12} A_1 + 360 a_0^2 a_2^3 \alpha_{12} A_1$$
$$+ 120 a_1^2 a_2^2 \alpha_{12} A_2 + 240 a_0 a_2^3 \alpha_{12} A_2 + 120 a_2^3 \alpha_{12} A_3 - 36 a_0 a_1^4 a_2 \alpha_{12} B_0$$
$$- 180 a_0^2 a_1^2 a_2^2 \alpha_{12} B_0 - 48 a_0^3 a_2^3 \alpha_{12} B_0 - 12 a_1^4 a_2 \alpha_{12} B_1 - 120 a_0 a_1^2 a_2^2 \alpha_{12} B_1$$
$$- 48 a_0^2 a_2^3 \alpha_{12} B_1 - 60 a_1^2 a_2^2 \alpha_{12} B_2 - 48 a_0 a_2^3 \alpha_{12} B_2 - 48 a_2^3 \alpha_{12} B_3 + 2 a_1^6 \alpha_{12} C_0$$
$$+ 36 a_0 a_1^4 a_2 \alpha_{12} C_0 + 48 a_0^2 a_1^2 a_2^2 \alpha_{12} C_0 + 18 a_1^4 a_2 \alpha_{12} C_1 + 48 a_0 a_1^2 a_2^2 \alpha_{12} C_1$$
$$+ 48 a_1^2 a_2^2 \alpha_{12} C_2 + 60 a_0 a_1^4 a_2^2 b_1 D_0 + 24 a_0 a_1^5 a_2 b_2 D_0 + 60 a_1^4 a_2^2 b_1 D_1 + 24 a_1^5 a_2 b_2 D_1$$
$$+ 13 a_1^6 a_2 b_1 D_0 + 2 a_1^7 b_2 D_0 + 84 a_1^5 a_2^2 b_0 + 14 a_1^6 a_2 b_1 + a_1^7 b_2 = 0,$$

(9) $$480 a_0^3 a_1 a_2^2 \alpha_{12} A_0 + 360 a_0^2 a_1 a_2^2 \alpha_{12} A_1 + 240 a_0 a_1 a_2^2 \alpha_{12} A_2 + 120 a_1 a_2^2 \alpha_{12} A_3$$
$$- 48 a_0^3 a_1 a_2^2 \alpha_{12} B_0 - 36 a_0^2 a_1^3 a_2 \alpha_{12} B_0 - 48 a_0^2 a_1 a_2^2 \alpha_{12} B_1 - 24 a_0 a_1^3 a_2 \alpha_{12} B_1$$
$$- 48 a_0 a_1 a_2^2 \alpha_{12} B_2 - 12 a_1^3 a_2 \alpha_{12} B_2 - 48 a_1 a_2^2 \alpha_{12} B_3 + 4 a_0 a_1^5 \alpha_{12} C_0 + 16 a_0^2 a_1^3 a_2 \alpha_{12} C_0$$
$$+ 2 a_1^5 \alpha_{12} C_1 + 16 a_0 a_1^3 a_2 \alpha_{12} C_1 + 16 a_1^3 a_2 \alpha_{12} C_2 + 12 a_0 a_1^5 a_1 b_2 D_0 + 2 a_0 a_1^6 b_2 D_0$$
$$+ 12 a_1^5 a_2 b_1 D_1 + 2 a_1^6 b_2 D_1 + a_1^7 b_1 D_0 + 14 a_1^6 a_2 b_0 + a_1^7 b_1 = 0,$$

(10) $$120 a_0^4 a_2^2 \alpha_{12} A_0 + 120 a_0^3 a_2^2 \alpha_{12} A_1 + 120 a_0^2 a_2^2 \alpha_{12} A_2 + 120 a_0 a_2^2 \alpha_{12} A_3 + 120 a_2^2 \alpha_{12} A_4$$
$$- 12 a_0^3 a_1^2 a_2 \alpha_{12} B_0 - 12 a_0^2 a_1^2 a_2 \alpha_{12} B_1 - 12 a_0 a_1^2 a_2 \alpha_{12} B_2 - 12 a_1^2 a_2 \alpha_{12} B_3$$
$$+ 2 a_0^2 a_1^4 \alpha_{12} C_0 + 2 a_0 a_1^4 \alpha_{12} C_1 + 2 a_1^4 \alpha_{12} C_2 + a_0 a_1^6 b_1 D_0 + a_1^6 b_1 D_1 + a_1^7 b_0 = 0.$$

Ќе претпоставиме дека се познати константите a_0, a_1, a_2, b_0, b_1, b_2 и во функција од нив да ги определиме A_0, A_1, A_2, A_3, A_4, B_0, B_1, B_2, B_3, C_0, C_1, C_2, D_0, D_1, бидејќи за определувањето на a_0, a_1, a_2, b_0, b_1, b_2, во функција од A_0, A_1, A_2, A_3, A_4, B_0, B_1, B_2, B_3, C_0, C_1, C_2, D_0, D_1 се добиваат равенки од повисоки степени чие решавање, во општ случај, е неизводливо.

Во зависност од константите a_0, a_1, a_2, b_0, b_1, b_2 во натамошната работа ќи ги разгледаме следните четири случаи:

1. $a_2 b_2 \neq 0$; 2. $a_2 = 0$, $b_2 \neq 0$; 3. $a_2 \neq 0$, $b_2 = 0$; 4. $a_2 = b_2 = 0$.

1. 1. $a_2 b_2 \neq 0$, $\alpha_{12} \neq 0$, $a_1 \neq 0$.

Во овој случај од равенката (1) добиваме

(1. 1. 1) $D_0 = -1,$

а равенките (2) и (3), според (1. 1. 1), стануваат соодветно

(1. 1. 2) $15A_0 - 6B_0 + 4C_0 - 8 = 0,$

(1. 1. 3) $30a_1 \alpha_{12} A_0 - 12a_1 \alpha_{12} B_0 + 8a_1 \alpha_{12} C_0 + 8a_2 b_2 D_1 - 8a_2 \alpha_{02} - 12a_1 \alpha_{12} = 0.$

Од (1. 1. 2) и (1. 1. 3) следува:

(1. 1. 4) $D_1 = (2a_2 \alpha_{02} - a_1 \alpha_{12}) / 2a_2 b_2.$

Равенките (4) и (5), врз основа од (1. 1. 1) и (1. 1. 4), се трансформираат соодветно во

(1. 1. 5) $$180a_1^2 b_2 A_0 + 120a_0 a_2 b_2 A_0 + 30a_2 b_2 A_1 - 75a_1^2 b_2 B_0 - 36a_0 a_2 b_2 B_0 - 12a_2 b_2 B_1 + 52a_1^2 b_2 C_0 + 16a_0 a_2 b_2 C_0 + 8a_2 b_2 C_1 + 16a_2^2 b_0 - 8a_1 a_2 b_1 - 108a_1^2 b_2 = 0.$$

