BIBLIOGRAFIJA

[1] R. P. Agnew, The Lototsky Method for Evaluation of Series, Michigan Math. J. 4 (1957), 105 — 128.

[2] J. Karamata, Teorija i praksa Stiltjesova integrala, Beograd (1949).

[3] B. Martić, O jednom skupu dvoparametarskih postupaka zbrojivosti i njihovim primjenama, 325 knjiga Rada Jugoslavenske akademije znanosti i umjetnosti, Zagreb (1962), str. 127 — 163.

[4] B. Martić, Jedna primedba o $S^{\alpha,\beta}$ postupcima zbirljivosti, Naučno društvo NRBIH, Radovi — XIX, Sarajevo (1962), str. 45 — 48.

[5] B. Martić, The Mutual Inclusion of $S^{\alpha,\beta}$ Methods of Summation, Publ. Inst. Math. Tome 2 (16), (1962) Beograd pp. 93 — 98.

[6] B. Martić, Note on the Summation of a classical divergent Series, Vesnik društva mat. i fiz. SRS (1963) Beograd pp. 17 — 20.

[7] V. Vučković, Eine neue Klasse von Polynomen und ihre Anwendungen in der Theorie der Limitierungsverfahren, Publ. Inst. Math. Acad. Serbe Sci. 12 (1958), Beograd S. 125 — 136.

TWO THEOREMS ON THE INCLUSIONS OF A CLASS OF SUMMATION METHODS

BRANISLAV MARTIĆ, Sarajevo

In [3], [4], [5] and [6] the author studied a class of summation methods. In the present paper we continue the research of these methods.

The sequence $\{s_n\}$ is said to be summable to s by the regular $S^{\alpha,\beta}$ methods if $S^{\alpha,\beta}\{s_n\} \to s$ as $n \to \infty$, where

$$S^{\alpha,\beta}\{s_n\} = \left\{\prod_{\nu=0}^{n-1}(\alpha+\beta+\nu)\right\}^{-1} \sum_{\nu=0}^{n} \sigma_\nu^n(\alpha)\,\beta^\nu s_\nu$$

and $\sigma_\nu^n(\alpha)$ is the coefficient of x^ν in $\prod_{\nu=0}^{n-1}(x+\alpha+\nu)$.

These methods have fundamental significance comparable to that of the classical methods of Cesàro, Abel, Euler, Borel and others; and contained as special case Vučković, Karamata-Stirling and Lototsky method.

In [3] the author proved

(1) $$S^{\alpha,\beta} \subset S^{\alpha+\varepsilon,\beta}; \qquad \varepsilon > 0,\ \alpha > -1,\ \beta > 0,\ \alpha+\beta \neq 0$$

and in [5]

(2) $$S^{\alpha,\beta} \subset S^{\alpha,\theta\beta}; \qquad \alpha \geqslant 0,\ \beta > 0,\ 0 < \theta < 1.$$

After that the author pointed out that (1) and (2) could be superposed to give

$$S^{\alpha,\beta} \subset S^{\alpha+\varepsilon,\theta\beta}; \qquad \alpha \geqslant 0,\ \beta > 0,\ \varepsilon > 0,\ 0 < \theta < 1$$

Here we give first the direct and shorter proof for

THEOREM I. *For every $\varepsilon > 0$, $0 < \theta < 1$ and each $\alpha \geqslant 0$ and $\beta > 0$ the $S^{\alpha,\beta}$ summability of a sequence implies its $S^{\alpha+\varepsilon,\theta\beta}$ summability to the same limit.*

As the $S^{\alpha,\beta}$ methods of summation are defined in terms of a sequence-to-sequence transformation, the author in [4] gave the series-to-series version of the $S^{\alpha,\beta}$ methods and proved theorem: The $S^{\alpha,\beta}$ sum of the series $\sum_{\nu=0}^{\infty} u_\nu$ is

$$(3) \qquad S^{\alpha,\beta}\left\{\sum_{\nu=0}^{\infty} u_\nu\right\} = u_0 + \sum_{\nu=0}^{\infty}\left\{\prod_{i=0}^{\nu}(\alpha+\beta+i)\right\}^{-1}\sum_{i=0}^{\nu}\sigma_i^{\nu}(\alpha)\,\beta^{i+1}u_{i+1}$$

if the series on the right is convergent.

Applying (3) to the series $o+u_0+u_1+\cdots$ gives the formula

$$S^{\alpha,\beta}\{o+u_0+u_1+\cdots\} = \sum_{\nu=0}^{\infty}\beta\left\{\prod_{i=0}^{\nu}(\alpha+\beta+i)\right\}^{-1}\sum_{i=0}^{\nu}\sigma_i^{\nu}(\alpha)\,\beta^{i}u_i$$

which has a more attractive right member than (3). Because of this circumstance, we introduce a class $S_1^{\alpha,\beta}$, $S_2^{\alpha,\beta}$, $S_3^{\alpha,\beta}, \cdots$ of methods for evaluation of series which are related to $S^{\alpha,\beta}$ in the same way that the Borel-Sannia methods B_0, B_1, $B_2, \cdots$ are related to the Borel method B. We will say that the series $u_0 + u_1 + u_2 + \cdots$ is evaluable $S_r^{\alpha,\beta}$ to s if the series $\underbrace{o+o+\cdots+o}_{r} + u_0+u_1+u_2+\cdots$ is evaluable $S^{\alpha,\beta}$ to s. We prove also

THEOREM 2. *For each $\alpha > -1$, $\beta > 0$ and $\alpha+\beta \neq 0$*

$$S^{\alpha,\beta} \subset S_1^{\alpha,\beta} \subset S_2^{\alpha,\beta} \subset \cdots$$

that is, the $S_r^{\alpha,\beta}$ summability of a series implies its $S_{r+1}^{\alpha,\beta}$ $(r=0, 1, 2, \cdots)$ summability to the same limit, but not conversely.