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ON OPERATIONS IN HOMOLOGY THEORIES

Let $\mathcal{E}=\{E_n\}$ and $\mathcal{F}=\{F_n\}$ be two Ω-spectra, $\mathcal{E}^*$ and $\mathcal{F}^*$ the corresponding cohomology theories, $\mathcal{E}_*$ and $\mathcal{F}_*$ the corresponding homology theories. Everything is in the category of CW-complexes. It is a wellknown fact that the operations $\mathcal{O}(n, m, \mathcal{E}^x, \mathcal{F}^x)$ from $\mathcal{E}^x$ to $\mathcal{F}^x$ of type (n, m) are in a 1—1 correspondence with the homotopy classes of maps from E_n to F_m.

Similarly, the stable operations $\mathcal{O}^s(k, \mathcal{E}^x, \mathcal{F}^x)$ from $\mathcal{E}^x$ to $\mathcal{F}^x$ of degree k correspond to the elements of $\underset{\text{delooping}}{\underleftarrow{\lim}} [E_n, F_{n+k}]$.

The question arises what about homology operations? A map from E_n to F_m obviously is not enough to define one. Nevertheless the following holds: The stable homology operations from $\mathcal{E}_x$ to $\mathcal{F}_x$ of degree k are in a 1—1 correspondance with the elements of $\underset{\text{delooping}}{\underleftarrow{\lim}} \underset{\substack{\text{fin-subcomplexes} \\ E_n^f \text{ of } E_n}}{\underleftarrow{\lim}} [E_n^f, F_{n-k}]$

One proves it by proceeding in the same way as G. WHITEHEAD in his paper "*Generalized homology theories*" Trans. Amer, Math. Soc. Vol **102** (1962) p. 227—283. We show that *each stable homology operation ψ of degree k defines a stable cohomology one of degree $-k$ in the category of finite complexes.*

Namely, for each finite CW-complex X, take its Spanier Whitehead dual Y. Let $u: X \wedge Y \to S^r$ be a duality map. Then define

$$\mathcal{E}^{r-n}(X) \to \mathcal{F}^{r-n-k}(X)$$

as the composition.

$$\mathcal{E}^{r-n}(X) \overset{\approx}{\to} \mathcal{E}_n(Y) \overset{\psi}{\to} \mathcal{F}_{n+k}(Y) \overset{\approx}{\to} \mathcal{F}^{r-n-k}(X)$$

Now stable cohomology operations of degree $-k$ in the category of finite WC-complexes correspond to the element of

$$\underset{\text{delooping}}{\underleftarrow{\lim}} \quad \underset{\substack{\text{fin-subcompl.} \\ E_n^f \text{ of } E_n}}{\underleftarrow{\lim}} \quad [E_n, F_{n-k}]$$

But since homology commutes with direct limits, this gives a classification of the stable homology operations in the category of *CW*-complexes and not only the finite ones.

The unstable homology operations do not seem to fit well in this picture and although I do not know of any interesting by themselves, they do exist. For example, J. F. ADAMS has constructed an analogue of the Postnikov operation in cohomology.

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