(1. 1. 6) $$360a_0 a_1 a_2 b_2 A_0 + 120a_1^3 b_2 A_0 + 90a_1 a_2 b_2 A_1 - 51a_1^3 b_2 B_0 - 108a_0 a_1 a_2 b_2 B_0 - 36a_1 a_2 b_2 B_1 + 44a_1^3 b_2 C_0 + 48a_0 a_1 a_2 b_2 C_0 + 24a_1 a_2 b_2 C_1 + 48a_1 a_2^2 b_0 - 24a_1^2 a_2 b_1 - 100a_1^3 b_2 = 0.$$

Од (1. 1. 5) и (1. 1. 6) имаме:

(1. 1. 7) $210 A_0 - 87 B_0 + 56 C_0 - 112 = 0,$

Равенките (1. 1. 2) и (1. 1. 7) не ќе бидат противречни, ако

(1. 1. 8) $B_0 = 0.$

Равенките (6) и (7), во врска со (1. 1. 1), (1. 14) и (1. 1. 8), стануваат соодветно

1. 1. 9) $$360a_0^2 a_2^2 b_2 A_0 + 720a_0 a_1^2 a_2 b_2 A_0 + 60a_1^4 b_2 A_0 + 180a_1^2 a_2 b_2 A_1 + 180a_0 a_2^2 b_2 A_1 + 60a_2^2 b_2 A_2 - 78a_1^2 a_2 b_2 B_1 - 48a_0 a_2^2 b_2 B_1 - 24 a_2^2 b_2 B_2 + 41 a_1^4 b_2 C_0 + 112a_0 a_1^2 a_2 b_2 C_0 + 16a_0^2 a_2^2 b_2 C_0 + 56a_1^2 a_2 b_2 C_1 + 16a_0 a_2^2 b_2 C_1 + 16a_2^2 b_2 C_2 + 120a_1^2 a_2^2 b_0 - 60a_1^3 a_2 b_1 - 110a_1^4 b_2 = 0.$$

(1. 1. 10) $$360a_0^2 a_2^2 b_2 A_0 + 120a_0 a_1^2 a_2 b_2 A_0 + 30a_1^2 a_2 b_2 A_1 + 180a_0 a_2^2 b_2 A_1 + 60a_2^2 b_2 A_2 - 18a_1^2 a_2 b_2 B_1 - 48a_0 a_2^2 b_2 B_1 - 24a_2^2 b_2 B_2 + 5a_1^4 b_2 C_0 + 32a_0 a_1^2 a_2 b_2 C_0 + 16a_1^2 a_2 b_2 C_1 + 16a_0 a_2^2 b_2 C_1 + 16a_2^2 b_2 C_2 + 40a_1^2 a_2^2 b_0 - 20a_1^3 a_2 b_1 - 18a_1^4 b_2 = 0.$$

Од (1. 1. 9) и (1. 1. 10) следува:

(1. 1. 11) $$300a_0 a_2 b_2 A_0 + 30a_1^2 b_2 A_0 + 75a_2 b_2 A_1 - 30a_2 b_2 B_1 + 18a_1^2 b_2 C_0 + 40a_0 a_2 b_2 C_0 + 20a_2 b_2 C_1 + 40a_1 a_2 b_0 - 46a_1^2 b_2 - 20a_1 a_2 b_1 = 0.$$

Од (1. 1. 5) и (1. 1. 11) добиваме:

(1. 1. 12) $15 A_0 + 4 C_0 - 8 = 0.$

Равенките (1. 1. 2) и (1. 1. 12), имајќи ја предвид (1. 1. 8), се еквивалентни.

Равенките (8) и (9), врз основа од (1. 1. 1), (1. 1. 4) и (1. 1. 8), стануваат соодветно

(1. 1. 13) $$720 a_0^2 a_1^2 a_2^2 b_2 A_0 + 480 a_0^3 a_2^3 b_2 A_0 + 360 a_0 a_1^2 a_2^2 b_2 A_1 + 360 a_0^2 a_2^3 b_2 A_1 + 120 a_1^2 a_2^2 b_2 A_2 + 240 a_0 a_2^3 b_2 A_2 + 120 a_2^3 b_2 A_3 - 12 a_1^4 a_2 b_2 B_1 - 120 a_0 a_1^2 a_2^2 b_2 B_1 - 48 a_0^2 a_2^3 b_2 B_1 - 60 a_1^2 a_2^2 b_2 B_2 - 48 a_0 a_2^3 b_2 B_2 - 48 a_2^3 b_2 B_3 + 2 a_1^6 b_2 C_0 + 36 a_0 a_1^4 a_2 b_2 C_0 + 48 a_0^2 a_1^2 a_2^2 b_2 C_0 + 18 a_1^4 a_2 b_2 C_1 + 48 a_0 a_1^2 a_2^2 b_2 C_1 + 48 a_1^2 a_2^2 b_2 C_2 + 60 a_1^4 a_2^2 b_0 - 30 a_1^5 a_2 b_1 - 13 a_1^6 b_2 = 0,$$

$$(1.\,1.\,14)\quad 480\,a_0^3 a_2^3 b_2 A_0 + 360\,a_0^2 a_2^3 b_2 A_1 + 240\,a_0 a_2^3 b_2 A_2 + 120\,a_2^3 b_2 A_3 - 48\,a_0^2 a_2^3 b_2 B_1 - 48\,a_0 a_2^3 b_2 B_2 - 12\,a_1^2 a_2^2 b_2 B_2 - 48\,a_2^3 b_2 B_3 + 4\,a_0 a_1^4 a_2 b_2 C_0 + 16\,a_0^2 a_1^2 a_2^2 b_2 C_0 + 2\,a_1^4 a_2 b_2 C_1 + 16\,a_0 a_1^2 a_2^2 b_2 C_1 + 16\,a_1^2 a_2^2 b_2 C_2 + 12\,a_1^4 a_2^2 b_0 - 6\,a_1^5 a_2 b_1 - a_1^6 b_2 = 0.$$

Од (1. 1. 13) и (1. 1. 14) следува:

$$(1.\,1.\,15)\quad 720\,a_0^2 a_2^2 b_2 A_0 + 360\,a_0 a_2^2 b_2 A_1 + 120\,a_2^2 b_2 A_2 - 96\,a_0 a_2^2 b_2 B_1 - 12\,a_1^2 a_2 b_2 B_1 - 48\,a_2^2 b_2 B_2 + 2\,a_1^4 b_2 C_0 + 32\,a_0 a_1^2 a_2 b_2 C_0 + 32\,a_0^2 a_2^2 b_2 C_0 + 16\,a_1^2 a_2 b_2 C_1 + 32\,a_0 a_2^2 b_2 C_1 + 32\,a_2^2 b_2 C_2 + 48\,a_1^2 a_2^2 b_0 - 25\,a_1^3 a_2 b_1 - 12\,a_1^4 b_2 = 0.$$

Од (1. 1. 9) и (1. 1. 15) добиваме:

$$(1.\,1.\,16)\quad 1440\,a_0 a_2 b_2 A_0 + 120\,a_1^2 b_2 A_0 + 360\,a_2 b_2 A_1 - 144\,a_2 b_2 B_1 + 80\,a_1^2 b_2 C_0 + 182\,a_0 a_2 b_2 C_0 + 96\,a_2 b_2 C_1 + 192\,a_2^2 b_0 - 95\,a_1 a_2 b_1 - 208\,a_1^2 b_2 = 0.$$

Од (1. 1. 5) и (1. 1. 16) следува равенката:

$$(1.\,1.\,17)\quad 15\,A_0 + 4\,C_0 - 8 = 0,$$

која е еквивалентна со равенката (1. 1. 12).

Равенката (10), според (1. 1. 1), (1. 1. 4) и (1. 1. 8), станува

$$(1.\,1.\,18)\quad 240\,a_0^4 a_2^3 b_2 A_0 + 240\,a_0^3 a_2^3 b_2 A_1 + 240\,a_0^2 a_2^3 b_2 A_2 + 240\,a_0 a_2^3 b_2 A_3 + 240\,a_2^3 b_2 A_4 - 24\,a_0^2 a_1^2 a_2^2 b_2 B_1 - 24\,a_0 a_1^2 a_2^2 b_2 B_2 - 24\,a_1^2 a_2^2 b_2 B_3 + 4\,a_0^2 a_1^4 a_2 b_2 C_0 + 4\,a_0 a_1^4 a_2 b_2 C_1 + 4\,a_1^4 a_2 b_2 C_2 + 2\,a_1^6 a_2 b_0 - a_1^7 b_1 = 0.$$

Од (1. 1. 12), (1. 1. 5), (1. 1. 9), (1. 1. 14) и (1. 1. 18) добиваме соодветно

$$A_0 = (8 - 4\,C_0)/15.$$

$$A_1 = (12\,a_2 b_2 B_1 - 180\,a_1^2 b_2 A_0 - 120\,a_0 a_2 b_2 A_0 - 52\,a_1^2 b_2 C_0 - 16\,a_0 a_2 b_2 C_0 - 8\,a_2 b_2 C_1 - 16\,a_2^2 b_0 + 8\,a_1 a_2 b_1 + 108\,a_1^2 b_2)/30\,a_2 b_2,$$

$$A_2 = (24\,a_2^2 b_2 B_2 - 180\,a_1^2 a_2 b_2 A_1 - 180\,a_0 a_2^2 b_2 A_1 + 78\,a_1^2 a_2 b_2 B_1 + 48\,a_0 a_2^2 b_2 B_1 - 360\,a_0^2 a_2^2 b_2 A_0 - 720\,a_0 a_1^2 a_2 b_2 A_0 - 60\,a_1^4 b_2 A_0 - 41\,a_1^4 b_2 C_0 - 112\,a_0 a_1^2 a_2 b_2 C_0 - 16\,a_0^2 a_2^2 b_2 C_0 - 56\,a_1^2 a_2 b_2 C_1 - 16\,a_0 a_2^2 b_2 C_1 - 16\,a_2^2 b_2 C_2 - 120\,a_1^2 a_2^2 b_0 + 60\,a_1^3 a_2 b_1 + 110\,a_1^4 b_2)/60\,a_2^2 b_2,$$

$$A_3 = (48\,a_2^3 b_2 B_3 - 240\,a_0 a_2^3 b_2 A_2 + 48\,a_0 a_2^3 b_2 B_2 + 12\,a_1^2 a_2^2 b_2 B_2 - 16\,a_1^2 a_2^2 b_2 C_2 - 360\,a_0^2 a_2^3 b_2 A_1 + 48\,a_0^2 a_2^3 b_2 B_1 + 24\,a_0 a_1^2 a_2^2 b_2 B_1 - 2\,a_1^4 a_2 b_2 C_1 - 16\,a_0 a_1^2 a_2^2 b_2 C_1 - 480\,a_0^3 a_2^3 b_2 A_0 - 4\,a_0 a_1^4 a_2 b_2 C_0 - 16\,a_0^2 a_1^2 a_2^2 b_2 C_0 - 12\,a_1^4 a_2^2 b_0 + 6\,a_1^5 a_2 b_1 + a_1^6 b_2)/120\,a_2^3 b_2,$$

$$A_4 = (24\,a_1^2 a_2^2 b_2 B_3 + 24\,a_0 a_1^2 a_2^2 b_2 B_2 + 24\,a_0^2 a_1^2 a_2^2 b_2 B_1 - 240\,a_0^4 a_2^3 b_2 A_0 - 240\,a_0^3 a_2^3 b_2 A_1 - 240\,a_0^2 a_2^3 b_2 A_2 - 240\,a_0 a_2^3 b_2 A_3 - 4\,a_0^2 a_1^4 a_2 b_2 C_0 - 4\,a_1^4 a_2 b_2 C_2 - 4\,a_0 a_1^4 a_2 b_2 C_1 - 2\,a_1^6 a_2 b_0 + a_1^7 b_1)/240\,a_2^3 b_2.$$

Константите B_1, B_2, B_3, C_0, C_1, C_2 се произволни.

1. 2. $a_2 b_2 \neq 0$, $\alpha_{12} \neq 0$, $a_1 = 0$.

Од равенката (1) во овој случај добиваме:

(1. 2. 1) $D_0 = -1$,

а од (2), според (1. 2. 1), ја добиваме равенката:

(1. 2. 2) $15 A_0 - 6 B_0 + 4 C_0 - 8 = 0$.

Од равенката (3), врз основа од (1. 2. 1), имаме

(1. 2. 3) $D_1 = \alpha_{02} / b_2$.

Равенката (4), според (1. 2. 1), станува:

(1. 2. 4) $60 a_0 A_0 + 15 A_1 - 18 a_0 B_0 - 6 B_1 + 8 a_0 C_0 + 4 C_1 + 8 a_0 - 8 D_1 = 0$.

Равенката (5) е задоволена.

Од (6), според (1. 2. 1). имаме

(1. 2. 5) $90 a_0^2 + 45 a_0 A_1 + 15 A_2 - 18 a_0^2 B_0 - 12 a_0 B_1 - 6 B_2 + 4 a_0^2 C_0 + 4 a_0 C_1 + 4 C_2 = 0$.

Равенката (7) е задоволена.

Од (8), врз основа од (1. 2. 1), ја добиваме релацијата

(1. 2. 6) $60 a_0^3 A_0 + 45 a_0^2 A_1 + 30 a_0 A_2 + 15 A_3 - 6 a_0^3 B_0 - 6 a_0^2 B_1 - 6 a_0 B_2 - 6 B_3 = 0$.

Равенката (9) е задоволена.

Равенката (10) станува

(1. 2. 7) $a_0^4 A_0 + a_0^3 A_1 + a_0^2 A_2 + a_0 A_3 + A_4 = 0$.

Од (1. 2. 2), (1. 2. 4), (1. 2. 5), (1. 2. 6) и (1. 2. 7) добиваме респективно

$$A_0 = (6 B_0 - 4 C_0 + 8) / 15.$$

$$A_1 = (18 a_0 B_0 + 6 B_1 - 60 a_0 A_0 - 8 a_0 C_0 84 C_1 - 8 a_0 + 8 D_1) / 15.$$

$$A_2 = (18 a_0^2 B_0 + 12 a_0 B_1 + 6 B_2 - 90 a_0^2 A_0 - 45 a_0 A_1 - 4 a_0^2 C_0 - 4 a_0 C_1 - 4 C_2) / 15,$$

$$A_3 = (2 a_0^3 B_0 + 2 a_0^2 B_1 + 2 a_0 B_2 + 2 B_3 - 20 a_0^3 A_0 - 15 a_0^2 A_1 - 10 a_0 A_2) / 5,$$

$$A_4 = -(a_0^4 A_0 + a_0^3 A_1 + a_0^2 A_2 + a_0 A_4).$$

Константите B_0, B_1, B_2, B_3, C_0, C_1, C_2 се произволни.

1. 3. $a_2 b_2 \neq 0$, $\alpha_{12} = 0$.

Во овој случај од равенката (1) добиваме

(1. 3. 1) $D_0 = -1$,

а равенката (2), според (1. 3. 1) е задоволена.

Од (3), врз основа од (1. 3. 1) следува:

(1. 3. 2) $D_1 = \alpha_{02} / b_2$.

Равенките (4), (5), (6), (7), (8), (9) и (10), во врска со (1. 3. 1) и (1. 3. 2) се задоволени.

Константите A_0, A_1, A_2, A_3, A_4, B_0, B_1, B_2, B_3, C_0, C_1, C_2 се произволни.

2. $a_2 = 0$, $b_2 \neq 0$.

Во овој случај равенките (1), (2), (3), (4), (5), (6) и (7) се задоволени.

Од (8) и (9) ги добиваме соодветно равенките:

(2. 1) $2 C_0 + 2 D_0 + 1 = 0$,

(2. 2) $4 a_0 b_2 C_0 + 2 b_2 C_1 + 2 a_0 b_2 D_0 + 2 b_2 D_1 + a_1 b_1 D_0 + a_1 b_1 = 0$.

Од (10) добиваме:

(2. 3) $2 a_0^2 b_2 C_0 + 2 a_0 b_2 C_1 + 2 b_2 C_2 + a_0 a_1 b_1 D_0 + a_1 b_1 D_1 + a_1^2 b_0 = 0$.

Од (2. 1), (2. 2) и (2. 3) добиваме соодветно:

$$C_0 = -(2D_0 + 1)/2$$

$$C_1 = -(2a_0 b_2 D_0 + 2b_2 D_1 + a_1 b_1 D_0 + a_1 b_1 + 4a_0 b_2 C_0)/2b_2,$$

$$C_2 = -(a_0 a_1 b_1 D_0 + a_1 b_1 D_1 + a_1^2 b_0 + 2a_0^2 b_2 C_0 + 2a_0 b_2 C_1)/2b_2.$$

Константите A_0, A_1, A_2, A_3, A_4, B_0, B_1, B_2, B_3, D_0, D_1 се произволни.

3. 1. $a_2 \neq 0$, $b_2 = 0$, $a_1 \neq 0$.

Равенката (1), во овој случај, е задоволена.

Равенките (2) и (3) стануваат соодветно

(3. 1. 1) $15A_0 - 6B_0 + 4C_0 - 8D_0 - 16 = 0,$

(3. 1. 2) $480a_1 b_1 A_0 - 192a_1 a_2^6 b_1 B_0 + 128a_1 b_1 C_0 - 256a_1 b_1 D_0 - 128a_2 b_0 - 448a_1 b_1 = 0.$

Од (3. 1. 1) и (3. 1. 2) следува:

(3. 1. 3) $a_1 b_1 = 2a_2 b_0.$

Равенките (4) и (5) стануваат соодветно

(3. 1. 4) $720a_1^2 A_0 + 480a_0 a_2 A_0 + 120a_2 A_1 - 300a_1^2 B_0 - 144a_0 a_2 B_0 - 48a_2 B_1 + 208a_1^2 C_0 + 64a_0 a_2 C_0 + 32a_2 C_1 - 64a_0 a_2 D_0 - 64a_2 D_1 - 432a_1^2 D_0 - 896a_1^2 = 0.$

(3. 1. 5) $1440a_0 a_2 A_0 + 480a_1^2 A_0 + 360a_2 A_1 - 204a_1^2 B_0 - 432a_0 a_2 B_0 - 144a_2 B_1 + 176a_1^2 C_0 + 192a_0 a_2 C_0 + 96a_2 C_1 - 192a_0 a_2 D_0 - 192a_2 D_1 - 400a_1^2 D_0 - 896a_1^2 = 0.$

Од (3. 1. 4) и (3. 1. 5) ја добиваме равенката

(3. 1. 6) $210A_0 - 87B_0 + 56C_0 - 112D_0 - 224 = 0.$

Равенките (3. 1. 1) и (3. 1. 6) не ќе бидат противречни, ако

(3. 1. 7) $B_0 = 0.$

Равенките (6) и (7), според (3. 1. 7), стануваат респективно

(3. 1. 8) $720a_0^2 a_2^2 A_0 + 1440a_0 a_1^2 a_2 A_0 + 120a_1^4 A_0 + 360a_0 a_2^2 A_1 + 360a_1^2 a_2 A_1 + 120a_2^2 A_2 - 156a_1^2 a_2 B_1 - 96a_0 a_2^2 B_1 - 48a_2^2 B_2 + 82a_1^4 C_0 + 224a_0 a_1^2 a_2 C_0 + 32a_0^2 a_2^2 C_0 + 112a_1^2 a_2 C_1 + 32a_0 a_2^2 C_1 + 32a_2^2 C_2 - 240a_0 a_1^2 a_2 D_0 - 240a_1^2 a_2 D_1 - 220a_1^4 D_0 - 560a_1^4 = 0,$

(3. 1. 9) $720a_0^2 a_2^2 A_0 + 240a_0 a_1^2 a_2 A_0 + 60a_1^2 a_2 A_1 + 360a_0 a_2^2 A_1 + 120a_2^2 A_2 - 36a_1^2 a_2 B_1 + 96a_0 a_2^2 B_1 - 48a_2^2 B_2 + 10a_1^4 C_0 + 64a_0 a_1^2 a_2 C_0 + 32a_0^2 a_2^2 C_0 + 32a_1^2 a_2 C_1 + 32a_0 a_2^2 C_1 + 32a_2^2 C_2 - 80a_0 a_1^2 a_2 D_0 - 80a_1^2 a_2 D_1 - 36a_1^4 D_0 - 112a_1^4 = 0.$

Од (3. 1. 8) и (3. 1. 9) следува:

(3. 1. 10) $1200a_0 a_2 A_0 + 120a_1^2 A_0 + 300a_2 A_1 - 120a_2 B_1 + 72a_1^2 C_0 + 160a_0 a_2 C_0 + 80a_2 C_1 - 160a_0 a_2 D_0 - 184a_1^2 D_0 - 448a_1^2 = 0.$

Од (3. 1. 4) и (3. 1. 10) ја добиваме равенката:

$$15A_0 + 4C_0 - 8D_0 - 16 = 0,$$

која, за $B = 0$, е идентична со (3. 1. 1).

Равенките (8) и (9), врз основа од (3. 1. 7), стануваат соодветно

(3. 1. 11) $720a_0^2 a_1^2 a_2^2 A_0 + 480a_0^3 a_2^3 A_0 + 360a_0 a_1^2 a_2^2 A_1 + 360a_0^2 a_2^3 A_1 + 120a_1^2 a_2^2 A_2 + 240a_0 a_2^3 A_2 + 120a_2^3 A_3 - 12a_1^4 a_2 B_1 - 120a_0 a_1^2 a_2^2 B_1 - 48a_0^2 a_2^3 B_1 - 60a_1^2 a_2^2 B_2$

$$- 48\, a_0 a_2^3 B_2 - 48\, a_2^3 B_3 + 2\, a_1^6 C_0 + 36\, a_0 a_1^4 a_2 C_0 + 48\, a_0^2 a_1^2 a_2^2 C_0 + 18\, a_1^4 a_2 C_1$$
$$+ 48\, a_0 a_1^2 a_2^2 C_1 + 48\, a_1^2 a_2^2 C_2 - 60\, a_0 a_1^4 a_2 D_0 - 60\, a_1^4 a_2 D_1 - 13\, a_1^6 D_0 - 56\, a_1^6 = 0,$$

(3. 1. 12) $$480\, a_0^3 a_2^3 A_0 + 360\, a_0^2 a_2^3 A_1 + 240\, a_0 a_2^3 A_2 + 120\, a_2^3 A_3 - 48\, a_0^2 a_2^3 B_1 - 24\, a_0 a_1^2 a_2^2 B_1$$
$$- 48\, a_0 a_2^3 B_2 - 12\, a_1^2 a_2^2 B_2 - 48\, a_2^3 B_3 + 4\, a_0 a_1^4 a_2 C_0 + 16\, a_0^2 a_1^2 a_2^2 C_0 + 2\, a_1^4 a_2 C_1$$
$$+ 16\, a_0 a_1^2 a_2^2 C_1 + 16\, a_1^2 a_2^2 C_2 - 12\, a_0 a_1^4 a_2 D_0 - 12\, a_1^4 a_2 D_1 - a_1^6 D_0 - 8\, a_1^6 = 0.$$

Од (3. 1. 11) и (3. 1. 12) следува:

(3. 1. 13) $$720\, a_0^2 a_2^2 A_0 + 360\, a_0 a_2^2 A_1 + 120\, a_2^2 A_2 - 12\, a_1^2 a_2 B_1 - 96\, a_0 a_2^2 B_1 - 48\, a_2^2 B_2$$
$$+ 2\, a_1^4 C_0 + 32\, a_0 a_1^2 a_2 C_0 + 32\, a_0^2 a_2^2 C_0 + 16\, a_1^2 a_2 C_1 + 32\, a_0 a_2^2 C_1 + 32\, a_2^2 C_2 - 48\, a_0 a_1^2 a_2 D_0$$
$$- 12\, a_1^4 D_0 - 48\, a_1^4 = 0.$$

Од (3. 1. 9) и (3. 1. 13) следува:

(3. 1. 14) $$240\, a_0 a_2 A_0 + 60\, a_2 A_1 - 24\, a_2 B_1 + 8\, a_1^2 C_0 + 32\, a_0 a_2 C_0 + 16\, a_2 C_1 - 32\, a_0 a_2 D_0 - 32\, a_2 D_1$$
$$- 24\, a_1^2 D_0 - 64\, a_1^2 = 0.$$

Од (3. 1. 4) и (3. 1. 14) ја добиваме релацијата:

$$15\, A_0 + 4\, C_0 - 8\, D_0 - 16 = 0,$$

која, за $B=0$, е идентична со (3. 1. 1).

Равенката (10), според (3. 1. 7), станува

(3. 1. 15) $$120\, a_0^4 a_2^3 b_1 A_0 + 120\, a_0^3 a_2^3 b_1 A_1 + 120\, a_0^2 a_2^3 b_1 A_2 + 120\, a_0 a_2^3 b_1 A_3 + 120\, a_2^3 b_1 A_4$$
$$- 12\, a_0^2 a_1^2 a_2^2 b_1 B_1 - 12\, a_0 a_1^2 a_2^2 b_1 B_2 - 12\, a_1^2 a_2^2 b_1 B_3 + 2\, a_0^2 a_1^4 a_2 b_1 C_0 + 2\, a_0 a_1^4 a_2 b_1 C_1$$
$$+ 2\, a_1^4 a_2 b_1 C_2 - a_0 a_1^6 b_1 D_0 - a_1^6 b_1 D_1 - a_1^7 b_0 = 0.$$

Од (3. 1. 1), врз основа од (3. 1. 7), добиваме:

$$A_0 = (8\, D_0 - 4\, C_0 + 16) / 15,$$

а од (3. 1. 4), (3. 1. 9), (3. 1. 12) и (3. 1. 15) добиваме респективно

$$A_1 = (6\, a_2 B_1 - 90\, a_1^2 A_0 - 60\, a_0 a_2 A_0 - 26\, a_1^2 C_0 - 8\, a_0 a_2 C_0 - 4\, a_2 C_1 + 8\, a_0 a_2 D_0$$
$$+ 8\, a_2 D_1 + 54\, a_1^2 D_0 + 112\, a_1^2) / 15\, a_2,$$

$$A_2 = (18\, a_1^2 a_2 B_1 + 48\, a_0 a_2^2 B_1 + 24\, a_2^2 B_2 - 360\, a_0^2 a_2^2 A_0 - 120\, a_0 a_1^2 a_2 A_0$$
$$- 30\, a_1^2 a_2 A_1 - 180\, a_0 a_2^2 A_1 - 5\, a_1^4 C_0 - 32\, a_0 a_1^2 a_2 C_0 - 16\, a_0^2 a_2^2 C_0 - 16\, a_1^2 a_2 C_1$$
$$- 16\, a_0 a_2^2 C_1 - 16\, a_2^2 C_2 + 40\, a_0 a_1^2 a_2 D_0 + 40\, a_1^2 a_2 D_1 + 18\, a_1^4 D_0 + 56\, a_1^4) / 60\, a_2^2,$$

$$A_3 = (a_0^2 a_2^3 B_1 + 24\, a_0 a_1^2 a_2^2 B_1 + 48\, a_0 a_2^3 B_2 + 12\, a_1^2 a_2^2 B_2 + 48\, a_2^3 B_3 - 480\, a_0^3 a_2^3 A_0$$
$$- 360\, a_0^2 a_2^3 A_1 - 240\, a_0 a_2^3 A_2 - 4\, a_0 a_1^4 a_2 C_0 - 16\, a_0^2 a_1^2 a_2^2 C_0 - 2\, a_1^4 a_2 C_1$$
$$- 16\, a_0 a_1^2 a_2^2 C_1 - 16\, a_1^2 a_2^2 C_2 + 12\, a_0 a_1^4 a_2 D_0 + 12\, a_1^4 a_2 D_1 + a_1^6 D_0 + 8\, a_1^6) / 120\, a_2^3,$$

$$A_4 = (12\, a_0^2 a_1^2 a_2^2 b_1 B_1 + 12\, a_0 a_1^2 a_2^2 b_1 B_2 + 12\, a_1^2 a_2^2 b_1 B_3 - 120\, a_0^4 a_2^3 b_1 A_0$$
$$- 120\, a_0^3 a_2^3 b_1 A_1 - 120\, a_0^2 a_2^3 b_1 A_2 - 120\, a_0 a_2^3 b_1 A_3 - 2\, a_0^2 a_1^4 a_2 b_1 C_0$$
$$- 2\, a_0 a_1^4 a_2 b_1 C_1 - 2\, a_1^4 a_2 b_1 C_2 + a_0 a_1^6 b_1 D_0 + a_1^6 b_1 D_1 + a_1^7 a_0) / 120\, a_2^3 b_1;$$

Константите B_1, B_2, B_3, C_0, C_1, C_2, D_0, D_1 се произволни.

3. 2. $a_2 \neq 0$, $b_2 = 0$, $a_1 = 0$.

Во овој случај равенката (1) е задоволена.

Равенката (2) станува:

(3. 2. 1) $15 A_0 - 6 B_0 + 4 C_0 - 8 D_0 - 16 = 0$

Од равенката (3) следува;

(3. 2. 2) $b_0 = 0.$

Равенката (4) станува:

(3. 2. 3) $60 a_0 A_0 + 15 A_1 - 18 a_0 B_0 - 6 B_1 + 8 a_0 C_0 + 4 C_1 - 8 a_0 D_0 - 8 D_1 = 0,$

а равенката (5) е задоволена.

Од равенката (6) добиваме:

(3. 2. 4) $90 a_0^2 A_0 + 45 a_0 A_1 + 15 A_2 - 18 a_0^2 B_0 - 12 a_0 B_1 - 6 B_2 + 4 a_0^2 C_0 + 4 a_0 C_1 + 4 C_2 = 0.$

Равенката (7) е задоволена.

Од (8) имаме:

(3. 2. 5) $60 a_0^3 A_0 + 45 a_0^2 A_1 + 30 a_0 A_2 + 15 A_3 - 6 a_0^3 B_0 - 6 a_0^2 B_1 - 6 a_0 B_2 - 6 B_3 = 0.$

Равенката (9) е задоволена, а равенката (10) станува:

(3. 2. 6) $a_0^4 A_0 + a_0^3 A_1 + a_0^2 A_2 + a_0 A_3 + A_4 = 0.$

Од (3. 2. 1), (3. 2. 3), (3. 2. 4), (3. 2. 5) и (3. 2. 6) добиваме соодветно

$A_0 = (6 B_0 - 4 C_0 + 8 D_0 + 16) / 15.$

$A_1 = (18 a_0 B_0 + 6 B_1 - 60 a_0 A_0 - 8 a_0 C_0 - 4 C_1 + 8 a_0 D_0 + 8 D_1) / 15,$

$A_2 = (18 a_0^2 b_0 + 12 a_0 B_1 + 6 B_2 - 90 a_0^2 A_0 - 45 a_0 A_1 - 4 a_0^2 C_0 - 4 a_0 C_1 - 4 C_2) / 15,$

$A_3 = (6 a_0^3 B_0 + 6 a_0^2 B_1 + 6 a_0 B_2 + 6 B_3 - 60 a_0^3 A_0 - 45 a_0^2 A_1 - 30 a_0 A_2) / 15,$

$A_4 = -(a_0^4 A_0 + a_0^3 A_1 + a_0^2 A_2 + a_0 A_3).$

Константите B_0, B_1, B_2, B_3, C_0, C_1, C_2, D_0, D_1 се произволни.

4. $a_2 = b_2 = 0.$

Равенките (1), (2), (3), (4), (5), (6), (7) и (8) се задоволени.

Од (9) добиваме:

(4. 1) $D_0 = -1,$

а од (10), според (4. 1), имаме:

$D_1 = \alpha_{01} / b_1.$

Константите A_0, A_1, A_2, A_3, A_4, B_0, B_1, B_2, B_3, C_0, C_1, C_2 се произволни.

На напред изнесените случаи им одговараат соодветно примерите;

I. 1. 1° $(64 x^4 - 480 x^3 + 956 x^2 - 510 x + 27) y^{IV} + 240 (x^2 + x + 1) y^{III}$
$+ 240 (x^2 + x + 1) y^{II} - 240 (x + 1) y^{I} + 240 y = 0,$

$x = 1 + t + t^2,$
$y = 1 - 4t + 2t^2;$

2° $(64 x^4 - 288 x^3 + 460 x^2 + 378 x - 785) y^{IV} + 240 (x^2 + 2x + 3) y^{III}$
$+ 240 (x^2 + 2x + 3) y^{II} - 240 (x - 3) y^{I} + 240 y = 0,$

$x = 1 + t - t^2,$
$y = 1 + t + t^2;$

3° $(128 x^4 - 352 x^3 + 360 x^2 - 162 x + 27) y^{IV} - 240 (x - 1) y^{I} + 240 y = 0,$

$x = 1 + t + t^2,$
$y = 1 + t - t^2;$

1. 2. 1^0 $(8x^4+16x^3-80x^2+56)y^{IV}-20(x^3+2x^2+3x+4)y^{III}$
$-20(5x^2+6x+7)y^{II}+5(4x-3)y^{I}-20y=0,$

$$x=1+t^2,$$
$$y=1+2t+4t^2;$$

2^0 $(2x^4+6x^3-26x^2+6x+12)y^{IV}-5(x^3+2x^2+3x+4)y^{III}$
$-5(5x^2+6x+7)y^{II}+5xy^{I}-5y=0,$

$$x=1+t^2,$$
$$y=1+t+t^2;$$

3^0 $(10x^4-22x^3+20x^2-8x)y^{IV}+15x^3y^{III}+15x^2y^{II}-15(x-2)y^{I}+15y=0,$

$$x=1-t^2,$$
$$y=1+t+t^2;$$

3. 1. 1^0 $(576x^4-1120x^3+876x^2+198x-351)y^{IV}+240(3x^2+2x+3)y^{III}$
$+240(x^2+2x+3)y^{II}+240(3x+2)y^{I}+240y=0,$

$$x=1+t+t^2,$$
$$y=1+2t;$$

2^0 $(1408x^4-4288x^3+4488x^2-968x-865)y^{IV}+240(4x^2+3x+2)y^{III}$
$+240(2x^2-3x-5)y^{II}+240(10x+7)y^{I}+240y=0,$

$$x=1+t-t^2,$$
$$y=1-2t;$$

3^0 $(16x^4-120x^3+332x^2-394x+164)y^{IV}+15y^{III}+15y^{II}+15y^{I}+15y=0,$

$$x=1-2t-t^2,$$
$$y=1+t;$$

3. 2. 1^0 $(30x^4-70x^3+54x^2+42x-56)y^{IV}+15(x^3+2x^2+3x+4)y^{III}$
$+15(2x^2+3x+4)y^{II}+15(2x+4)y^{I}+15y=0,$

$$x=1-t^2,$$
$$y=t;$$

2^0 $(34x^4-66x^3+54x^2+14x-36)y^{IV}+15(3x^3+2x^2+x+4)y^{III}$
$+15(2x^2+x+3)y^{II}+15(x+2)y^{I}+15y=0;$

$$x=1+t^2,$$
$$y=t.$$

ILIJA A. ŠAPKAREV

SUR UNE ÉQUATION DIFFÉRENTIELLE LINÉAIRE* DU QUATRIÈME ORDRE

Résimé

Dans cette Note est donnée la solution du probléme de Mitrinović que voici: Examiner si équation différentielle

(a) $$(A_0x^4+A_1x^3+A_2x^2+A_3x+A_4)\frac{d^4y}{dx^4}+(B_0x^3+B_1x^2+B_2x+B_3)\frac{d^3y}{dx^3}$$
$$+(C_0x^2+C_1x+C_2)\frac{d^2y}{dx^2}+(D_0x+D_1)\frac{dy}{dx}+y=0$$

* Un résimé de cet article a paru.

admet des solutions particulières de la forme suivante

(b)
$$x = a_0 + a_1 t + a_2 t^2,$$
$$y = b_0 + b_1 t + b_2 t^2,$$

où

(c)
$$A_0,\ A_1,\ A_2,\ A_3,\ A_4,\ B_0,\ B_1\ B_2,\ B_3,\ C_0,\ C_1,\ C_2,\ D_0,\ D_1,$$
$$a_0,\ a_1,\ a_2,\ b_0,\ b_1,\ b_2$$

sont des constantes convenablement choisies.

Puisque

$$\frac{dy}{dx} = \frac{\dot y}{\dot x},\quad \frac{d^2y}{dx^2} = \frac{\dot x\ddot y - \ddot x\dot y}{\dot x^3},\quad \frac{d^3y}{dx^3} = \frac{\dot x(\dot x\dddot y - \dddot x\dot y) - 3\ddot x(\dot x\ddot y - \ddot x\dot y)}{\dot x^5},$$

$$\frac{d^4y}{dx^4} = \frac{\dot x^2(\ddot x\dddot y - \dddot x\ddot y + \dot x\ddddot y - \ddddot x\dot y) - 7\dot x\ddot x(\dot x\dddot y - \dddot x\dot y) - 3(\dot x\ddot y - \ddot x\dot y)(\dot x\dddot x - 5\ddot x^2)}{\dot x^7}$$

l' équation (a) devient

(d)
$$(A_0 x^4 + A_1 x^3 + A_2 x^2 + A_3 x + A_4)\,[\dot x^2(\ddot x\dddot y - \dddot x\ddot y + \dot x\ddddot y - \ddddot x\dot y) - 7\dot x\ddot x(\dot x\dddot y - \dddot x\dot y)$$
$$-3(\dot x\ddot y - \ddot x\dot y)(\dot x\dddot x - 5\ddot x^2)] + (B_0 x^3 + B_1 x^2 + B_2 x + B_3)\,[\dot x^3(\dot x\dddot y - \dddot x\dot y)$$
$$-3\dot x^2\ddot x(\dot x\ddot y - \ddot x\dot y)] + (C_0 x^2 + C_1 x + C_2)\dot x^4(\dot x\ddot y - \ddot x\dot y) + (D_0 x + D_1)\dot x^6\dot y + \dot x^7 y = 0.$$

Remplaçon dans l' équation (d) x, y donnés par (b) et ses derivées. Nous obtenons ainsi une équation algébrique du neuvième dergé en t, laquelle sera nommée: Équation (E).

Pour que (b) soit une solution de l' équation (a). il faut et il suffit que le polynôme de l' équation (E) s' annulle identiquement. On obtient de cette manière les dix équation entre les coefficients (c).

On peut déterminer $A_0,\ A_1,\ A_2,\ A_3,\ A_4,\ B_0,\ B_1,\ B_2,\ B_3,\ C_0,\ C_1,\ C_2,\ D_0,\ D_1$ en fonction des $a_k,\ b_k\ (k=0,\ 1,\ 2)$.

L' équation (a) a des solutions particulières de la forme (b) dans les cas suivants:

1\. 1. $a_2 b_2 \neq 0,\ \alpha_{12} \neq 0,\ a_1 \neq 0,$

$$D_0 = -1,\quad D_1 = (2a_2\alpha_{02} - a_1\alpha_{12})/2a_2 b_2,\quad B_0 = 0,\quad A_0 = (8-4C_0)/15.$$

$$A_1 = (12a_2 b_2 B_1 - 180a_1^2 b_2 A_0 - 120a_0 a_2 b_2 A_0 - 52a_1^2 b_2 C_0 - 16a_0 a_2 b_2 C_0$$
$$-8a_2 b_2 C_1 - 16a_2^2 b_0 + 8a_1 a_2 b_1 + 108a_1^2 b_2)/30a_2 b_2,$$

$$A_2 = (24a_2^2 b_2 B_2 - 180a_1^2 b_2 A_1 - 180a_0 a_2^2 b_2 A_1 + 78a_1^2 a_2 b_2 B_1 + 48a_0 a_2^2 b_2 B_1$$
$$-360a_0^2 a_2^2 b_2 A_0 - 720a_0 a_1^2 a_2 b_2 A_0 - 60a_1^4 b_2 A_0 - 41a_1^4 b_2 C_0 - 112a_0 a_1^2 a_2 b_2 C$$
$$-16a_0^2 a_2^2 b_2 C_0 - 56a_1^2 a_2 b_2 C_1 - 16a_0 a_2^2 b_2 C_1 - 16a_2^2 b_2 C_2 - 120a_1^2 a_2^2 b_0$$
$$+ 60a_1^3 a_2 b_1 + 110a_1^4 b_2)/60a_2^2 b_2,$$

$$A_3 = (48a_2^3 b_2 B_3 - 240a_0 a_2^3 b_2 A_2 + 48a_0 a_2^3 b_2 B_2 + 12a_1^2 a_2^2 b_2 B_2 - 16a_1^2 a_2^2 b_2 C_2$$
$$-360a_0^2 a_2^3 b_2 A_1 + 48a_0^2 a_2^3 b_2 B_1 + 24a_0 a_1^2 a_2^2 b_2 B_1 - 2a_1^4 a_2 b_2 C_1 - 16a_0 a_1^2 a_2^2 b_2 C_1$$
$$-480a_0^3 a_2^3 b_2 A_0 - 4a_0 a_1^4 a_2 b_2 C_0 - 16a_0^2 a_1^2 a_2^2 b_2 C_0 - 12a_1^4 a_2^2 b_0 + 6a_1^5 a_2 b_1$$
$$+ a_1^6 b_2)/120a_2^3 b_2,$$

$$A_4 = (24a_1^2 a_2^2 b_2 B_3 + 24a_0 a_1^2 a_2^2 b_2 B_2 + 24a_0^2 a_1^2 a_2^2 b_2 B_1 - 240a_0^4 a_2^3 b_2 A_0$$
$$-240a_0^3 a_2^3 b_2 A_1 - 240a_0^2 a_2^3 b_2 A_2 - 240a_0 a_2^3 b_2 A_3 - 4a_0^2 a_1^4 a_2 b_2 C_0$$
$$-4a_0 a_1^4 a_2 b_2 C_1 - 4a_1^4 a_2 b_2 C_2 - 2a_1^6 a_2 b_0 + a_1^7 b_1)/240a_2^3 b_2,$$

B_1, B_2, B_3, C_0, C_1, C_2 sont des constantes arbitraires et

$$\alpha_{ik} = \begin{vmatrix} a_i & a_k \\ b_i & b_k \end{vmatrix} \quad (i, k = 0, 1, 2);$$

1\. 2. $a_2 b_2 \neq 0$, $\alpha_{12} \neq 0$. $a_1 = 0$, $D_0 = -1$, $D_1 = \alpha_{02} / b_2$,

$$A_0 = (6 B_0 - 4 C_0 + 8) / 15,$$

$$A_1 = (18 a_0 B_0 + 6 B_1 - 60 a_0 A_0 - 8 a_0 C_0 - 4 C_1 - 8 a_0 + 8 D_1) / 15,$$

$$A_2 = (18 a_0^2 B_0 + 12 a_0 B_1 + 6 B_2 - 90 a_0^2 A_0 - 45 a_0 A_1 - 4 a_0^2 C_0 - 4 a_0 C_1 - 4 C_2) / 15,$$

$$A_3 = (2 a_0^3 B_0 + 2 a_0^2 B_1 + 2 a_0 B_2 + 2 B_3 - 20 a_0^3 A_0 - 15 a_0^2 A_1 - 10 a_0 A_2) / 5,$$

$$A_4 = -(a_0^4 A_0 + a_0^3 A_1 + a_0^2 A_2 + a_0 A_3),$$

B_0, B_1, B_2, B_3, C_0, C_1, C_2 sont des constantes arbitraires;

1\. 3. $a_2 b_2 \neq 0$, $\alpha_{12} = 0$,

$$D_0 = -1, \quad D_1 = \alpha_{02} / b_2.$$

A_0, A_1, A_2, A_3, A_4, B_0, B_1, B_2, B_3, C_0, C_1, C_2 sont des constantes arbitraires;

2\. $a_2 = 0$, $b_2 \neq 0$,

$$C_0 = -(2 D_0 + 1) / 2,$$

$$C_1 = -(2 a_0 b_2 D_0 + 2 b_2 D_1 + a_1 b_1 D_0 + a_1 b_1 + 4 a_0 b_2 C_0) / 2 b_2,$$

$$C_2 = -(a_0 a_1 b_1 D_0 + a_1 b_1 D_1 + a_1^2 b_0 + 2 a_0^2 b_2 C_0 + 2 a_0 b_2 C_1) / 2 b_2,$$

A_0, A_1, A_2, A_3, A_4, B_0, B_1, B_2, B_3, D_0, D_1 sont des constantes arbitraires;

3\. 1. $a_2 \neq 0$, $b_2 = 0$, $a_1 \neq 0$, $a_1 b_1 = 2 a_2 b_0$,

$$A_0 = (8 D_0 - 4 C_0 + 16) / 15, \quad B_0 = 0,$$

$$A_1 = (6 a_2 B_1 - 90 a_1^2 A_0 - 60 a_0 a_2 A_0 - 26 a_1^2 C_0 - 8 a_0 a_2 C_0 - 4 a_2 C_1 + 8 a_0 a_2 D_0 + 8 a_2 D_1 + 54 a_1^2 D_0 + 112 a_1^2) / 15 a_2,$$

$$A_2 = (18 a_1^2 a_2 B_1 + 48 a_0 a_2^2 B_1 + 24 a_2^2 B_2 - 360 a_0^2 a_2^2 A_0 - 120 a_0 a_1^2 a_2 A_0 - 30 a_1^2 a_2 A_1 - 180 a_0 a_2^2 A_1 - 5 a_1^4 C_0 - 32 a_0 a_1^2 a_2 C_0 - 16 a_0^2 a_2^2 C_0 - 16 a_1^2 a_2 C_1 - 16 a_0 a_2^2 C_1 - 16 a_2^2 C_2 + 40 a_0 a_1^2 a_2 D_0 + 40 a_1^2 a_2 D_1 + 18 a_1^4 D_0 + 56 a_1^4) / 60 a_2^2,$$

$$A_3 = (48 a_0^2 a_2^3 B_1 + 24 a_0 a_1^2 a_2^2 B_1 + 48 a_0 a_2^3 B_2 + 12 a_1^2 a_2^2 B_2 + 48 a_2^3 B_3 - 480 a_0^3 a_2^3 A_0 - 360 a_0^2 a_2^3 A_1 - 240 a_0 a_2^3 A_2 - 4 a_0 a_1^4 a_2 C_0 - 16 a_0^2 a_1^2 a_2^2 C_0 - 2 a_1^4 a_2 C_1 - 16 a_0 a_1^2 a_2^2 C_1 - 16 a_1^2 a_2^2 C_2 + 12 a_0 a_1^4 a_2 D_0 + 12 a_1^4 a_2 D_1 + a_1^6 D_0 + 8 a_1^6) / 120 a_2^3,$$

$$A_4 = (12 a_0^2 a_1^2 a_2^2 b_1 B_1 + 12 a_0 a_1^2 a_2^2 b_1 B_2 + 12 a_1^2 a_2^2 b_1 B_3 - 120 a_0^4 a_2^3 b_1 A_0 - 120 a_0^3 a_2^3 b_1 A_1 - 120 a_0^2 a_2^3 b_1 A_2 - 120 a_0 A_2^3 b_1 A_3 - 2 a_0^2 a_1^4 a_2 b_1 C_0 - 2 a_0 a_1^4 a_2 b_1 C_1 - 2 a_1^4 a_2 b_1 C_2 + a_0 a_1^6 b_1 D_0 + a_1^6 b_1 D_1 + a_1^7 b_0) / 120 a_2^3 b_1,$$

B_1, B_2, B_3, C_0, C_1, C_2, D_0, D_1 sont des constantes arbitraires;

3\. 2. $a_2 \neq 0$, $b_2 = 0$, $a_1 = 0$, $b_0 = 0$,

$$A_0 = (6 B_0 - 4 C_0 + 8 D_0 + 16) / 15,$$

$$A_1 = (18 a_0 B_0 + 6 B_1 - 60 a_0 A_0 - 8 a_0 C_0 - 4 C_1 + 8 a_0 D_0 + 8 D_1) / 15,$$

$$A_2 = (18 a_0^2 B_0 + 12 a_0 B_1 + 6 B_2 - 90 a_0^2 A_0 - 45 a_0 A_1 - 4 a_0^2 C_0 - 4 a_0 C_1 - 4 C_2) / 15,$$

$$A_3 = (6a_0^3 B_0 + 6a_0^2 B_1 + 6a_0 B_2 + 6 B_3 - 60a_0^3 A_0 - 45a_0^2 A_1 - 30a_0 A_2) / 15,$$

$$A_4 = -(a_0^4 A_0 + a_0^3 A_1 + a_0^2 A_2 + a_0 A_3),$$

B_0, B_1, B_2, B_3, C_0, C_1, C_2, D_0, D_1 sont des constantes arbitraires;

4. $a_2 = b_2 = 0,$

$$D_0 = -1, \ D_1 = \alpha_{01} / b_1,$$

A_0, A_1, A_2, A_3, A_4, B_0, B_1, B_2, B_3, C_0, C_1, C_2 sont des constantes arbitraires.